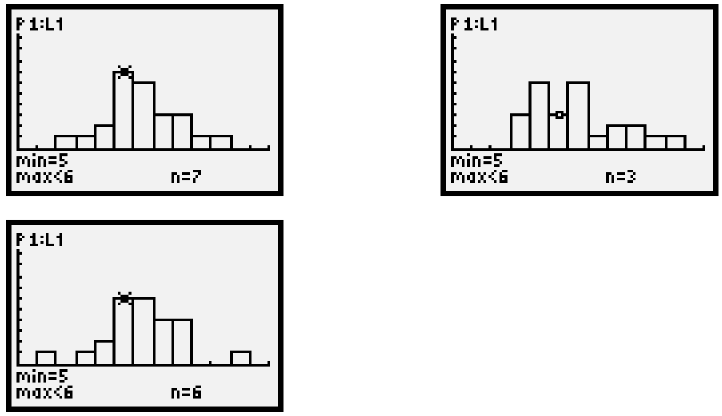
Selected Solutions — Chapter 8

Exploring with a Graphing Calculator, page 464

1. Graphs may vary. Below are 3 possible graphs.



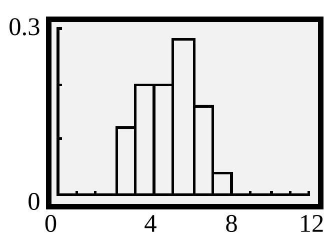
3. Tables may vary. Below is one possible table.

Number of heads	0	1	2	3	4	5	6	7	8	9	10	11	12
In 25 repetitions	0	0	0	3	5	5	7	4	1	0	0	0	0
In next 25 repetitions	0	1	0	1	2	4	5	7	4	1	0	0	0
In next 25 repetitions	0	0	0	0	3	6	7	7	1	1	0	0	0
In next 25 repetitions	0	0	0	1	5	6	7	2	3	1	0	0	0

5. Tables may vary. Below is one possible table.

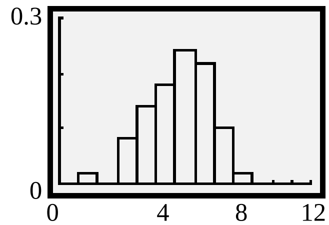
Number of heads	0	1	2	3	4	5	6	7	8	9	10	11	12
In 25 repetitions	0	0	0	3	5	5	7	4	1	0	0	0	0
In 50 repetitions	0	1	0	4	7	9	12	11	5	1	0	0	0
In 75 repetitions	0	1	0	4	10	15	19	18	6	2	0	0	0
In 100 repetitions	0	1	0	5	15	21	26	20	9	3	0	0	0

6.

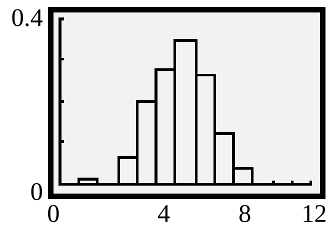


Selected Solutions — Chapter 8

7. **i)** Frequency graph for 50 repetitions



ii) Frequency graph for 75 repetitions

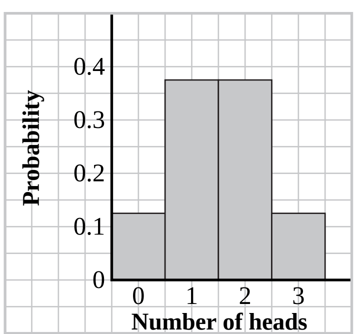


8. As the number of repetitions increases, the graph starts to look more symmetric around the tallest bar.

8.1 Exercises, page 471

2. **a)**

3.

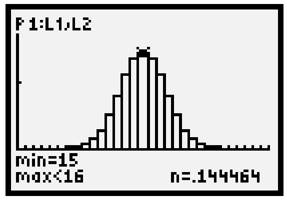


4. Both histograms are symmetrical. The histogram on page 468 has 13 bars, while my histogram has 4 bars. The sums of the areas of the bars in both histograms are 1.

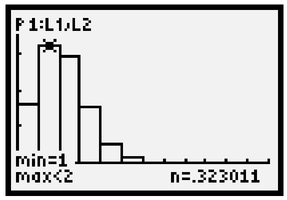
5. **a)** There would be 30 bars instead of 13, but otherwise the histogram would be similar. It would be symmetric about the greatest probability.

