# Selected Solutions — Chapter 9

***Investigate, page 518***

**1. a)** The edge of the shadow forms a circle.

**b)** The circle becomes larger as I move farther from the wall, and smaller as I move closer to the wall.

**c)** The circle becomes smaller and smaller until it is the same size as the clear plastic cover over the bulb.

**2. a)** An ellipse looks like an elongated circle.

**b)** It becomes more elongated until the pointed ends split apart and move away from one another.

**c)** The ellipse turns into a hyperbola.

**3. a)** The hyperbola is curved around the flashlight and then moves off into two straight lines directed away from the light source.

**b)** The two lines come closer together and become almost vertical.

**c)** The hyperbola turns into an ellipse.

**4. a)** The angle of elevation is about 45˚.

**b)** The curve appears to be a parabola.

**c)** The flashlight is positioned at an angle of about 45˚ to the wall.

**5. a)** The ellipse becomes larger as I move farther from the wall, and smaller as I move closer to the wall.

**b)** The ellipse is at its minimum size when the flashlight touches the wall.

**c)** The hyperbola becomes larger as I move farther from the wall, and smaller as I move closer to the wall. The hyperbola is at its minimum size when the flashlight touches the wall.

***9.1 Exercises, page 522***

**4.** Explanations may vary. Suppose that the cone has nappes and generators of infinite length. Then every plane will intersect the cone, whether very close or very far from the vertex. As the planes move farther away from the vertex, the circles get larger. As the plane move closer to the vertex, the circles get smaller. For the plane that crosses the vertex, the circle is a point.

**5. a)** The conic sections on the other plane will be circles or a single point. If the second plane is closer to the vertex than the first, then the circle will be smaller. If the second plane is further from the vertex than the first, then the circle will be larger. If the second plane intersects the cone at the vertex, then the conic section will be a point.

# Selected Solutions — Chapter 9

**b) i)** The conic sections on the other plane will be ellipses or a single point. If the second plane is closer to the vertex than the first, then the ellipse will be smaller. If the second plane is further from the vertex than the first, then the ellipse will be larger. If the second plane intersects the cone at the vertex, then the conic section will be a point.

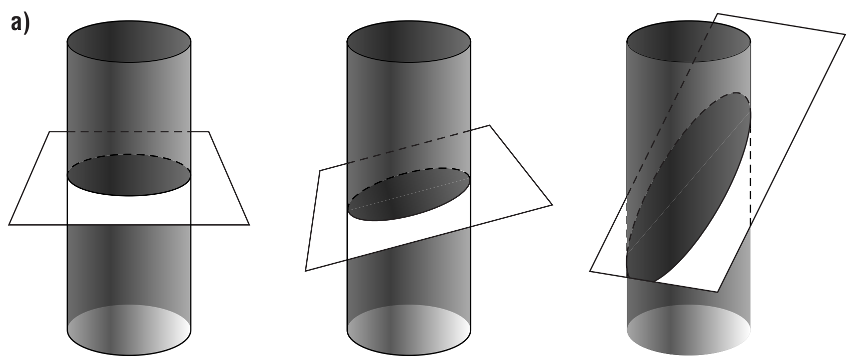
**ii)** The conic sections on the other plane will be parabolas or a single point. If the second plane is closer to the vertex than the first, then the parabola will be smaller. If the second plane is further from the vertex than the first, then the parabola will be larger. If the second plane intersects the cone at the vertex, then the conic section will be a point.

**iii)** The conic sections on the other plane will be hyperbolas or a pair of intersecting lines. If the second plane is closer to the vertex than the first, then the hyperbola will be larger. If the second plane is further from the vertex than the first, then the hyperbola will be smaller. If the second plane bisects the cone (intersects the cone at the vertex), then the conic section will be two intersecting lines.

**8.** Descriptions may vary. If a plane is perpendicular to and touching the vertex, it intersects the cone in a point. If a plane bisects the cone and passes through the vertex, it forms a single line as it intersects the base of the cone and forms two intersecting lines as it cuts the generators.

**9.** Explanations may vary. A single orbit of a moving comet may exceed the lifetime of interested scientists. Tracking and identifying specific comets as they journey far beyond the sun also presents a sizable challenge.

**10. a)**



**b)** No. The last diagram in part a looks like it might be a hyperbola or parabola, but if the cylinder is made tall enough, the plane will intersect it as in the second diagram of part a.

**c)** Descriptions may vary. Visualize shining a light source on a ball at an angle greater than 45° with the floor. The shadow of the ball forms an ellipse on the floor.

# Selected Solutions — Chapter 9

**11.** **a)** **i)** When the angle of elevation is 0˚ a circle is formed.

**ii)** When the angle of elevation is between 0˚ and 45˚ an ellipse is formed.

**iii)** When the angle of elevation is 45˚ a parabola is formed.

**iv)** When the angle of elevation is between 45˚ and 90˚ a hyperbola is formed.

**b)** When the angle of elevation is between 90˚ and 120˚ an ellipse is formed. This is just pointing the flashlight in the opposite direction.

**12.** **a)** The edge of the shadow is a circle when $\theta$ is 0˚ and 180˚.

**b)** The edge of the shadow is an ellipse when $\theta$ is between 0˚ and 45˚, between 135˚ and 225˚, and between 315˚ and 360˚.

**c)** The edge of the shadow is a parabola when $\theta$ is 45˚, 135˚, 225˚, and 315˚.

**d)** The edge of the shadow is a hyperbola when $\theta$ is between 45˚ and 135˚, and between 225˚ and 315˚.

**13.** **a)** A circle is produced when $\beta$ is 0˚ and 180˚.

**b)** An ellipse is produced when $\beta$ is between 0˚ and 45˚, between 135˚ and 225˚, and between 315˚ and 360˚.

**c)** A parabola is produced when $\beta$ is 45˚, 135˚, 225˚, and 315˚.

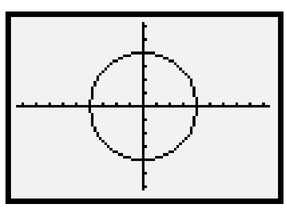
**e)** A hyperbola is produced when $\beta$ is between 45˚ and 135˚, and between 225˚ and 315˚.

**14.** circle: the plane is perpendicular to the central axis
ellipse: the plane is between perpendicular to and 45˚ to the central axis
parabola: the plane is parallel to a generator
hyperbola: the plane is between 45˚ to the central axis and parallel to a generator

**15.** The second diagram in exercise 10, part a shows how an ellipse is formed when a plane intersects a cylinder.

***Investigate, page 527***

**1.** **a)**



The graph is a circle of radius 4 centred at the origin, since, according to the formula, the distance from any point on the curve to the origin is 4.

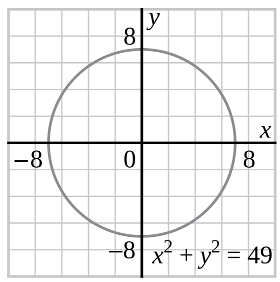
**b)** For positive constants, the graph remains a circle. For a zero constant, the graph is the point (0, 0). For negative constants, there are no points that satisfy the equation.

# Selected Solutions — Chapter 9

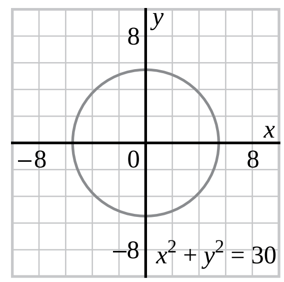
**2. a)** The square root of $k$ provides the radius of the circle. If $k > 0$, then the radius of the circle is $\sqrt{k}$. If $k < 0$ then $\sqrt{k}$ is undefined and the circle cannot be drawn.

**b)** Suppose that $k > 0$. As $k$ increases, the radius of the circle increases. As $k$ decreases to 0, the radius of the circle decreases until it becomes a single point. If $k < 0$, then the circle cannot be drawn.

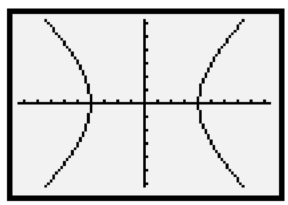
**3. a)**




**b)**




**4. a)**



The graph is a hyperbola with vertices (–4, 0) and (4, 0). Two branches reach out from each of the two vertices. In the first and third quadrants, the branches approach the $y = x$ line. In the second and fourth quadrants, the branches approach the $y = -x$ line.

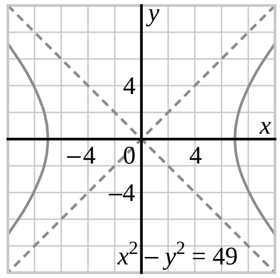
**b)** For positive constants, the graph is a hyperbola with vertices on the $x$-axis. For a zero constant, the graph is the lines $y = \pm x$. For negative constants, the graph is a hyperbola with vertices on the $y$-axis.

**5. a)** $k$ provides the location of the vertices of the hyperbola.

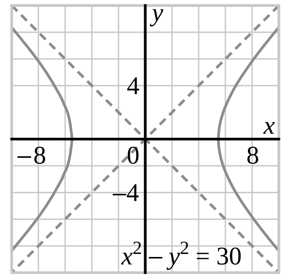
**b)** $k > 0$: As $k$ increases, the vertices move farther away from the origin along the $x$-axis.
$k = 0$: The graph is the lines $y = \pm x$.
$k < 0$: As $k$ decreases, the vertices move farther away from the origin along the $y$-axis.

**6.** Descriptions may vary. For $k > 0$, the graph of $x^2 + y^2 = k$ is an ellipse and the graph $x^2 - y^2 = k$ is a hyperbola. For $k < 0$, the graph of $x^2 + y^2 = k$ is not defined but the graph of $x^2 - y^2 = k$ is a hyperbola.

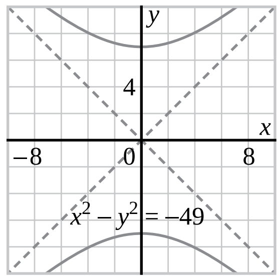
**7. a)**



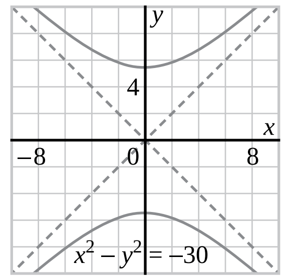

**b)**




**c)**

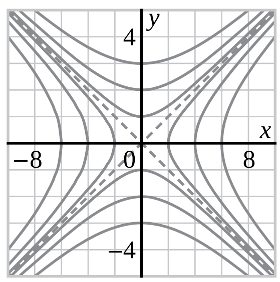



**d)**

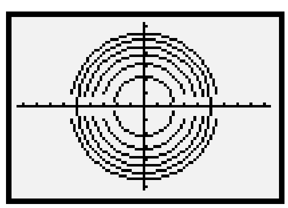


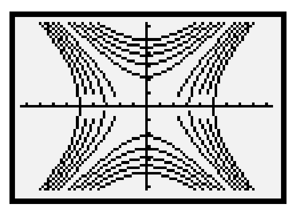

### *9.2 Exercises, page 532*

**4.**




**6.**





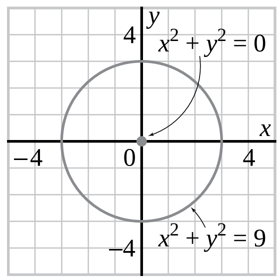
The first screen shows the graphs of $x^2 + y^2 = k$ where k = 5, 10, 15, 20, and 25. The second screen shows the graphs of $x^2 - y^2 = k$ where $k = \pm 5,\ \pm 10,\ \pm 15,\ \pm 20$, and $\pm 25$.

# Selected Solutions — Chapter 9

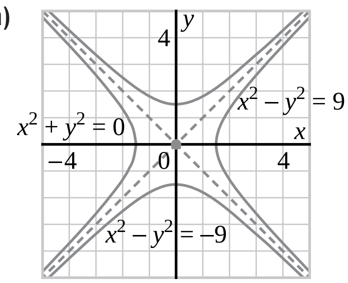
**7. e)** Consider a plane that is perpendicular to the axis of a cone and that intersects the cone in a circle. As the distance between the plane and the vertex decreases, the radius of the circle of intersection also decreases. When the plane intersects the vertex in a single point, the radius of the circle is 0.

Consider a plane that is perpendicular to the base of a cone and that intersects the cone in a hyperbola. As the distance between the plane and the vertex decreases, the two branches approach each other. When the constant term becomes negative, the cone and plane diagram on page 521 is rotated 90˚ so that the plane is parallel with the flat surface.

**9. a)**




**10. a)**




**b)** When $k = 9$, the graph is a rectangular hyperbola with vertices $(\pm3,\ 0)$. When $k = 0$, the graph is the lines $y = \pm x$. When $k = -9$, the graph is a rectangular hyperbola with vertices $(0,\ \pm3)$.

**11.** The cone must be a right circular cone. This means the maximum angle formed by the generators, at the vertex of the cone, is 90˚. If the cone is not a right circular cone, then the hyperbola will be skewed.

**12.** No. If the cone is not a right circular cone, the hyperbola will be skewed so that it is no longer a rectangular hyperbola.

**13. a)** To ensure that the conic section is circular, $A = B$. If $A > 0$ then clearly $B > 0$ and so $C < 0$. If $A < 0$ then it is also true that $B < 0$ and so $C > 0$. This ensures that the radius, $\sqrt{C}$, is defined.

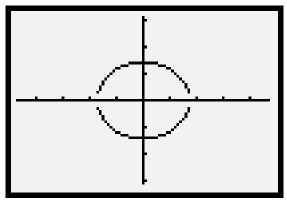
# Selected Solutions — Chapter 9

**b) i)** To ensure that the conic section is a rectangular hyperbola, $A = -B$. To ensure that the vertices of the hyperbola are on the $x$-axis, $C$ must have the same sign as $B$. Thus, if $A > 0$ then $B < 0$ and $C < 0$. If $A < 0$ then $B > 0$ and $C > 0$.

**ii)** To ensure that the conic section is a rectangular hyperbola, $A = -B$. To ensure that the vertices of the hyperbola are on the $y$-axis, $C$ must have the same sign as $A$. Thus, if $A > 0$ then $B < 0$ and $C > 0$. If $A < 0$ then $B > 0$ and $C < 0$.