Selected Solutions — Chapter 8

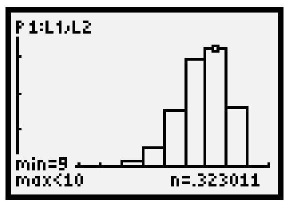
b)



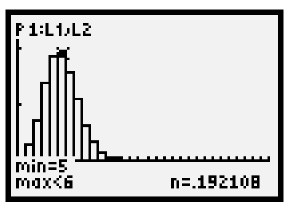
6. a)



b) The histogram would be a horizontal reflection in the 6th bar of the histogram in part a.



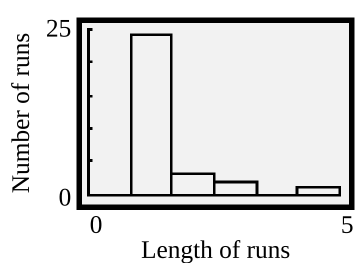
c) The histogram would become more symmetrical as the number of times the die is rolled increases. The tallest bar would be at $n \div 6$, where n is the number of rolls.



d) The probablities in both graphs lie between 0 and 1 and they add up to 1. Part a has a greater distribution of probabilities than the *Example*.

8. a) Answers may vary. Example: 24 runs of length 1, 3 runs of length 2, 2 runs of length 3, 0 runs of length 4 , and 1 run of length 5.

b)



There is greater frequency of a run of 1 head than all the others. The frequency decreases as the number of heads increases.

9. a) Answers may vary. The number of run lengths is from 0 to 100, although very imporbable outside the range of 1 to 5. The number of runs in 100 trials will vary with each experiment.

b) No. Since there are more than two results that can occur, it cannot be a binomial distribution.

10. Probability distribution must add up to 1.

For $P(x) = \frac{x}{10}$ for $x = 1, 2, 3$, and 4.

For each x value, solve the probability.

$P(1) = \frac{1}{10} = 0.10$

$P(2) = \frac{2}{10} = 0.20$

$P(3) = \frac{3}{10} = 0.30$

$P(4) = \frac{4}{10} = 0.40$

The sum of $P(1) + P(2) + P(3) + P(4)$

$= 0.10 + 0.20 + 0.30 + 0.40$

$= 1.0$

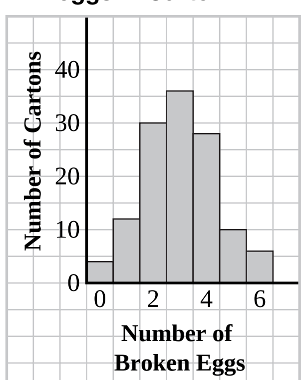
Since the probabilities all add up to 1.0, it is a probability. distribution.

Investigate, page 473

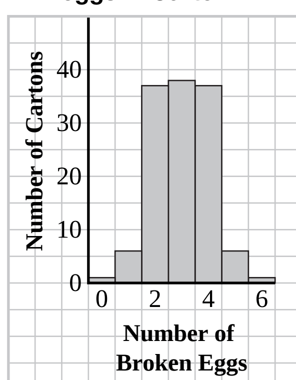
1. Answers may vary. Carton A because the number of broken eggs appear to be distributed evenly.

2.

Frequency of broken eggs in Carton A



Frequency of broken eggs in Carton B



Answers may vary. The frequencies in the graph for Carton A appear to be less.

3. The means, medians, and modes are the same for both cartons. It is impossible to tell which is better.

4. Answers may vary. No, since I choose Carton A which seemed to look like the better choice from both the chart and the graph.

Selected Solutions — Chapter 8

8.2 Exercises, page 477

9. Answers may vary. An example might be as follows.

a) 60 football player were asked their weight in kilograms. The results are listed.

85	98	96	111	82	101
92	96	92	79	96	92
77	111	85	87	92	114
85	94	72	95	110	85
94	87	112	92	115	79
115	71	110	84	120	86
105	92	92	104	87	97
86	97	85	102	98	93
104	89	102	89	89	98
118	104	98	85	92	101

b) $\text{Mean} = \sum \frac{x_i}{n}$

$= \frac{5699}{60} = 95 \text{ kg}$

$\text{Standard Deviation} = \sqrt{\sum \frac{(x_i - \mu)^2}{n}}$

$= 11.18$

The average or mean weight of the 60 football players is about 95 kg.

The standard deviation is the measure of how the 60 weights are spread out around 59 kg or the mean of the weights.

Investigate, page 479

The table from exercise 5 on page 465 is shown below.

Number of heads	0	1	2	3	4	5	6
In 25 repetitions	0	0	0	3	5	5	7
In 50 repetitions	0	1	0	4	7	9	12
In 75 repetitions	0	1	0	4	10	15	19
In 100 repetitions	0	1	0	5	15	21	26

Number of heads	7	8	9	10	11	12
In 25 repetitions	4	1	0	0	0	0
In 50 repetitions	11	5	1	0	0	0
In 75 repetitions	18	6	2	0	0	0
In 100 repetitions	20	9	3	0	0	0

1. a) From the table the total number of heads when the experiment was repeated 25 times is 132.

b) The mean number of heads that appeared when the experiment was repeated 25 times is 132 ÷ 25 = 5.28.

Selected Solutions — Chapter 8

2. a) From the table the total number of heads when the experiment was repreated 50 times is 284. The mean number of heads that appeared is $284 \div 50 = 5.68$.

b) From the table the total number of heads when the experiment was repreated 75 times is 434. The mean number of heads that appeared is $434 \div 75 = 5.79$.

c) From the table the total number of heads when the experiment was repreated 100 times is 576. The mean number of heads that appeared is $576 \div 100 = 5.76$.

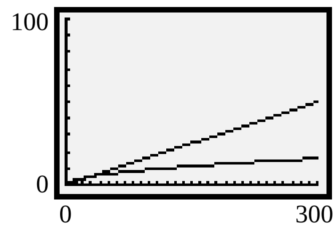
3. The mean is approaching 6. It is reasonable to expect 6 heads when tossing 12 coins, since the probability of heads is $\frac{1}{2}$.

4. $n = 12$ represents the number of times the coin is tossed and $p = 0.5$ represents the probability of a head appearing when the coin is tossed.

8.3 Exercises, page 483

5. a) Both the mean and standard deviation will increase.

c) For part a:



As the number of experiments increases, the graph of the mean increases at a steady rate while the graph of the standard deviation levels off.

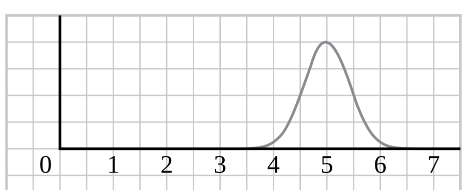
8.4 Exercises, page 488

1. a) Part ii: The masses of the posts should not vary much, so they would have a small standard deviation.

b) Part iii: The ages of people would be quite spread out, so they would have a larger standard deviation.

c) Part i: The marks would be such that most people would get about 50% on the test, as in part i.

2. a)

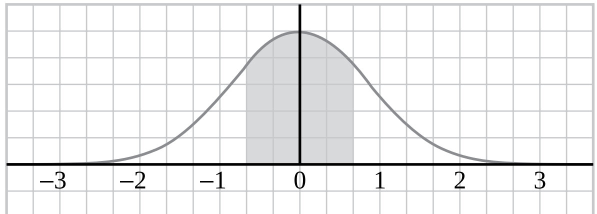


Selected Solutions — Chapter 8

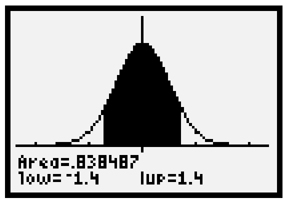
8.5 Exercises, page 492

3. Diagrams may vary.

For exercise 1, part f:



For exercise 2, part c:



4. **a)** The area approaches 0 as the range gets smaller and smaller.

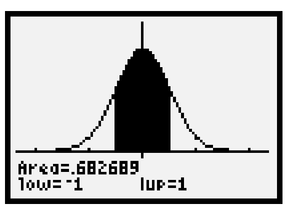
b) The area approaches 0 as the range gets smaller and smaller.

c) The area for $a < z < b$ is the same as the area for $-b < z < -a$.