***Investigate, page 535***

**1. a)**



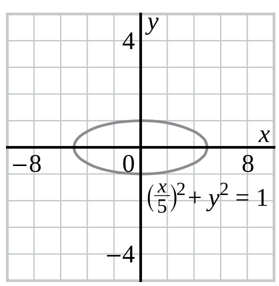
**2.** $(\pm a,\ 0)$ are the $x$-intercepts and $(0,\ \pm b)$ are the $y$-intercepts.
Set $y = 0$ to find the $x$-intercepts.

$$\left(\frac{x}{a}\right)^2 = 1$$
$$\frac{x}{a} = \pm 1$$
$$x = \pm a$$

Similarly, set $x = 0$ to find the $y$-intercepts.

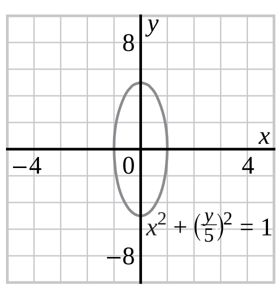
$$\left(\frac{y}{b}\right)^2 = 1$$
$$\frac{y}{b} = \pm 1$$
$$y = \pm b$$

**4. a)**



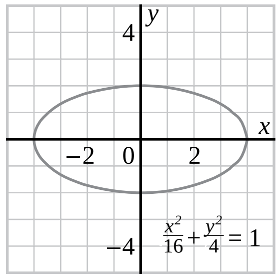

This is an ellipse with $x$-intercepts $\pm 5$ and $y$-intercepts $\pm 1$.

**b)**



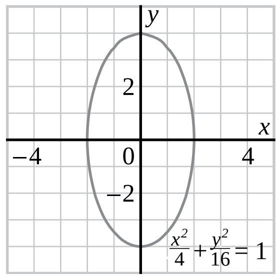

This is an ellipse with $x$-intercepts $\pm 1$ and $y$-intercepts $\pm 5$.

**c)**



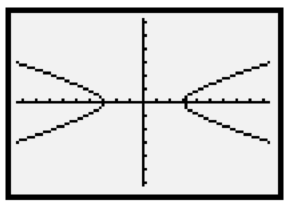

This is an ellipse with $x$-intercepts $\pm 4$ and $y$-intercepts $\pm 2$.

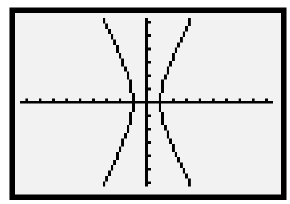
**d)**




This is an ellipse with $x$-intercepts $\pm 2$ and $y$-intercepts $\pm 4$.

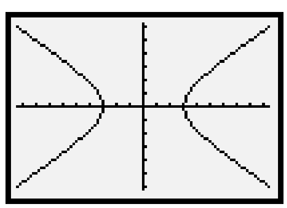
**5. a)**





The graph of $\left(\frac{x}{3}\right)^2 - y^2 = 1$ is a horizontal expansion by a factor of 3 of the graph of $x^2 - y^2 = 1$. This is because if the point $(x, y)$ satisfies $x^2 - y^2 = 1$, the point $(3x, y)$ satisfies the equation $\left(\frac{x}{3}\right)^2 - y^2 = 1$. The graph of $x^2 = \left(\frac{y}{2}\right)^2 = 1$ is a vertical expansion by a factor of 2 of the graph of $x^2 - y^2 = 1$. This is because if the point $(x, y)$ satisfies $x^2 - y^2 = 1$, the point $(x, 2y)$ satisfies the equation $x^2 = \left(\frac{y}{2}\right)^2 = 1$.

**b)**



The graph of $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is a horizontal expansion by a factor of 3 and a vertical expansion by a factor of 2 of the graph of $x^2 - y^2 = 1$. This is because $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is the same as $\left(\frac{x}{3}\right)^2 - \left(\frac{y}{2}\right)^2 = 1$. Use the results of part a to describe how the graphs are related.

# Selected Solutions — Chapter 9

**6.** $\pm a$ are the $x$-intercepts. Set $y = 0$ to find the $x$-intercepts.

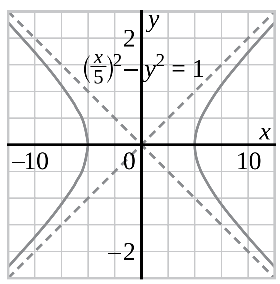
$$\frac{x^2}{a^2} = 1$$
$$x^2 = a^2$$
$$x = \pm a$$

$a$ and $b$ also give us information about the asymptotes. Consider what happens to the branches of the hyperbola as the value 1, on the right-hand side of the equation, decreases to 0. The hyperbola will approach the asymptotes. When the constant term is 0, the equation represents the asymptotes. Thus, set the left-hand side of the equation equal to 0 and solve for $y$ to find the equations of the asymptotes in terms of $a$ and $b$.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$
$$b^2x^2 - a^2y^2 = 0$$
$$b^2x^2 = a^2y^2$$
$$y^2 = \frac{b^2x^2}{a^2}$$
$$y = \pm\frac{b}{a}x$$

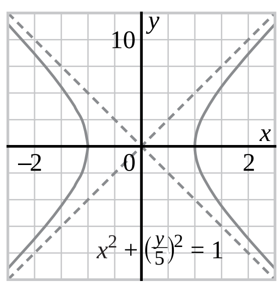
$a$ also provides us with the horizontal expansion factor and $b$ provides us with the vertical expansion factor of the graph $x^2 - y^2 = 1$.

**8. a)**




This is a hyperbola with $x$-intercepts $\pm 5$ and asymptotes $y = \pm\frac{1}{5}x$.

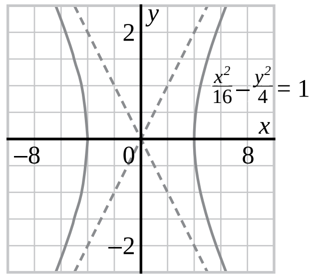
**b)**




This is a hyperbola with $x$-intercepts $\pm 1$ and asymptotes $y = \pm 5x$.

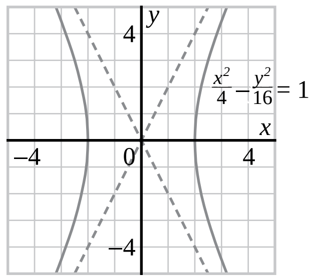
# Selected Solutions — Chapter 9

**c)**



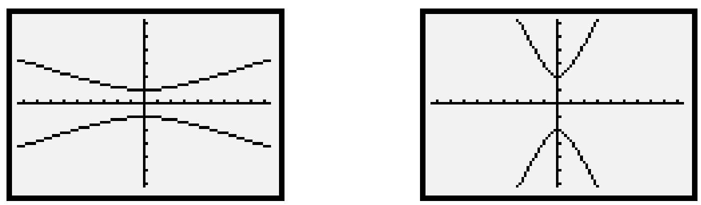

This is a hyperbola with $x$-intercepts $\pm 4$ and asymptotes $y = \pm\frac{1}{2}x$.

**d)**



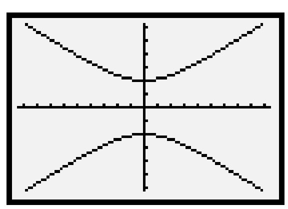

This is a hyperbola with $x$-intercepts $\pm 2$ and asymptotes $y = \pm 2x$.

**7. a)**



The graph of $\left(\frac{x}{3}\right)^2 - y^2 = -1$ is a horizontal expansion by a factorof 3 of the graph of $x^2 - y^2 = -1$. This is because if the point $(x, y)$ satisfies $x^2 - y^2 = -1$, the point $(3x, y)$ satisfies $\left(\frac{x}{3}\right)^2 - y^2 = -1$.

**b)**



The graph of $\frac{x^2}{9} - \frac{y^2}{4} = -1$ is a horizontal expansion by a factor of 3 and a vertical expansion by a factor of 2 of the graph of $x^2 - y^2 = -1$. This is because $\frac{x^2}{9} - \frac{y^2}{4} = -1$ is the same as $\left(\frac{x}{3}\right)^2 - \left(\frac{y}{2}\right)^2 = -1$. Use the results of part a to describe how the graphs are related.

# Selected Solutions — Chapter 9

**10.** $\pm b$ are the $y$-intercepts. Set $x = 0$ to find the $y$-intercepts.

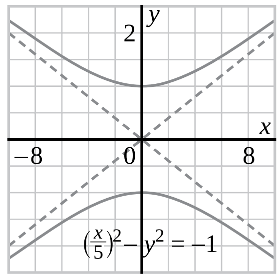
$$\frac{y^2}{b^2} = 1$$

$$y^2 = b^2$$

$$y = \pm b$$

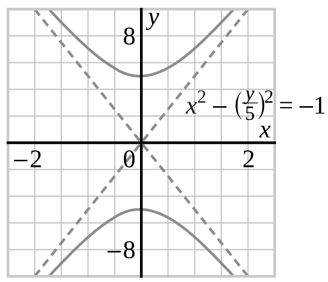
As in question 5, the equations of the asymptotes are $y = \pm\frac{b}{a}x$.

**12. a)**



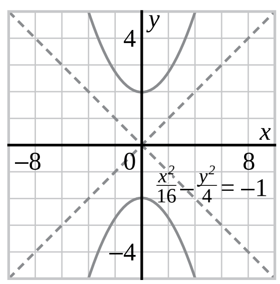

This is a hyperbola with $y$-intercepts $\pm 1$ and asymptotes $y = \pm\frac{1}{5}x$.

**b)**



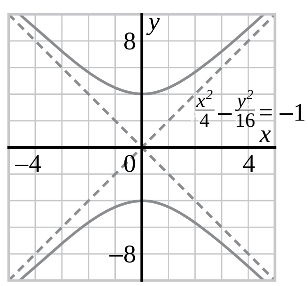

This is a hyperbola with $y$-intercepts $\pm 5$ and asymptotes $y = \pm 5x$.

**c)**




This is a hyperbola with $y$-intercepts $\pm 2$ and asymptotes $y = \pm\frac{1}{2}x$.

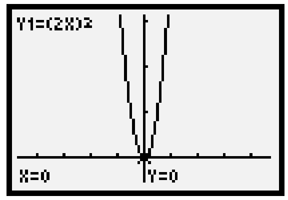
**d)**




This is a hyperbola with $y$-intercepts $\pm 4$ and asymptotes $y = \pm 2x$.

# Selected Solutions — Chapter 9

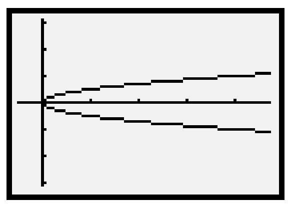
**13. a)**




The graph of $y = (2x)^2$ is a horizontal compression by a factor of $\frac{1}{2}$ of the graph of $y = x^2$. This is because if $(x,\ y)$ satisfies $y = x^2$, then $\left(\frac{1}{2}x,\ y\right)$ satisfies the equation $y = (2x)^2$.

**c)** The graph of $y = (kx)^2$ is a horizontal compression or expansion by a factor of $\frac{1}{k}$ of the graph of $y = x^2$. Suppose the point $(x,\ y)$ satisfies the graph of $y = x^2$. Consider compressing the horizontal or $x$ component of this point so that it will be a point on the graph of $y = (kx)^2$. Thus, $\left(\frac{1}{k}x,\ y\right)$ satisfies the equation $y = (kx)^2$.

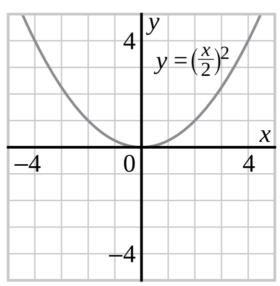
**14. a)**



The graph of $y = (2x)^2$ is a vertical compression by a factor of $\frac{1}{2}$ of the graph of $x = y^2$. This is because if $(x,\ y)$ satisfies the equation $x = y^2$, then $\left(x,\ \frac{1}{2}y\right)$ satisfies the equation $x = (2y)^2$.

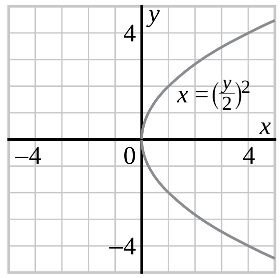
**c)** The graph of $x = (ky)^2$ is a vertical compression or expansion by a factor of $\frac{1}{k}$ of the graph of $x = y^2$. Suppose the point $(x,\ y)$ satisfies the equation $x = y^2$, then $\left(x,\ \frac{1}{k}y\right)$ satisfies the equation $x = (ky)^2$.

**16. a)**




This is a parabola with vertex (0, 0). Its axis of symmetry is the $y$-axis and it opens upward.

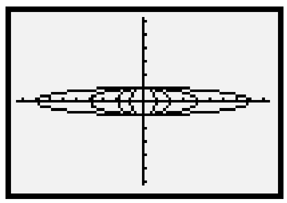
**b)**



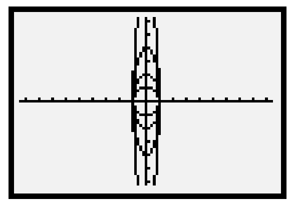

This is a parabola with vertex (0, 0). Its axis of symmetry is the $x$-axis and it opens to the right.

### *9.3 Exercises, page 542*

**2. a)**



**b)**

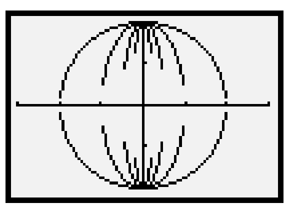


**3.** Answers may vary. For part a:

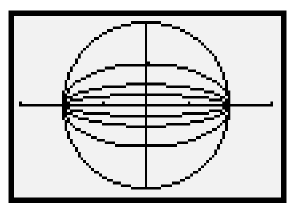
**a)** The ellipse becomes wider and narrower. The $x$-intercepts move farther away from the origin and the $y$-intercepts remain fixed. Eventually, the ellipse becomes two straight lines at $y = \pm 1$.

**b)** For an ellipse, the plane intersects the cone through both sides at an angle. As the number in the denominator increases, the angle becomes larger and larger, until the plane is lying on the side of the cone, and no longer forms an ellipse, but a straight line.

**4. a)**



**b)**



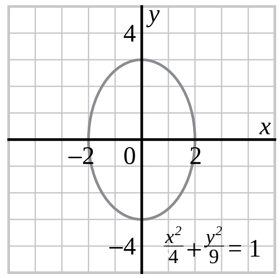
**5.** Answers may vary. For part b:

**a)** The ellipse becomes narrower. The $y$-intercepts move very close to (0, 0) and the $x$-intercepts stay the same. Eventually, the ellipse becomes the straight line $y = 0$, defined on the interval $-1 \le x \le 1$.

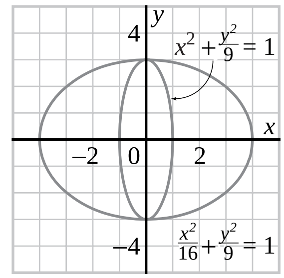
**b)** For an ellipse, the plane intersects the cone through both sides at an angle. As the number in the denominator increases, the angle becomes larger and larger, until the plane is lying on the side of the cone, and no longer forms an ellipse, but a straight line.

# Selected Solutions — Chapter 9

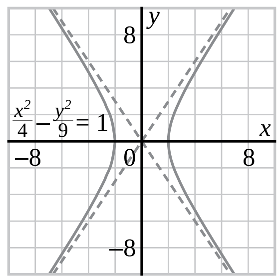
**6. a)**



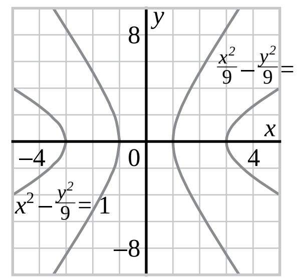

**b)** Diagrams may vary.



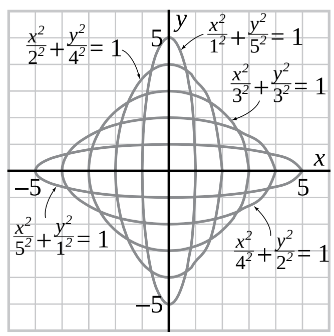

**7. a)**




**b)** Diagrams may vary.

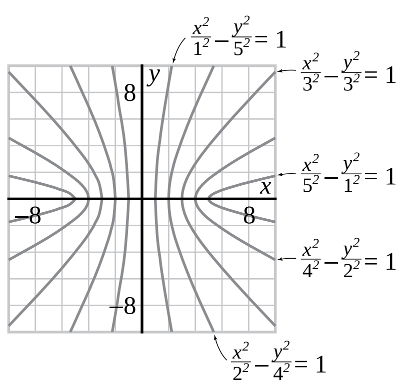



**8. a)**




The graphs are all ellipses, with *y*-intercepts getting closer to the origin, and *x*-intercepts moving farther from the origin.

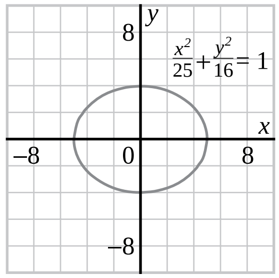
**b)**




The graphs are all hyperbolas, with *x*-intercepts moving farther from the origin, and asymptotes with slopes getting less steep.

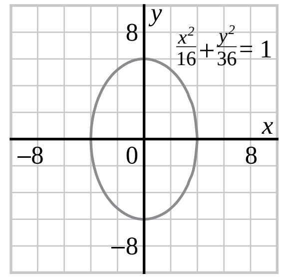
# Selected Solutions — Chapter 9

**14. a)**



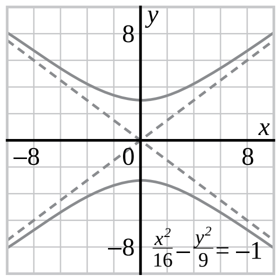

This is an ellipse with $x$-intercepts $\pm5$ and $y$-intercepts $\pm4$.

**b)**



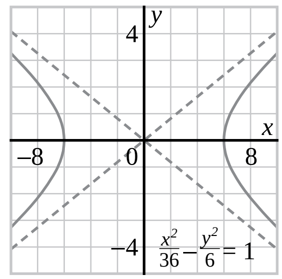

This is an ellipse with $x$-intercepts $\pm4$ and $y$-intercepts $\pm6$.

**c)**



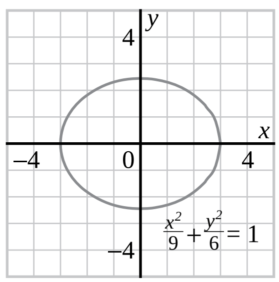

This is a hyperbola with $y$-intercepts $\pm3$ and asymptotes $y=\pm\frac{3}{4}x$.

**d)**



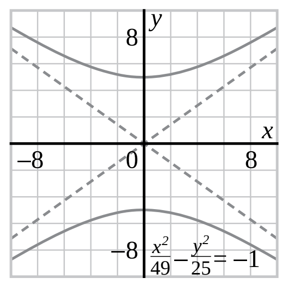

This is a hyperbola with $x$-intercepts $\pm6$ and asymptotes $y=\pm\frac{\sqrt{6}}{6}x$.

**e)**



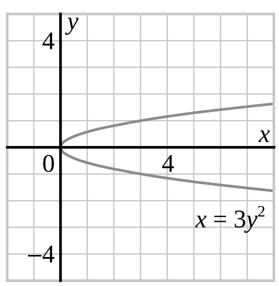

This is an ellipse with $x$-intercepts $\pm3$ and $y$-intercepts $\pm\sqrt{6}$.

**f)**



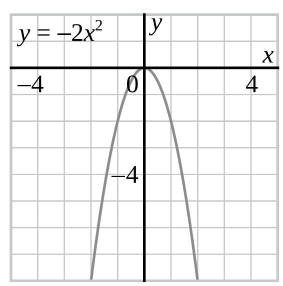

This is a hyperbola with $y$-intercepts $\pm5$ and asymptotes $y=\pm\frac{5}{7}x$.

**g)**




This is a parabola with vertex (0, 0). Its axis of symmetry is the $x$-axis and it opens to the right.

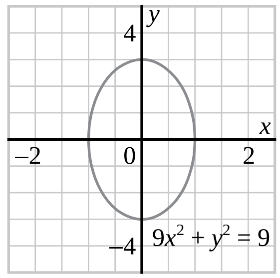
**h)**




This is a parabola with vertex (0, 0). Its axis of symmetry is the $y$-axis and it opens downward.

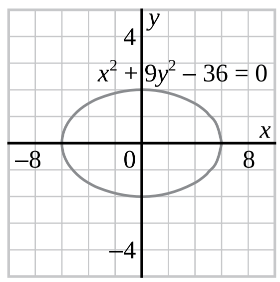
# Selected Solutions — Chapter 9

**i)**



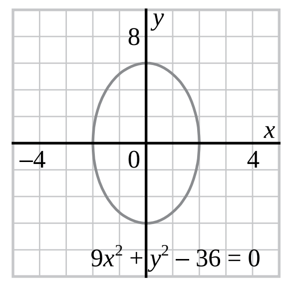

This is an ellipse with $x$-intercepts $\pm1$ and $y$-intercepts $\pm3$.

**15. a)**



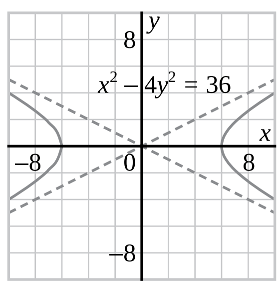

This is an ellipse with $x$-intercepts $\pm6$ and $y$-intercepts $\pm2$.

**b)**



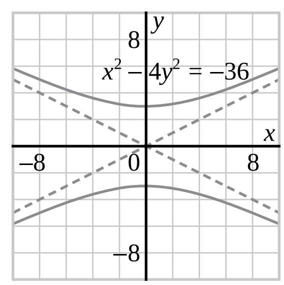

This is an ellipse with $x$-intercepts $\pm2$ and $y$-intercepts $\pm6$.

**c)**



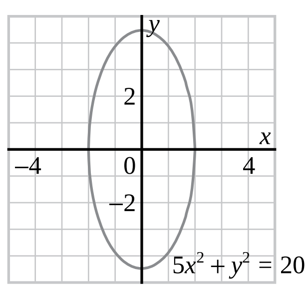

This is a hyperbola with $x$-intercepts $\pm6$ and asymptotes $y = \pm\frac{1}{2}x$.

**d)**



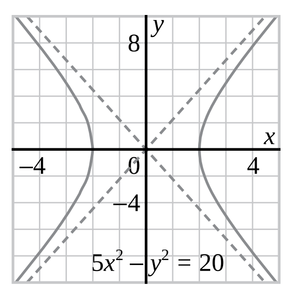

This is a hyperbola with $y$-intercepts $\pm3$ and asymptotes $y = \pm\frac{1}{2}x$.

**e)**




This is an ellipse with $x$-intercepts $\pm2$ and $y$-intercepts $\pm2\sqrt{5}$.

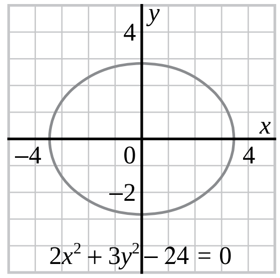
**f)**




This is a hyperbola with $x$-intercepts $\pm2$ and asymptotes $y = \pm\sqrt{5}x$.

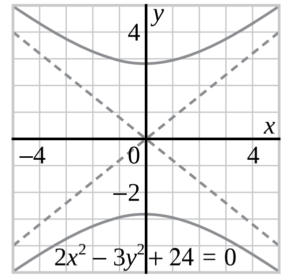
# Selected Solutions — Chapter 9

**g)**



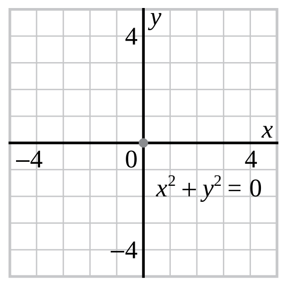

This is an ellipse with $x$-intercepts $\pm 2\sqrt{3}$ and $y$-intercepts $\pm 2\sqrt{2}$.

**h)**

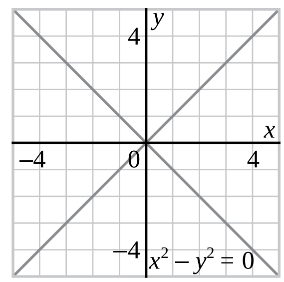



This is a hyperbola with $y$-intercepts $\pm 2\sqrt{2}$ and asymptotes $y = \pm\frac{\sqrt{6}}{3}x$.

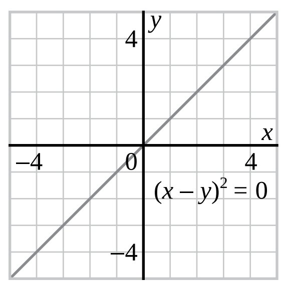
**16. a) i)**




**ii)**




**iii)**




**b) i)** The plane would slice vertically through the vertex of the cone.

**ii)** The plane would pass through the vertex of the cone and run parallel to a generator of the cone.

**iii)** The plane would be perpendicular to the axis of the cone and intersect the vertex in a single point.

**17. a)** This design is a series of 26 ellipses and 1 circle of radius 3.5. It is a more complex version of the diagram in question 8 part a. The $x$-intercepts range from $\pm 0.25$ to $\pm 6.75$ and move away from (0, 0) in 0.25 unit steps. Correspondingly, the $y$-intercepts range from $\pm 6.75$ to $\pm 0.25$ and gradually move towards the origin in 0.25 unit steps. Thus, the equation of the ellipses is $\frac{x^2}{a^2} + \frac{y^2}{(7 - a)^2} = 1$, where $a = 0.25, 0.50, 0.75, \ldots, 6.75$.

**b)** This design is a series of 46 ellipses and 1 circle of radius 24. It is very similar to the diagram in part a, but includes more equations. The $x$-intercepts range from $\pm 1$ to $\pm 47$ and move away from (0, 0) in single unit steps. Correspondingly, the $y$-intercepts range from $\pm 47$ to $\pm 1$ and gradually move towards the origin in single unit steps. Thus, the equation of the ellipses is $\frac{x^2}{a^2} + \frac{y^2}{(48 - a)^2} = 1$, where $a = 1, 2, 3, \ldots, 47$.

# Selected Solutions — Chapter 9

**18.** The area will be $4\left(\frac{a^2b^2}{(a^2+b^2)}\right)$ for part a and $4\left(\frac{a^2b^2}{(a^2-b^2)}\right)$ for part b.

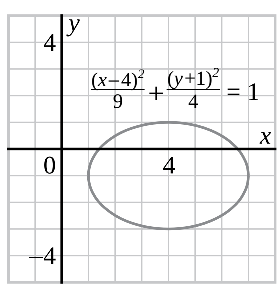
**20.** Let the two equations of the parabolas be $(x-a)^2 + by = c$ and $(y-d)^2 + ex = f$. Since the $x$ term is squared in the first equation, its axis of symmetry must be parallel with the $y$-axis. Similarly, the axis of symmetry of the second equation is parallel with the $x$-axis, given that the $y$ term is squared. Thus, the axes of symmetry of these two parabolas are perpendicular. Add the two equations.
$(x-a)^2 + (y-d)^2 + ex + by = c + f$
This is the equation of a circle with centre $(a, d)$ and that is horizontally and vertically compressed. Thus, all points that satisfy the equations of the two parabolas also satisfy the equation of the circle.

### *9.4 Exercises, page 548*

**2. a)** Each equation represents the graph of a circle with radius 1. The first equation has centre $(0, -4)$. The graph of the second equation is a vertical translation of the first 2 units up. Its centre is $(0, -2)$. Follow this pattern for the remaining 3 circles. The centres of all of the graphs are of the form $(0, k)$, where $k = -4, -2, 0, 2, 4$.