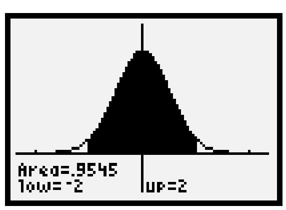
d) The area for $z < a$ plus the area for $z > a$ is equal to 1. The area for $z < a$ is equal to the area for $z > a$.

5. Answers may vary. Using a graphing calculator:

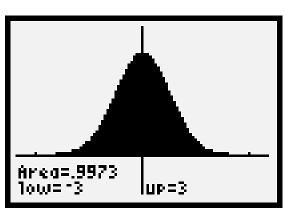
Find the area for $-1 < z < 1$ using ShadeNorm. It is 0.683, or about 68%.



Find the area for $-2 < z < 2$ using ShadeNorm. It is 0.954, or about 95%.

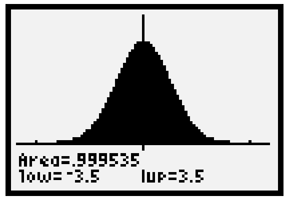


Find the area for $-3 < z < 3$ using ShadeNorm. It is 0.997, or 99.7%.



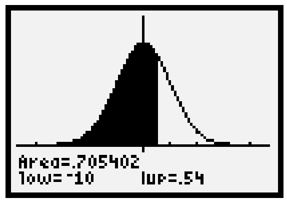
Selected Solutions — Chapter 8

9. b)

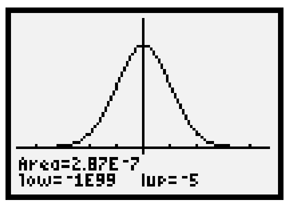


c) Answers may vary. The graph is a standard normal curve. The graph is symmetric about the mean. The mean is 0. The total shaded area is about 99.95%.

10. a) Use ShadeNorm(–10,0.54) to find the area, which is 0.7054, the same as on page 491.

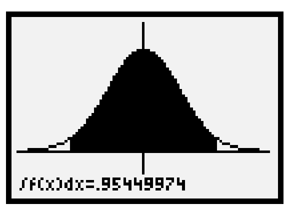


b) Use ShadeNorm(–10^99,–5) to find the area. Notice that the calculator screen does not show any shading that is because the area for this region, which is 2.87×10^{-7}, is a very small amount.

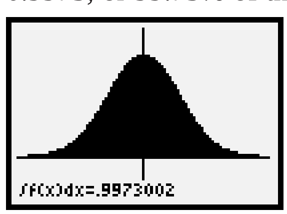


Exploring with a Graphing Calculator, page 496

3. a) i) Graph the standard normal curve as in exercise 1. Press CALC **7**. Press **–2** ENTER, then **2** ENTER. The area is about 0.9545, or 95.45% of the total area.

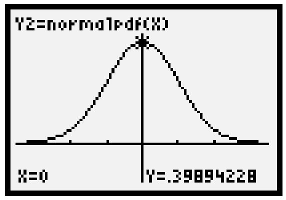


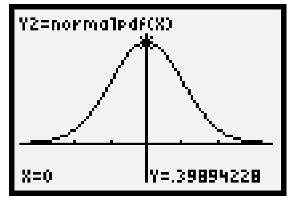
ii) Press CALC **7**. Press **–3** ENTER, then **3** ENTER. The area is about 0.9973, or 99.73% of the total area.



Selected Solutions — Chapter 8

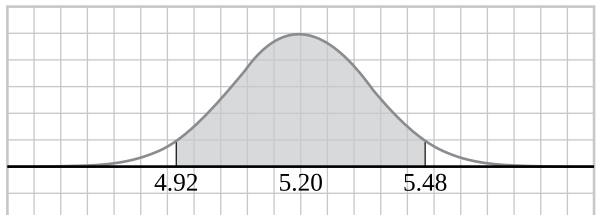
4. Notice that the two graphs are the same.





8.6 Exercises, page 501

2. a)



Modelling Distribution of Body Temperatures

- The sample size may be too small to be normally distributed.
- If a much larger sample was involved, the temperatures would more likely be normally distributed.
- Yes, the body temperatures in a group of 148 could be exactly normally distributed but it would be highly coincidental.