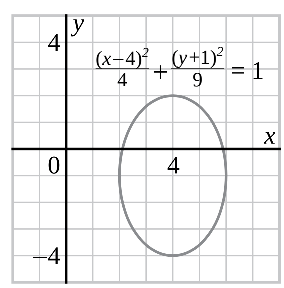
**b)** Each equation represents the graph of a circle with radius 2. The first equation has centre $(8, -2)$. The graph of the second equation is a horizontal translation of the first 4 units left on the line $y = -2$. Its centre is $(4, -2)$. Follow this pattern for the remaining 3 circles. The centres of all of the graphs are of the form $(k, -2)$, where $k = -8, -4, 0, 4, 8$.

**3. a)**




This is an ellipse with centre $(4, -1)$ and vertices $(1, -1)$, $(7, -1)$, $(4, 1)$, and $(4, -3)$.

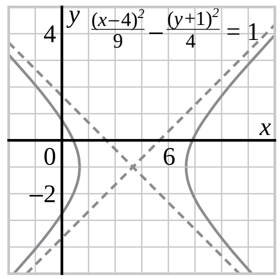
**b)**




This is an ellipse with centre $(4, -1)$ and vertices $(2, -1)$, $(6, -1)$, $(4, 2)$, and $(4, -4)$.

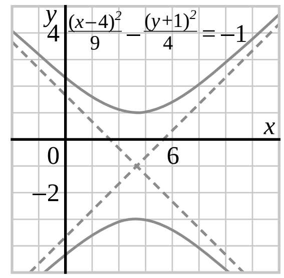
# Selected Solutions — Chapter 9

c)



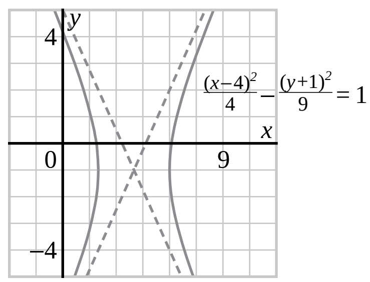

This is a hyperbola with centre (4, −1). Its vertices are (1, −1), and (7, −1). Its asymptotes $y = \frac{2x - 11}{3}$ and $y = \frac{-2x + 5}{3}$.

d)



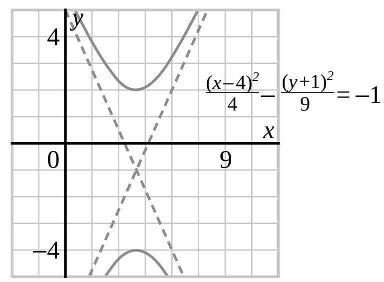

This is a hyperbola with centre (4, −1) and vertices (2, −1), and (6, −1). Its asymptotes are $y = -\frac{3}{2}x + 5$ and $y = \frac{3}{2}x - 7$.

e)



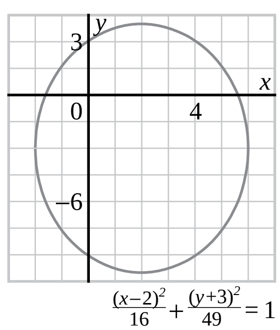

This is a hyperbola with centre (4, −1) and vertices (4, 1) and (4, −3). Its asymptotes are $y = \frac{2x - 11}{3}$ and $y = \frac{-2x + 5}{3}$.

f)

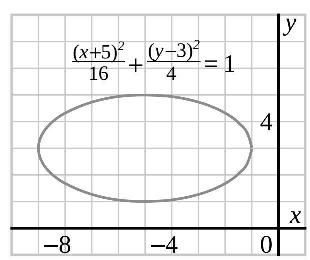



This is hyperbola with centre (4, −1) and vertices (4, 2) and (4, −4). Its asymptotes are $y = -\frac{3}{2}x + 5$ and $y = \frac{3}{2}x - 7$.

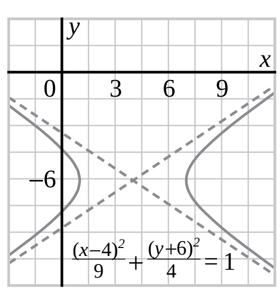
4. a)



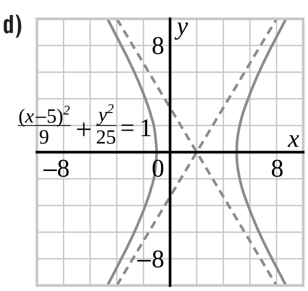

b)




c)

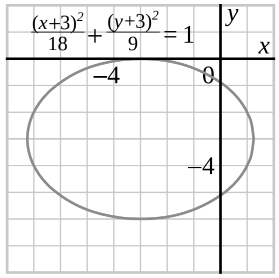



d)

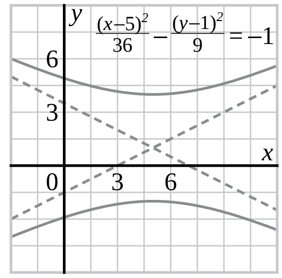


# Selected Solutions — Chapter 9

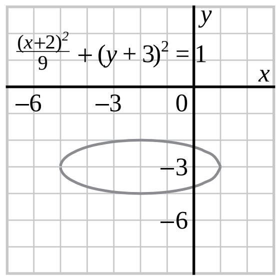
e)



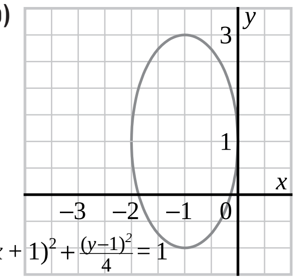

f)




5. a)

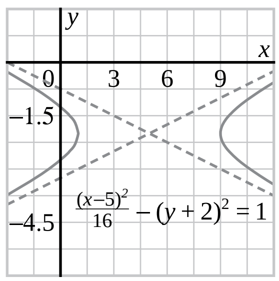



b)

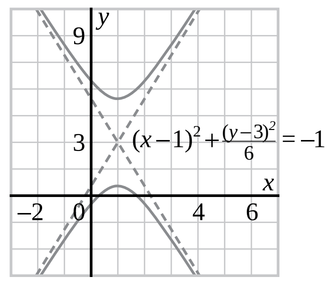



$(x+1)^2 + \frac{(y-1)^2}{4} = 1$

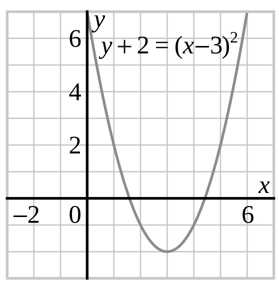
c)



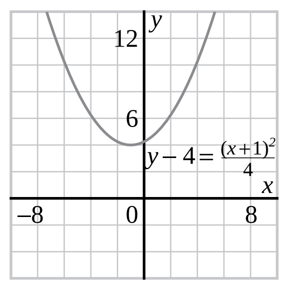

d)



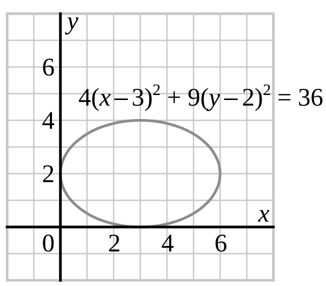

e)



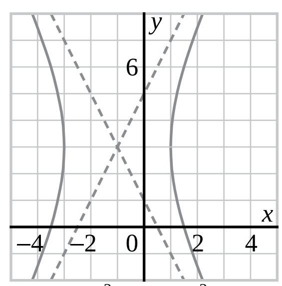

f)




8. a)

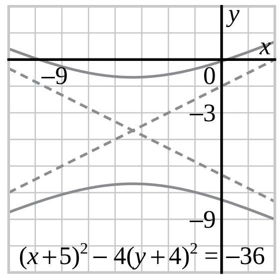



b)

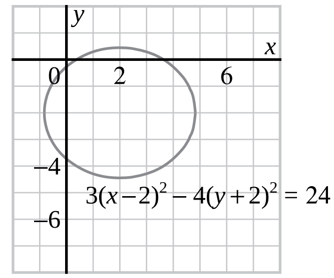



$4(x+1)^2 - (y-3)^2 = 16$

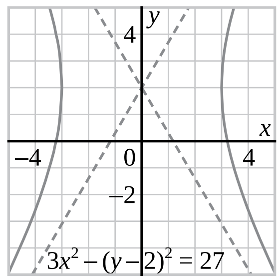
c)



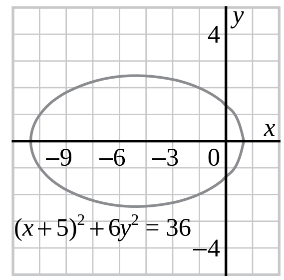

d)



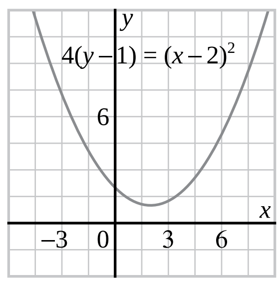

e)



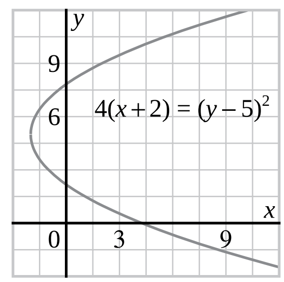

f)




g)




h)




**9.** **a)** These instructions apply to the TI-83 graphing calculator.

**1.** Enter the numbers –20, –15, ..., 0, ... 15, 20 into L1.

**2.** Enter L1 + √( 4 – $x^2$/16) into Y1 and –Y1 + 2L1 into Y2.

**3.** Adjust the graphing window so that $-10 \le x \le 10$ and $-25 \le y \le 25$.

**b)** These instructions apply to the TI-83 graphing calculator.

**1.** Enter the numbers –30, –25, ..., 0, ... 25, 30 into L1.

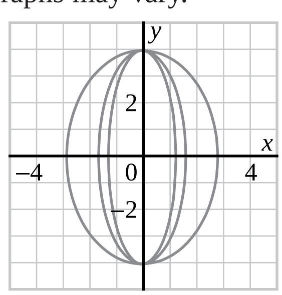
**2.** Enter L1 + √($x^2$ – 1) into Y1 and –Y1 + 2L1 into Y2.

**3.** Adjust the graphing window so that $-15 \le x \le 15$ and $-30 \le y \le 30$.

### *Investigate, page 550*

**1.** **a)** Graphs may vary.

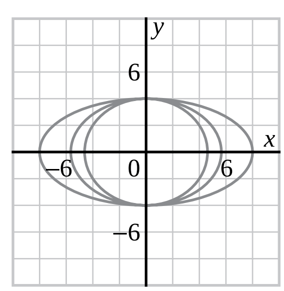
i)




$Ax^2 + y^2 - 16 = 0$

$A = 2,\ 6,\ 10$

ii)

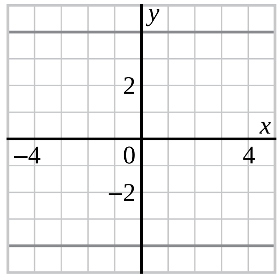



$Ax^2 + y^2 - 16 = 0$

$A = \frac{1}{4},\ \frac{1}{2},\ \frac{3}{4}$

# Selected Solutions — Chapter 9

iii)




$Ax^2 + y^2 - 16 = 0,\ A = 0$

**b)** For $A > 1$, the equation represents an ellipse with $y$-intercepts ±4. The graph of $Ax^2 + y^2 - 16 = 0$ is a horizontal compression by a factor of $\sqrt{A}$ of the graph of $x^2 + y^2 - 16 = 0$.

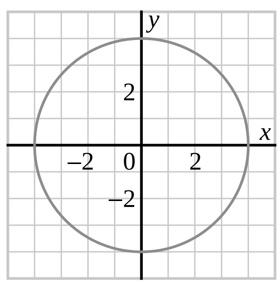
For $0 < A < 1$, the equation represents an ellipse with $y$-intercepts ±4. The graph of $Ax^2 + y^2 - 16 = 0$ is a horizontal expansion by a factor of $\sqrt{A}$ of the graph of $x^2 + y^2 - 16 = 0$.

For $A = 0$, the equation represents the two horizontal lines $y = \pm 4$.

**4.** The results would be different because they apply to the opposite axis. Take the graphs in exercises 1, 2, and 3 and reflect them in the $y = x$ line. These graphs represent solutions for the equation $x^2 + Cy^2 + F = 0$.

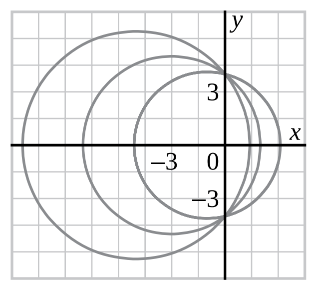
**5. a)** Graphs may vary.

i)



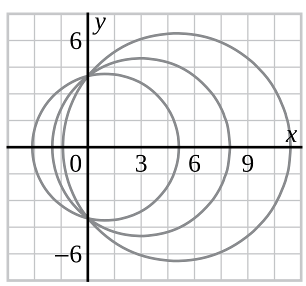

$x^2 + y^2 + Dx - 16 = 0,$
$D = 0$

ii)




$x^2 + y^2 + Dx - 16 = 0,$
$D = 2,\ 6,\ 10$

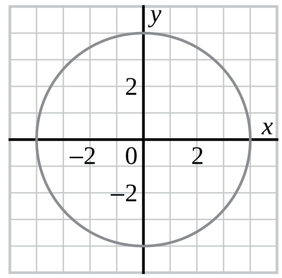
iii)




$x^2 + y^2 + Dx - 16 = 0,\ D = -2,\ -6,\ -10$

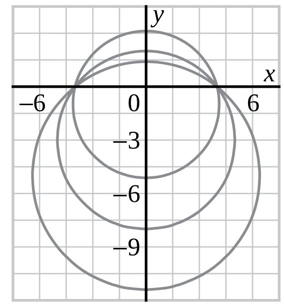
**b)** Graphs may vary.

**i)**



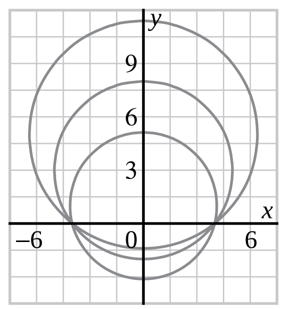

$x^2 + y^2 + Ey - 16 = 0$
$E = 0$

**ii)**




$x^2 + y^2 + Ey - 16 = 0$
$E = 2,\ 6,\ 10$

**iii)**




$x^2 + y^2 + Ey - 16 = 0$
$E = -2,\ -6,\ -10$

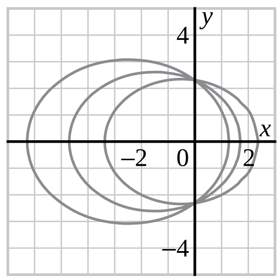
**c)** For part a:
For $D = 0$: The equation represents a circle with centre (0, 0) and radius 4.
For $D > 0$: The graph of $x^2 + y^2 + Dx - 16 = 0$ is a translation to the left and a vertical and horizontal expansion of the graph of $x^2 + y^2 - 16 = 0$.
For $D < 0$: The graph of $x^2 + y^2 + Dx - 16 = 0$ is a translation to the right and a vertical and horizontal expansion of the graph of $x^2 + y^2 - 16 = 0$.

For part b:
For $E = 0$: The equation represents a circle with centre (0, 0) and radius 4.
For $E > 0$: The graph of $x^2 + y^2 + Ey - 16 = 0$ is a translation down and a vertical and horizontal expansion of the graph of $x^2 + y^2 - 16 = 0$.
For $E < 0$: The graph of $x^2 + y^2 + Ey - 16 = 0$ is a translation up and a vertical and horizontal expansion of the graph of $x^2 + y^2 - 16 = 0$.

# Selected Solutions — Chapter 9

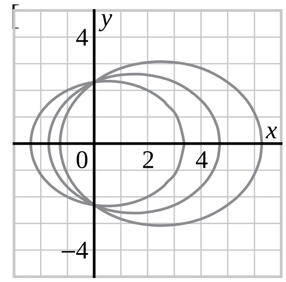
**6. a)** Graphs may vary.

**i)**




$2x^2 + 3y^2 + Dx - 16 = 0$
$D = 2,\ 6,\ 10$

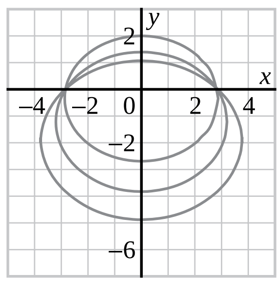
**ii)**




$2x^2 + 3y^2 + Dx - 16 = 0$
$D = -2,\ -6,\ -10$

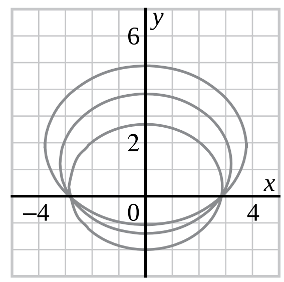
**b)** Graphs may vary.

**i)**




$2x^2 + 3y^2 + Ey - 16 = 0$
$E = 2,\ 6,\ 10$

**ii)**



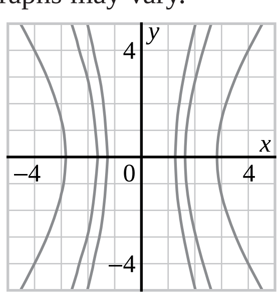

$2x^2 + 3y^2 + Ey - 16 = 0$
$E = -2,\ -6,\ -10$

For $D > 0$: The graph of $2x^2 + 3y^2 + Dx - 16 = 0$ is a translation to the left and a vertical and horizontal expansion of the graph of $2x^2 + 3y^2 - 16 = 0$.
For $D < 0$: The graph of $2x^2 + 3y^2 + Dx - 16 = 0$ is a translation to the right and a vertical and horizontal expansion of the graph of $2x^2 + 3y^2 - 16 = 0$.

For part b:
For $E = 0$: The equation represents a circle with centre (0, 0) and radius 4.
For $E > 0$: The graph of $2x^2 + 3y^2 + Ey - 16 = 0$ is a translation down and a vertical and horizontal expansion of the graph of $2x^2 + 3y^2 - 16 = 0$.
For $E < 0$: The graph of $2x^2 + 3y^2 + Ey - 16 = 0$ is a translation up and a vertical and horizontal expansion of the graph of $2x^2 + 3y^2 - 16 = 0$.

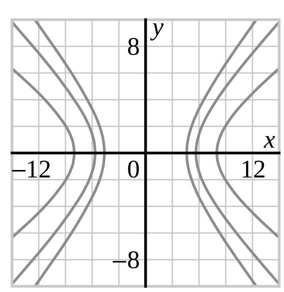
**8. a)** Graphs may vary.

**i)**




$Ax^2 - y^2 - 16 = 0$
$A = 2,\ 6,\ 10$

**ii)**

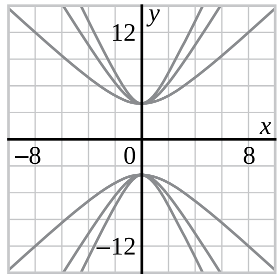



$Ax^2 - y^2 - 16 = 0$
$A = \frac{1}{4},\ \frac{1}{2},\ \frac{3}{4}$

# Selected Solutions — Chapter 9

**b)** Graphs may vary.

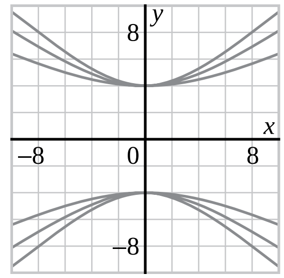
**i)**




$Ax^2 - y^2 + 16 = 0$

$A = 2,\ 6,\ 10$

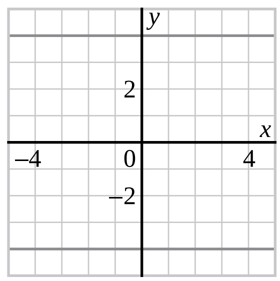
**ii)**




$Ax^2 - y^2 + 16 = 0$

$A = \frac{1}{4},\ \frac{1}{2},\ \frac{3}{4}$

**iii)**




$Ax^2 - y^2 + 16 = 0,\ A = 0$

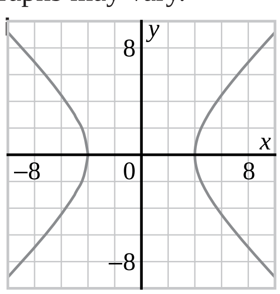
**c)** For part a:
For $A > 1$: The graph of $Ax^2 - y^2 - 16 = 0$ is a horizontal compression by a factor of $\sqrt{A}$ of the graph of $x^2 - y^2 - 16 = 0$.
For $0 < A < 1$: The graph of $Ax^2 - y^2 - 16 = 0$ is a horizontal expansion by a factor of $\sqrt{A}$ of the graph of $x^2 - y^2 - 16 = 0$.
For $A = 0$: There are no points that satisfy the equation.

For part b:
For $A > 1$: The graph of $Ax^2 - y^2 + 16 = 0$ is a horizontal compression by a factor of $\sqrt{A}$ of the graph of $x^2 - y^2 + 16 = 0$.
For $0 < A < 1$: The graph of $Ax^2 - y^2 + 16 = 0$ is a horizontal expansion by a factor of $\sqrt{A}$ of the graph of $x^2 - y^2 + 16 = 0$.
For $A = 0$: The graph is the two straight lines $y = \pm 4$.

**11.** The results would be different because they apply to the opposite axis. Take the graphs in exercises 8, 9, and 10 and reflect them in the $y = x$ line. These graphs represent solutions for the equation $x^2 + Cy^2 + F = 0$.

**12. a)** Graphs may vary.

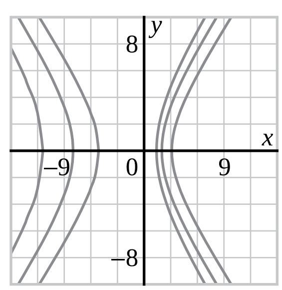
**i)**




$x^2 - y^2 + Dx - 16 = 0$

$D = 0$

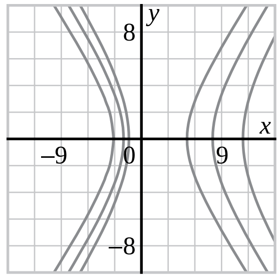
**ii)**




$x^2 - y^2 + Dx - 16 = 0$

$D = 2,\ 6,\ 10$

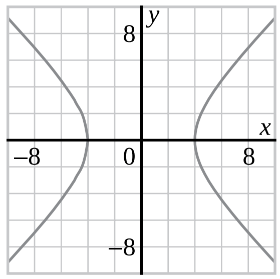
iii)




$x^2 - y^2 + Dx - 16 = 0$, $D = -2, -6, -10$

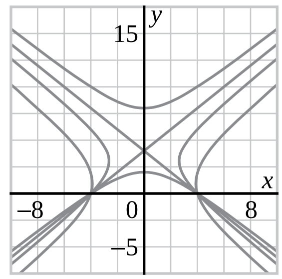
**b)** Graphs may vary.

i)



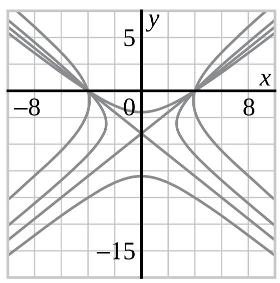

$x^2 - y^2 + Ey - 16 = 0$
$E = 0$

ii)




$x^2 - y^2 + Ey - 16 = 0$
$E = 2, 6, 8, 10$

iii)




$x^2 - y^2 + Ey - 16 = 0$
$E = -2, -6, -8, -10$

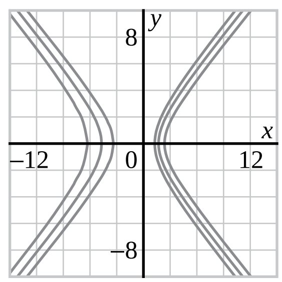
**c)** For part a:
For $D = 0$: The equation represents a hyperbola with vertices $(\pm 4, 0)$ and asymptotes $y = \pm x$.
For $D > 0$: The graph of $x^2 - y^2 + Dx - 16 = 0$ is a translation to the left and a horizontal and vertical expansion of the graph of $x^2 - y^2 - 16 = 0$.
For $D < 0$: The graph of $x^2 - y^2 + Dx - 16 = 0$ is a translation to the right and a horizontal and vertical expansion of the graph of $x^2 - y^2 - 16 = 0$.

# Selected Solutions — Chapter 9

For part b:
For $E = 0$: The equation represents a hyperbola with vertices $(\pm 4,\ 0)$ and asymptotes $y = \pm x$.
For $0 < E < 8$: The graph of $x^2 - y^2 + Ey - 16 = 0$ is a translation down and a horizontal and vertical expansion of the graph of $x^2 - y^2 - 16 = 0$. The vertices of the hyperbola are on the $x$-axis. When $E = 8$, the graph is the two straight lines $y = \pm x + 4$. For $E > 8$, the vertices of the hyperbola are on the $y$-axis.
For $E < 0$: The graph of $x^2 - y^2 + Ey - 16 = 0$ is a translation up and a horizontal and vertical expansion of the graph of $x^2 - y^2 - 16 = 0$. The vertices of the hyperbola are on the $x$-axis. When $E = -8$, the graph is the two straight lines $y = \pm x - 4$. For $E < -8$, the vertices of the hyperbola are on the $y$-axis.

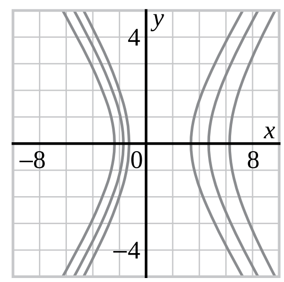
**13. a)** Graphs may vary.

**i)**




$2x^2 - 3y^2 + Dx - 16 = 0$
$D = 2,\ 6,\ 10$

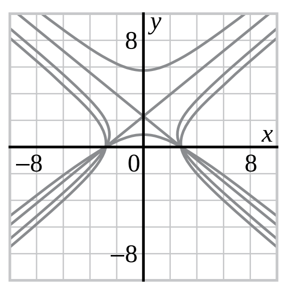
**ii)**




$2x^2 - 3y^2 + Dx - 16 = 0$
$D = -2,\ -6,\ -10$

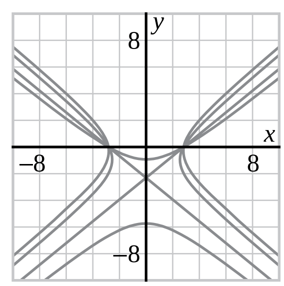
**b)** Graphs may vary.

**i)**




$2x^2 - 3y^2 + Ey - 16 = 0$
$D = 2,\ 6,\ 10,\ 8\sqrt{3},\ 20$

**ii)**




$2x^2 - 3y^2 + Ey - 16 = 0$
$D = -2,\ -6,\ -10,\ -8\sqrt{3},\ -20$

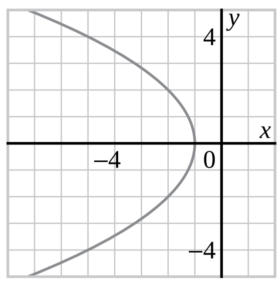
**c)** For part a:
For $D > 0$: The graph of $3x^2 - 2y^2 + Dx - 16 = 0$ is a translation to the left and a horizontal and vertical expansion of the graph of $3x^2 - 2y^2 - 16 = 0$.
For $D < 0$: The graph of $3x^2 - 2y^2 + Dx - 16 = 0$ is a translation to the right and a horizontal and vertical expansion of the graph of $3x^2 - 2y^2 - 16 = 0$.