6. No. Using the *z*-scores for the range of men's heights, we find that the men's heights cover 97.56% of the population. Using the *z*-scores for the range of women's heights, we find that the women's heights cover only 94.061% of the population. Also the range for women's heights are within two standard deviations of the mean while the men's heights are not.

13. The curve is an area curve, so at one point *a*, the area is 0. It is just a representation of probabilities, so probabilities for one point cannot be found.

Mathematical Modelling, page 504

1. $z = \frac{x - \mu}{\sigma}$

$= \frac{50 - 46}{14}$

$\doteq 0.29$

$P(x > 50) = P(z > 0.29)$

Use the tables on pages 494 and 495, or a graphing calculator.

$P(z > 0.29) = 1 - P(z < -0.29)$

$= 1 - 0.6141$

$= 0.3859$

About 38.6% of the students had scores of 50% or more.

Selected Solutions — Chapter 8

2. **a)** The mean is increased by 10 to 56, but the standard deviation stays at 14.

b) $z = \frac{50 - 56}{14}$

$\doteq -0.43$

$P(x > 50) = P(z > -0.43)$

Use the tables on pages 494 and 495 or a graphing calculator.

$P(z > -0.43) = 1 - P(z < -0.43)$

$= 1 - 0.3372$

$= 0.6628$

About 66% of the students had scores of 50% or more.

3. Find a for $P(z > a) = 0.90$.

$P(z < a) = 1 - P(z > a)$

$= 1 - 0.90$

$= 0.10$

Using the TI-83 graphing calculator, press DISTR **3** 0.10) ENTER to display invNorm(.1) = –1.28.

Thus $P(z < -1.28) = 0.10$.

$z = \frac{x - \mu}{\sigma}$

$-1.28 = \frac{50 - \mu}{14}$

$\mu \doteq 68$

For a mean score of 68, every score should be increased by about (68 – 46) or 22 points.

4. Answers may vary. It is unfair to the students with scores higher than 100 – 22, or 78.

5. **a)** $\mu = 62, \sigma = 10$

$z = \frac{x - \mu}{\sigma}$

$= \frac{50 - 62}{10}$

$= -1.2$

$P(x > 50) = P(z > -1.2)$

$= 1 - P(z < -1.2)$

$= 1 - 0.1151$

$= 0.8849$

88.49% of the scores are passes.

b) **i)** $z = \frac{55 - 46}{14}$

$\doteq 0.6429$

$P(x < 55) = P(z < 0.6429)$

$= 0.7399$

73.99% of the raw scores are less than 55.

Selected Solutions — Chapter 8

ii) For the converted scores, $\mu = 62$ and $\sigma = 10$.

$$z = \frac{x - \mu}{\sigma}$$
$$0.6429 = \frac{x - 62}{10}$$
$$x = 68.429$$

The converted score is about 68%.

c) i)
$$z = \frac{35 - 46}{14}$$
$$\doteq -0.7857$$
$$-0.7857 = \frac{x - 62}{10}$$
$$x = 54.143$$

The converted score is about 54%.

ii)
$$z = \frac{75 - 46}{14}$$
$$\doteq 2.0714$$
$$2.0714 = \frac{x - 62}{10}$$
$$x = 82.714$$

The converted score is about 83%.

d) The lower the score, the more the score increases, which is unfair to people with higher scores.

6.
$$z = \frac{x - \mu}{\sigma}$$
$$= \frac{50 - 62}{10}$$
$$= -1.2$$
$$P(x > 50) = P(z > -1.2)$$
$$= 1 - P(z < -1.2)$$
$$= 1 - 0.1151$$
$$\doteq 0.8849$$

About 88% of the scores are passes.

7. a) Find a such that $P(z > a) = 0.95$

$$P(z < a) = 1 - P(z > a)$$
$$= 1 - 0.95$$
$$= 0.05$$

Using the TI-83 graphing calculator, press DISTR **3** 0.05) ENTER to display invNorm(.05) = –1.6449. Thus, $P(z > -1.6449) = 0.95$

$$z = \frac{x - \mu}{\sigma}$$
$$-1.6449 = \frac{50 - 62}{\sigma}$$
$$\sigma \doteq 7.295$$

The standard deviation is about 7.3.

b) The z-score for a raw score of 55 is 0.6429, from exercise 5b.

$$0.6429 = \frac{x - 62}{7.3}$$
$$x \doteq 66.69$$

The converted score is about 67%.