# Selected Solutions — Chapter 9

For part b:
For $0 < E < 8\sqrt{3}$: The graph of $3x^2 - 2y^2 + Ey - 16 = 0$ is a translation down and a horizontal and vertical expansion of the graph of $3x^2 - 2y^2 - 16 = 0$. The hyperbola has a horizontal transverse axis. When $E = 8\sqrt{3}$, the graph is the two straight lines $\frac{\sqrt{6}x + 4\sqrt{3}}{3}$ and $\frac{-\sqrt{6}x + 4\sqrt{3}}{3}$. For $E > 8\sqrt{3}$, the hyperbola has a vertical transverse axis.
For $-8\sqrt{3} < E < 0$: The graph of $3x^2 - 2y^2 + Ey - 16 = 0$ is a translation up and a horizontal and vertical expansion of the graph of $x^2 - y^2 - 16 = 0$. The hyperbola has a horizontal transverse axis. When $E = -8\sqrt{3}$, the graph is the two straight lines $\frac{\sqrt{6}x + 4\sqrt{3}}{3}$ and $\frac{-\sqrt{6}x + 4\sqrt{3}}{3}$. For $E < -8\sqrt{3}$, the hyperbola has a vertical transverse axis.

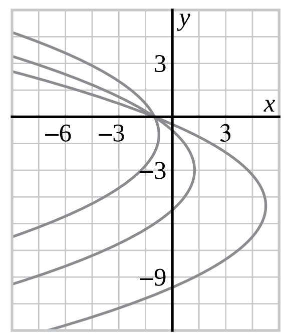
**15. a)** Graphs may vary.

**i)**



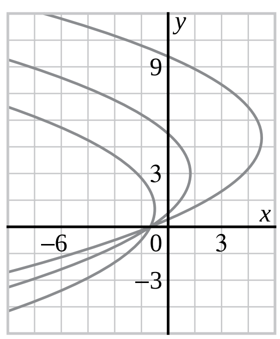

$y^2 + 4x + Ey + 4 = 0$
$E = 0$

**ii)**




$y^2 + 4x + Ey + 4 = 0$
$E = 2,\ 6,\ 10$

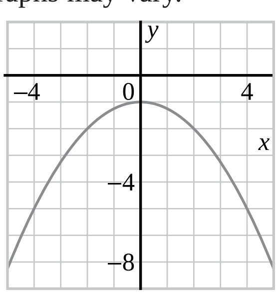
**iii)**




$y^2 + 4x + Ey + 4 = 0$
$E = -2,\ -6,\ -10$

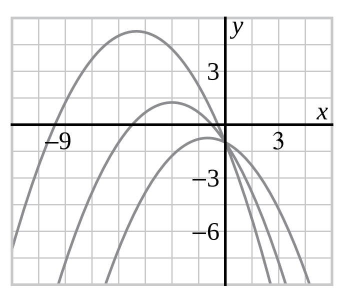
**b)** Graphs may vary.

**i)**

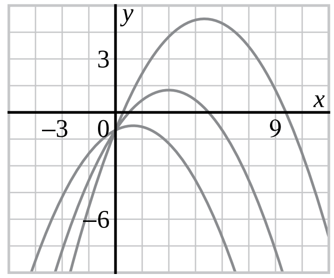



$x^2 + 4y + Dx + 4 = 0$
$D = 0$

**ii)**




$x^2 + 4y + Dx + 4 = 0$
$D = 2,\ 6,\ 10$

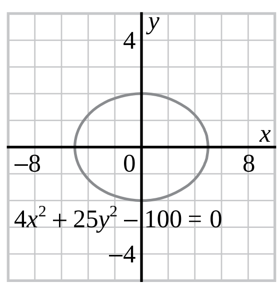
iii) 


$x^2 + 4y + Dx + 4 = 0$, $D = -2, -6, -10$

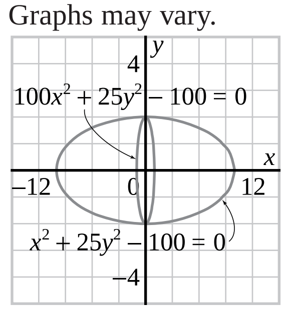
**c)** For part a:
For $E = 0$: The equation represents a parabola with vertex (−1, 0), axis of symmetry the $x$-axis, and opening to the left.
For $E > 0$: The graph of $y^2 + 4x + Ey + 4 = 0$ is a translation down and a vertical and horizontal expansion of the graph of $y^2 + 4x + 4 = 0$.
For $E < 0$: The graph of $y^2 + 4x + Ey + 4 = 0$ is a translation up and a vertical and horizontal expansion of the graph of $y^2 + 4x + 4 = 0$.

For part b:
For $D = 0$: The equation represents a parabola with vertex (0, −1), axis of symmetry the $y$-axis, and opening down.
For $D > 0$: The graph of $x^2 + 4y + Dx + 4 = 0$ is a translation left and a vertical and horizontal expansion of the graph of $x^2 + 4y + 4 = 0$.
For $D < 0$: The graph of $x^2 + 4y + Dx + 4 = 0$ is a translation right and a vertical and horizontal expansion of the graph of $x^2 + 4y + 4 = 0$.

### *9.5 Exercises, page 555*

**1. a)** 

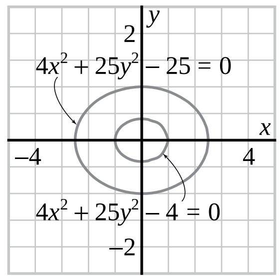

**b)** Graphs may vary.




**d)** If the coefficient of $x^2$ is replaced with a negative number, the equation represents a hyperbola with vertical transverse axis. If the coefficient of $y^2$ is replaced with a negative number, the equation represents a hyperbola with horizontal transverse axis.

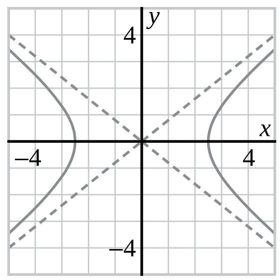
# Selected Solutions — Chapter 9

**2. a)** Graphs may vary.

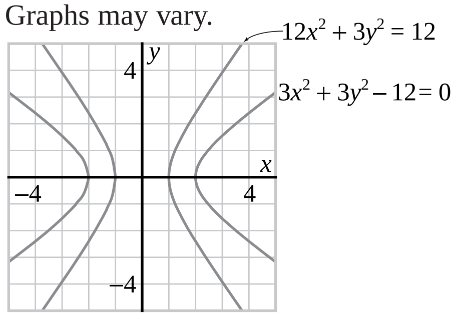



**b)** If the constant is replaced with a positive number, there are no points that satisfy the equation, since 3 positive numbers cannot sum to 0.

**3. a)**

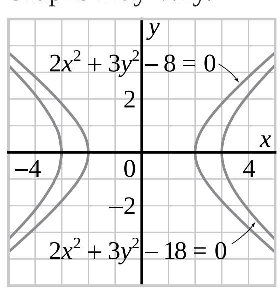



**b)** Graphs may vary.




**d)** If the coefficient of $x^2$ is replaced with a negative number, there are no points that satisfy the equation, since 3 negative numbers cannot sum to 0. If the coefficient of $y^2$ is replaced with a negative number, the equation represents an ellipse.

**4. a)** Graphs may vary.

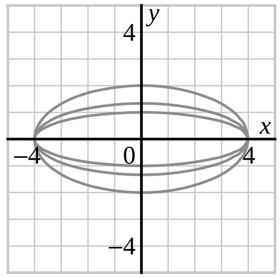



**b)** If the constant is replaced with a positive number, the equation represents a hyperbola with vertical transverse axis, since the graph has $y$-intercepts and no $x$-intercepts.

# Selected Solutions — Chapter 9

**5. a)** Graphs may vary.

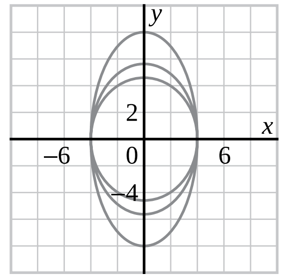
**i)**




$x^2 + Cy^2 - 16 = 0$

$C = 4,\ 9,\ 16$

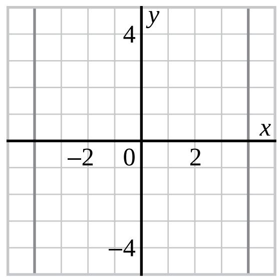
**ii)**




$x^2 + Cy^2 - 16 = 0$

$\frac{3}{4},\ \frac{1}{2},\ \frac{1}{4}$

**iii)**

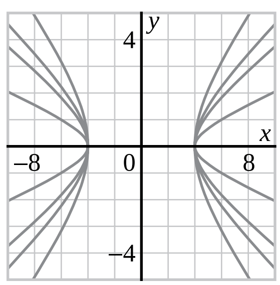



$x^2 + Cy^2 - 16 = 0,\ C = 0$

**b)** For $C > 1$, the graph is a vertical compression by a factor of $\frac{1}{\sqrt{C}}$ of the graph of $x^2 + y^2 - 16 = 0$. For $0 < C < 1$, the graph is a vertical expansion by a factor of $\frac{1}{\sqrt{C}}$ of the graph of $x^2 + y^2 - 16 = 0$. For $C = 0$, the graph is the lines $x = \pm 4$.

**7. a)** Graphs may vary.

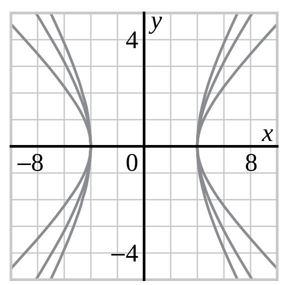
**i)**




$x^2 + Cy^2 - 16 = 0$

$C = -2,\ -4,\ -6,\ -20$

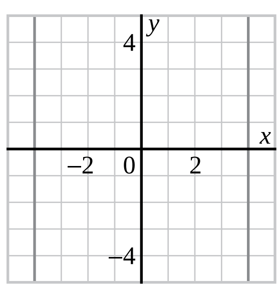
**ii)**




$x^2 + Cy^2 - 16 = 0$

$C = -\frac{3}{4},\ -\frac{1}{2},\ -\frac{1}{4}$

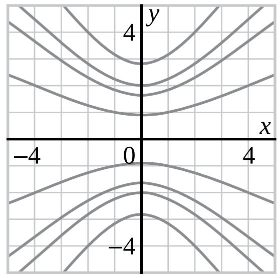
**iii)**




$x^2 + Cy^2 - 16 = 0,\ C = 0$

**b)** Graphs may vary.

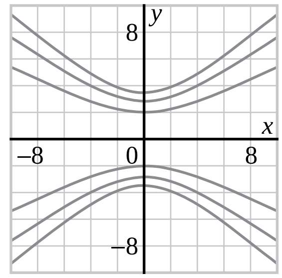
**i)**




$x^2 + Cy^2 + 16 = 0$

$C = -2,\ -4,\ -6,\ -20$

**ii)**




$x^2 + Cy^2 + 16 = 0$

$C = -\frac{3}{4},\ -\frac{1}{2},\ -\frac{1}{4}$

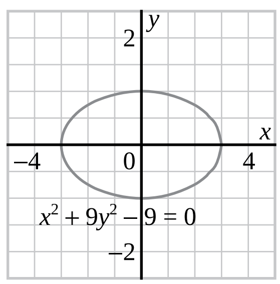
**c)** For part a:

For $C < -1$, the graph is a vertical compression by a factor of $\frac{1}{\sqrt{-C}}$ of the graph of $x^2 + y^2 - 16 = 0$. For $-1 < C < 0$, the graph is a vertical expansion by a factor of $\frac{1}{\sqrt{-C}}$ of the graph of $x^2 - y^2 - 16 = 0$. For $C = 0$, the graph is the lines $x = \pm 4$.

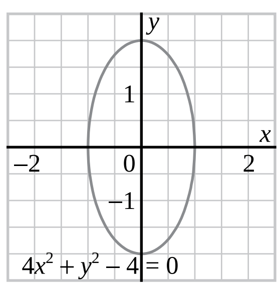
For part b:

For $C < -1$, the graph is a vertical compression by a factor of $\frac{1}{\sqrt{-C}}$ of the graph of $x^2 - y^2 + 16 = 0$. For $-1 < C < 0$, the graph is a vertical expansion by a factor of $\frac{1}{\sqrt{-C}}$ of the graph of $x^2 - y^2 - 16 = 0$. For $C = 0$, there are no points that satisfy the equation.

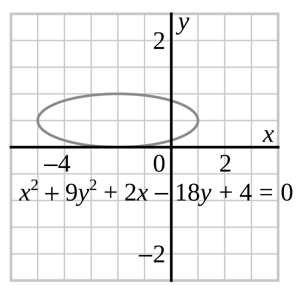
**9. a)**




**d) i)**

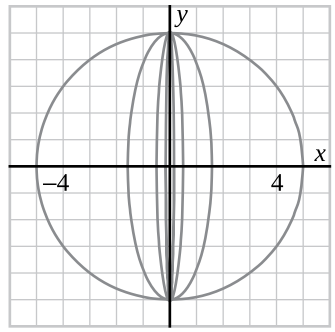



**ii)**



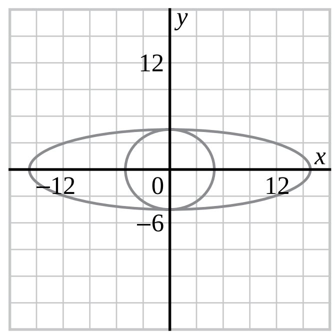
# Selected Solutions — Chapter 9

**10. b) i)**

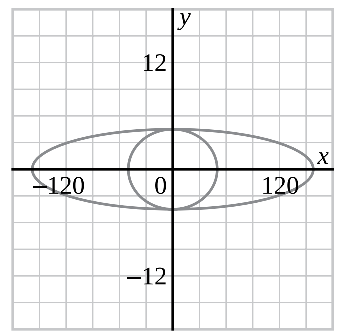



$Ax^2 + y^2 - 25 = 0,\ A = 1,\ 10,\ 100,\ 1000$

**ii)**




$Ax^2 + y^2 - 25 = 0,$
$A = 1,\ 0.1$

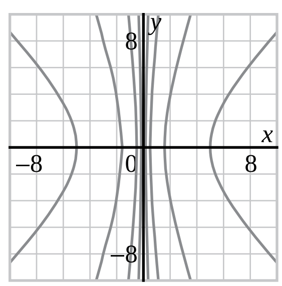



$Ax^2 + y^2 - 25 = 0,$
$A = 0.01,\ 0.001$

**c)** The graph becomes the two lines $y = \pm 5$.

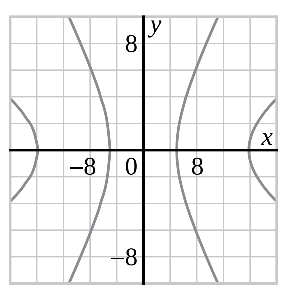
**d)** The graph becomes a hyperbola with vertical transverse axis and vertices $(\pm 5,\ 0)$.

**12. b) i)**

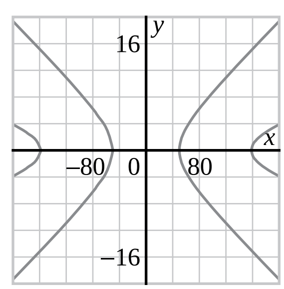



$Ax^2 - y^2 - 25 = 0,\ A = 1,\ 10,\ 100,\ 1000$

**ii)**




$Ax^2 - y^2 - 25 = 0$
$A = 1,\ 0.1$



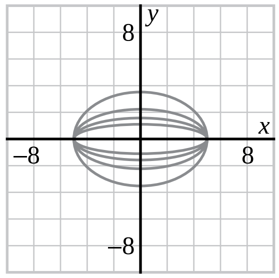

$Ax^2 - y^2 - 25 = 0$
$A = 0.01,\ 0.001$

**c)** There are no points that satisfy the equation when the coefficient of $x$ is 0.

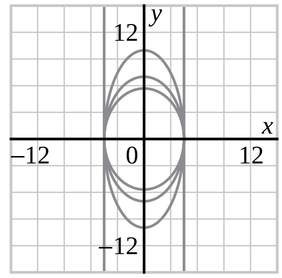
**d)** There are no points that satisfy the equation when the coefficient of $x$ is negative.

# Selected Solutions — Chapter 9

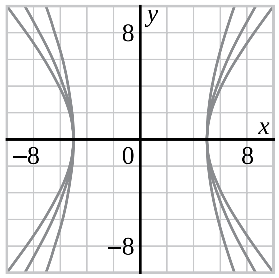
**14. a)** $x^2 + Cy^2 - 25 = 0$



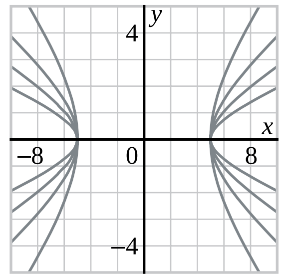

$C = 2,\ 5,\ 10,\ 20$




$C = \frac{3}{4},\ \frac{1}{2},\ \frac{1}{4},\ 0$

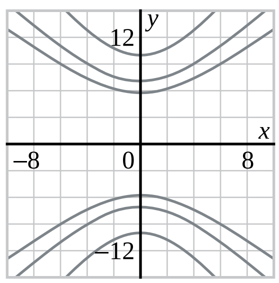



$C = -\frac{1}{4},\ -\frac{1}{2},\ -\frac{3}{4}$

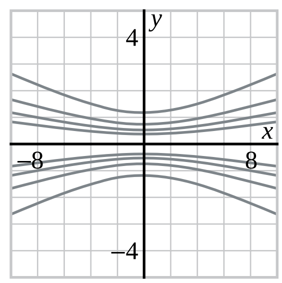



$C = -2,\ -5,\ -10,\ -20$

**b)** $x^2 + Cy^2 + 25 = 0$



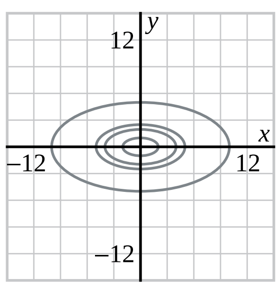

$C = -\frac{1}{4},\ -\frac{1}{2},\ -\frac{3}{4}$




$C = -2,\ -5,\ -10,\ -20$

When $C = 0$, there are no points that satisfy the equation.

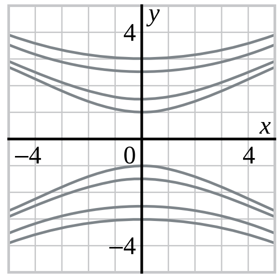
**15. a)** $x^2 + 4y^2 + F = 0,\ F = -4,\ -16,\ -25,\ -100$



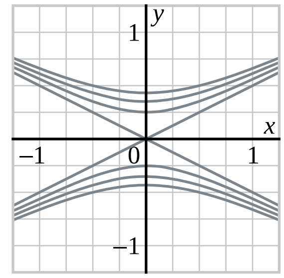

When $F = 0$, there are no points that satisfy the equation.

# Selected Solutions — Chapter 9

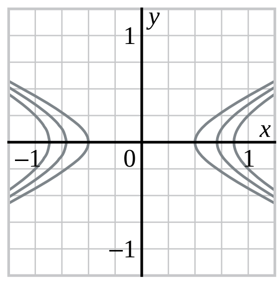
**b)** $x^2 - 4y^2 + F = 0$



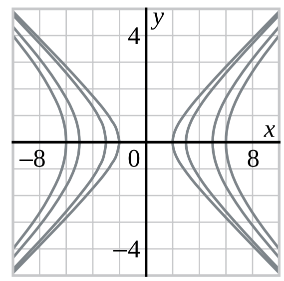

$F = 4,\ 9,\ 25,\ 36$




$F = \frac{3}{4},\ \frac{1}{2},\ \frac{1}{4},\ 0$

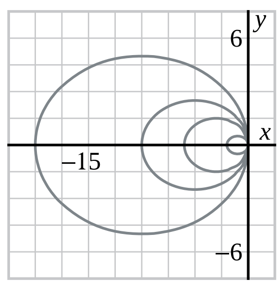



$F = -\frac{1}{4},\ -\frac{1}{2},\ -\frac{3}{4}$

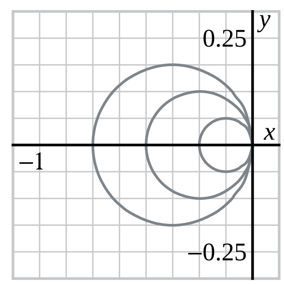



$F = -4,\ -9,\ -25,\ -36$

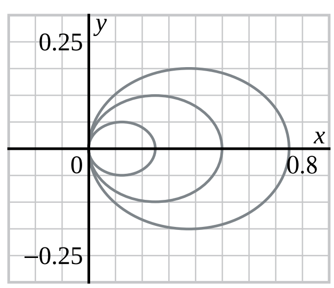
**16. a)** $x^2 + 4y^2 + Dx = 0$



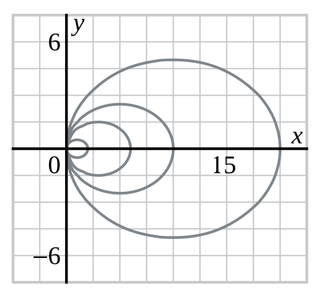

$D = 2,\ 6,\ 10,\ 20$




$D = \frac{3}{4},\ \frac{1}{2},\ \frac{1}{4},\ 0$

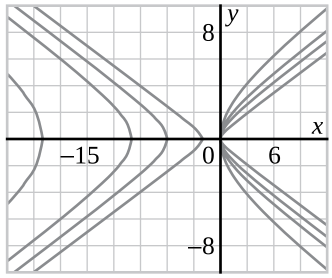



$D = -\frac{1}{4},\ -\frac{1}{2},\ -\frac{3}{4}$

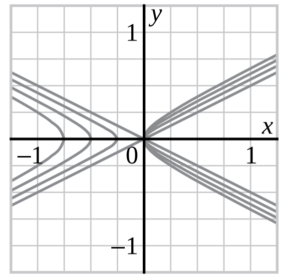



$D = -2,\ -6,\ -10,\ -20$

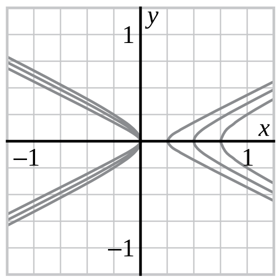
**b)** $x^2 - 4y^2 + Dx = 0$



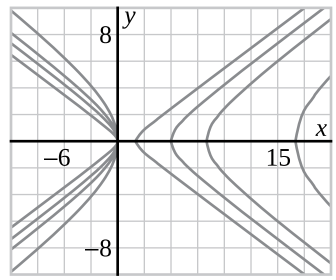

$D = 2,\ 6,\ 10,\ 20$




$D = \frac{3}{4},\ \frac{1}{2},\ \frac{1}{4},\ 0$

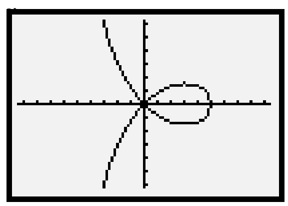



$D = -\frac{1}{4},\ -\frac{1}{2},\ -\frac{3}{4}$

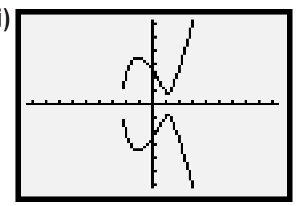



$D = -2,\ -6,\ -10,\ -20$

**19. b)**



**ii)**



**20. a) i)** The graph is a horizontal line with $y$-intercept $-\frac{C}{B}$.

**ii)** The graph is a vertical line with $x$-intercept $-\frac{C}{A}$.

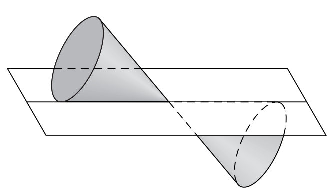
**b) i)** The graph is a horizontal line with $y$-intercept $-\frac{F}{E}$.

**ii)** The graph is a parabola with horizontal axis of symmetry and vertex $\left(\frac{B^2}{4C} - F,\ \frac{E}{2C}\right)$.

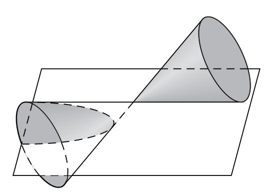
**iii)** The graph is a vertical line with $x$-intercept $-\frac{F}{D}$.

**iv)** The graph is a parabola with vertical axis of symmetry and vertex $\left(\frac{D}{2A},\ \frac{D^2}{4A} - F\right)$.

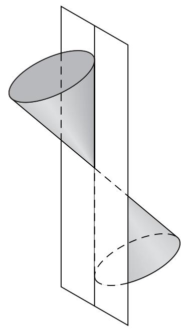
**c) i)**



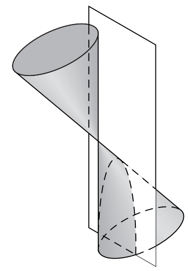
**ii)**



iii)



iv)



***Mathematical Modelling, page 558***

**6.** **a)** The $x$-coordinate is twice the $y$-coordinate.

**c)** Answers may vary. The tangent was drawn using visual estimation. Measurements may not have been accurate. Results were rounded.

**7.** **a)** The width of the dish is 60 cm, so the $x$-coordinate of $H$ is half that, or 30. The depth is 7 cm, so the $y$-coordinate is 7.

**b)** Substitute (30, 7) into the equation $y = ax^2$.

$7 = a(30)^2$

$a = \frac{7}{900}$

The equation is $y = \frac{7}{900}x^2$.

**c)** Substitute $(2p,\ p)$ into the equation $y = \frac{7}{900}x^2$.

$p = \frac{7}{900}(2p)^2$

$p = \frac{7p^2}{225}$

$7p^2 - 225p = 0$

$p(7p - 225) = 0$

$p = 0$ or $p = \frac{225}{7}$.

$p$ cannot be 0, so $p = \frac{225}{7}$.

**d)** The receiver should be located at the focus, $F$, whose coordinates are $\left(0,\ \frac{225}{7}\right)$.

**8.** **a)** The coordinates of $H$ are $\left(\frac{d}{2},\ h\right)$.

**b)** Substitute $\left(\frac{d}{2},\ h\right)$ into the equation $y = ax^2$.

$h = a\left(\frac{2}{d}\right)^2$

$a = \frac{4h}{d^2}$

The equation is $y = \frac{4h}{d^2}x^2$.

# Selected Solutions — Chapter 9

**c)** Substitute $(2p,\ p)$ into the equation $y = \frac{4h}{d^2}x^2$.

$p = \frac{4h}{d^2}(2p)^2$

$p = \frac{16h}{d^2}p^2$

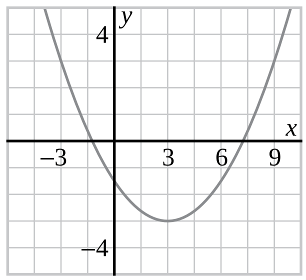
$p = 0$ or $p = \frac{d^2}{16h}$.

$p$ cannot be 0, so $p = \frac{d^2}{16h}$.

**d)** The receiver should be located at $F$, whose coordinates are $\left(0,\ \frac{d^2}{h}\right)$.

### *9.6 Exercises, page 563*

**4. b)**




**5.** A translation is represented by replacing $x$ with $(x - a)$ and $y$ with $(y - b)$.

$$A(x - a)^2 + C(y - b)^2 + D(x - a) + E(y - b) + F = 0$$
$$Ax^2 - 2Aax + Aa^2 + Cy^2 - 2Cby + Cb^2 + Dx - Da + Ey - Eb + F = 0$$
$$Ax^2 + Cy^2 + (D - 2Aa)x + (E - 2Cb)y + Aa^2 + Cb^2 - Da - Eb + F = 0$$

From the translated equation, the expression $Ax^2 + Cy^2$ does not change.

**6.** No. The equation of a circle does not change when it is rotated about its centre.

### *Problem Solving, page 565*

**1. a)** P is a point outside the sphere. $PF_1$ and $PQ_1$ are tangents to the top sphere. Thus, $PF_1 = PQ_1$. Similarly, $PF_2 = PQ_2$, since $PF_2$ and $PQ_2$ are tangents to the bottom sphere.

**b)** $PF_1 + PF_2 = PQ_1 + PQ_2$

Since P is on the line $Q_1Q_2$, $PQ_1 + PQ_2 = Q_1Q_2$, which is constant. Thus, $PF_1 + PF_2$ is constant.

**2. a)** Visualize a small sphere starting at the vertex of the cone. Expand the sphere as you move it toward the plane, so it remains tangent to the inside of the cone. It will eventually touch the plane in one spot, that is, it will be tangent to the plane.

**b)** Visualize a large sphere starting below the plane. Compress the sphere as you move it toward the plane, so it remains tangent to the inside of the cone. It will eventually touch the plane in one spot, that is, it will be tangent to the plane.

# Selected Solutions — Chapter 9

**3.** **a)** **i)** The ellipse is expanded vertically and horizontally if the string is longer, and compressed vertically and horizontally if the string is shorter.

**ii)** If the thumbtacks are farther apart, the ellipse is expanded horizontally and compressed vertically. If the thumbtacks are closer together, the ellipse is compressed horizontally and expanded vertically.

**b)** For part i, the plane moves away from the vertex for a larger ellipse and toward the vertex for a smaller ellipse. For part ii, the plane becomes closer to parallel to the generator of the cone for the foci farther apart and closer to perpendicular to the axis of the cone for the foci closer together.

**4.** **a)** Since $PF_1 + PF_2$ is constant for any point P, find the sum for $P(a,\ 0)$.

$PF_1 = a - c$

$PF_2 = a + c$

$$\text{Thus, } PF_1 + PF_2 = a - c + a + c = 2a$$

**b)** $\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$

**c)**

$$\begin{aligned}
\sqrt{(x-c)^2 + y^2} &= 2a - \sqrt{(x+c)^2 + y^2} \\
(x-c)^2 + y^2 &= 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + (x+c)^2 + y^2 \\
4a\sqrt{(x+c)^2 + y^2} &= 4cx + 4a^2 \\
a\sqrt{(x+c)^2 + y^2} &= cx + a^2 \\
a^2[(x+c)^2 + y^2] &= c^2x^2 + 2a^2cx + a^4 \\
a^2x^2 + 2a^2cx + a^2c^2 + a^2y^2 &= c^2x^2 + 2a^2cx + a^4 \\
(a^2 - c^2)x^2 + a^2y^2 &= a^4 - a^2c^2 \\
&= a^2(a^2 - c^2) \\
\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} &= 1
\end{aligned}$$

**d)** Let $b^2 = a^2 - c^2$.

Thus, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

**5.** The graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is a horizontal compression or expansion by a factor of $a$ and a vertical compression or expansion by a factor of $b$ of the graph of $x^2 + y^2 = 1$.

**6.** **a)** Find the perpendicular bisector of the major axis to find the minor axis. Place the centre of the ellipse at the origin of a coordinate grid. Let the coordinates of the foci be $F_1(c,\ 0)$ and $F_2(-c,\ 0)$, where $c > 0$. Let the coordinates of the vertex on the positive $y$-axis be $B(0, b)$. Since B is a point on the ellipse, $BF_1 + BF_2 = 2a$. By symmetry, $BF_1 = BF_2$, so $BF_1 = BF_2 = a$. Thus, construct a right triangle $BF_1O$ with hypotenuse $BF_1 = a$. The length of $c$ is thus $\sqrt{a^2 - b^2}$. Find $F_2$ similarly.

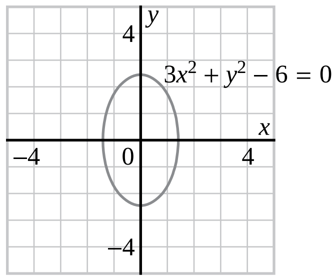
**b)** See part a.

**7.** A circle is formed when a plane intersects a cone perpendicular to its axis. Tilt the plane up or down to form an ellipse.

# Selected Solutions — Chapter 9

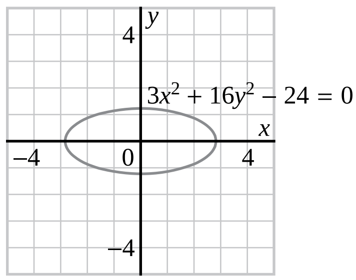
***9 Review, page 568***

**2. a)**

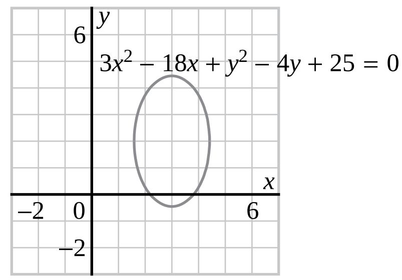



**b)** The graph of $3x^2 + y^2 - 6 = 0$ is an ellipse with centre (0, 0), $x$-intercepts $\pm\sqrt{2}$, and $y$-intercepts $\pm\sqrt{6}$.

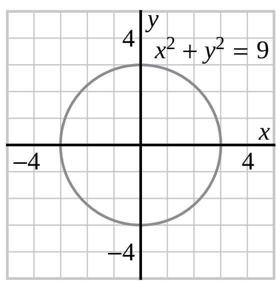
**d) i)**



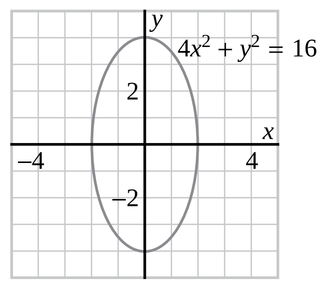

**d) ii)**



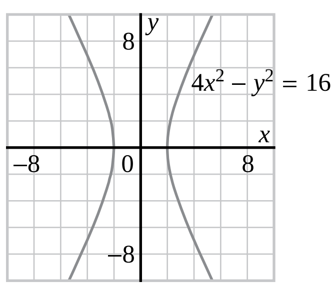

**3. a)**



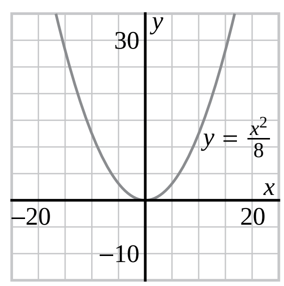

**b)**



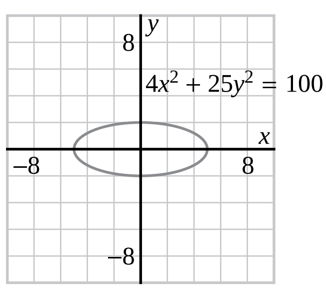

**c)**



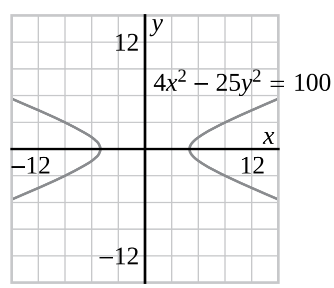

**d)**




**e)**




**f)**



# Selected Solutions — Chapter 9

**4.** From exercise 5 on page 563, the expression $Ax^2 + Cy^2$ does not change when the graph of a conic section is translated. Also, the type of conic section does not change after a translation. Thus, translate the graph of $Ax^2 + Cy^2 + Dx + Ey + F = 0$ so that $D$ and $E$ disappear. For the new equation, the type of conic section depends only on $Ax^2 + Cy^2$. Thus, in the original equation, the type of conic section depends only on $Ax^2 + Cy^2$.

**5.** The conic sections defined by $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ do not have vertical and horizontal axes. Their axes are rotated.

### *9 Cumulative Review, page 569*

**3. c)** I assumed I could not draw a segment shorter than 1 mm. The side lengths of the squares are in the sequence 1, $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2\sqrt{2}}, \ldots$. This is a geometric sequence with $a = 1$ and $r = \frac{1}{\sqrt{2}}$.

$t_n = \left(\frac{1}{\sqrt{2}}\right)^{n-1}$

$t_n = 1$ mm, or 0.001 m

I solved the equation $0.001 = \left(\frac{1}{\sqrt{2}}\right)^{n-1}$ to find $n$.

I took the logarithm of each side:

$\log 0.001 = (n-1)\log\frac{1}{\sqrt{2}}$

$n = \frac{-3}{\log\frac{1}{\sqrt{2}}}$

$\doteq 21$

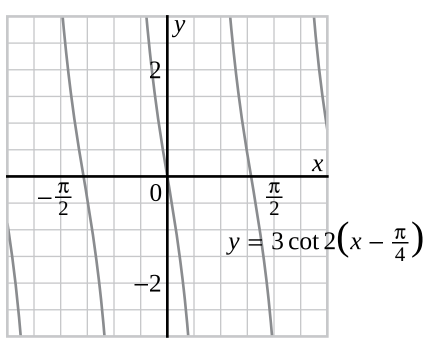
The sum of the side lengths is

$4(1) + 4\left(\frac{1}{\sqrt{2}}\right) + 4\left(\frac{1}{2}\right) + 4\left(\frac{1}{2\sqrt{2}}\right) + \ldots$, which is a geometric series with $a = 4$ and $r = \frac{1}{\sqrt{2}}$.

I found $S_{21} = \frac{4\left(1 - \left(\frac{1}{\sqrt{2}}\right)^{21}\right)}{1 - \frac{1}{\sqrt{2}}}$

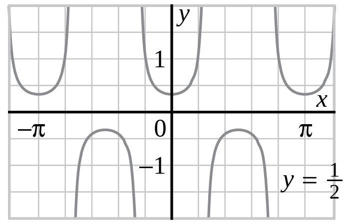
$\doteq 13.65$

**6. a)**



# Selected Solutions — Chapter 9

**b)**




**7. a)** $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

$$\begin{aligned}\text{Left side} &= \sin^4 x - \cos^4 x\\ &= (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)\\ &= (1)(\sin^2 x - (1 - \sin^2 x))\\ &= 2\sin^2 x - 1\\ &= \text{Right side}\end{aligned}$$

**b)** $\dfrac{\csc x}{\sec^2 x} = \csc x - \sin x$

$$\begin{aligned}\text{Left side} &= \frac{\csc x}{\sec^2 x}\\ &= \frac{\frac{1}{\sin x}}{\frac{1}{\cos^2 x}}\\ &= \frac{\cos^2 x}{\sin x}\\ &= \frac{1 - \sin^2 x}{\sin x}\\ &= \csc x - \sin x\\ &= \text{Right side}\end{aligned}$$

# Selected Solutions — Chapter 9

**c)** $\dfrac{\sin x + \tan x}{1 + \cos x} = \tan x$

$$\text{Left side} = \frac{\sin x + \tan x}{1 + \cos x}$$

$$= \frac{\sin x + \dfrac{\sin x}{\cos x}}{1 + \cos x}$$

$$= \frac{\sin x \left(\dfrac{\cos x + 1}{\cos x}\right)}{1 + \cos x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$= \text{Right side}$$

**d)** $\dfrac{\cos x}{1 - \sin x} + \dfrac{\cos x}{1 + \sin x} = 2 \sec x$

$$\text{Left side} = \frac{\cos x}{1 - \sin x} + \frac{\cos x}{1 + \sin x}$$

$$= \cos x \left(\frac{1 + \sin x + 1 - \sin x}{1 - \sin^2 x}\right)$$

$$= \frac{2 \cos x}{\cos^2 x}$$

$$= \frac{2}{\cos x}$$

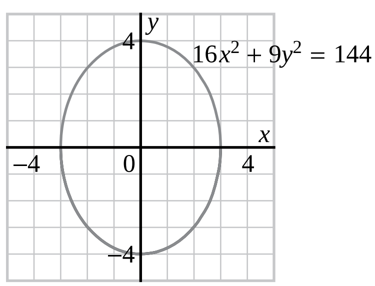
$$= 2 \sec x$$

$$= \text{Right side}$$

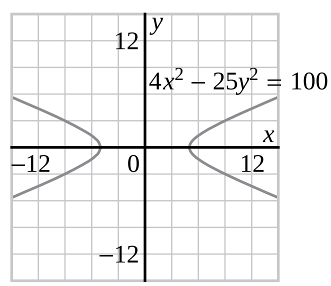
**8.** Explanations may vary. For part a:
I started with the left side, $\sin^4 x - \cos^4 x$, and factored the difference of squares to get $(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$, or $(1)(\sin^2 x - (1 - \sin^2 x))$, using the Pythagorean identity. This simplifies to $2 \sin^2 x - 1$, which is equal to the right side.

**15.** The probability of left-eye dominant people is similar to the probability of left-handed people.

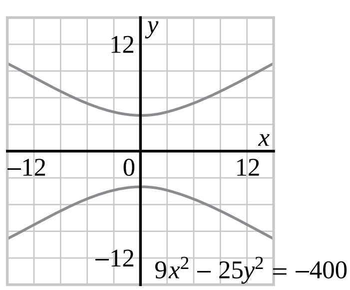
**21. a)**



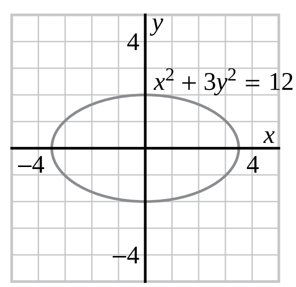

**b)**




**c)**

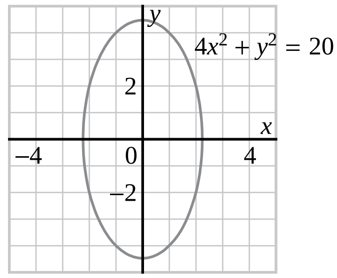



**d)**

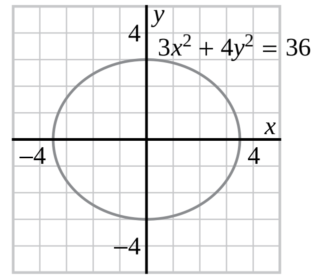


# Selected Solutions — Chapter 9

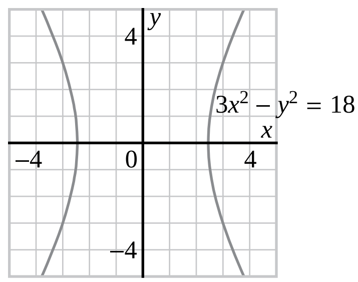
e)



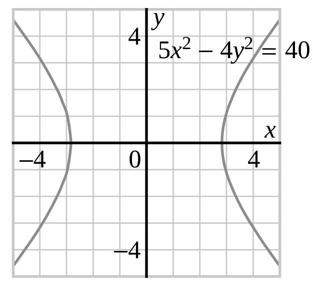

f)




g)




h)




i)