Selected Solutions — Chapter 8

c) i) The z-score for a raw score of 35 is –0.7857, from exercise 5c.

$$-0.7857 = \frac{x - 62}{7.3}$$
$$x \approx 56.26$$

The converted score is about 56%.

ii) The z-score for a raw score of 75 is 2.0714, from exercise 5c.

$$2.0714 = \frac{x - 62}{7.3}$$
$$x \doteq 77.12$$

The converted score is about 77%.

d) The lower the score, the more the score increases, which is unfair to people with higher scores.

8. Find z for the raw score, x, by using the formula $z = \frac{x - \mu}{\sigma}$, for μ and σ of the raw data. Find the converted score, x_c, using the formula $z = \frac{x_c - \mu_c}{\sigma_c}$, for μ_c and σ_c of the converted data, and z from above.

9. By this method, the percent score is still the same, so it does not make a difference.

10. a) The z-score for a raw score of 55 is 0.6429, from exercise 5b.

$$0.6429 = \frac{x - 500}{200}$$
$$x = 628.58$$

The converted score is 628.

b) i) The z-score for a raw score of 35 is –0.7857, from exercise 5c.

$$-0.7857 = \frac{x - 500}{200}$$
$$x = 342.86$$

The converted score is 342.

ii) The z-score for a raw score of 75 is 2.0714, from exercise 5c.

$$2.0714 = \frac{x - 500}{200}$$
$$x = 914.29$$

The converted score is 914.

c) $z = \frac{400 - 500}{200}$

$= -0.5$

$P(z > -0.5) = 0.6914$

69.14% of the converted scores are over 400.

11. a) Find a for $P(z > a) = 0.90$.

$$P(z < a) = 1 - P(z > a)$$
$$= 1 - 0.90$$
$$= 0.10$$

Using the TI-83 graphing calculator, press DISTR **3** 0.10) ENTER to display invNorm(.1) = –1.28. Thus $P(z < -1.28) = 0.10$.

$$z = \frac{x - \mu}{\sigma}$$
$$-1.28 = \frac{400 - 500}{\sigma}$$
$$\sigma = 78.125$$

The standard deviation is 78.

Selected Solutions — Chapter 8

b) For a raw score of 55, $z = 0.6429$.

$$0.6429 = \frac{x - 500}{78}$$
$$x \doteq 550.15$$

The converted score is about 550.

c) i) For a raw score of 35, $z = -0.7857$.

$$-0.7857 = \frac{x - 500}{78.125}$$
$$x \doteq 438.62$$

The converted score is about 439.

ii) For a raw score of 75, $z = 2.0714$.

$$2.0714 = \frac{x - 500}{78.125}$$
$$x \doteq 661.83$$

The converted score is about 662.

12. Explanations may vary. The raw score is converted to a *z*-score using the mean and standard deviation of the raw data. The converted score is found using the *z*-score and the mean and standard deviation of the converted data.

8.7 Exercises, page 510

4. d) Explanations may vary. The more times an experiment is repeated the closer it conforms to expected probabilities.

5. d) Explanations may vary. The more times an experiment is repeated the closer it conforms to expected probabilities.

6. Explanations may vary. For exercise 5:

a) It would make a lot of difference for part a, a little bit of difference for part b, and no difference for part c. Therefore, as the number of trials increase the less the affect there would be on the estimated probabilities if the continuity correction were not applied.

b) For part a: The probability is 27% instead of 40%. For part b: The probability is 73% instead of 75%. For part c: The probability is about the same.

c) The continuity correction has less of an effect as the number of repetitions increases.

Selected Solutions — Chapter 8

Modelling a Binomial Distribution with a Normal Distribution, page 511

- Problems where the probability is known are appropriate for using a normal model.
- A normal model gives only approximate results because it does not follow the histogram of the binomial distribution exactly.
- Answers may vary.

  For exercise 1a:

  $n = 400,\ p = 0.4$

  $np = (400)(0.4)$
  $= 160 > 5$

  $n(1 - p) = (400)(1 - 0.4)$
  $= (400)(0.6)$
  $= 240 > 5$

  For exercise 1c:

  $p = 0.1, n = 120$

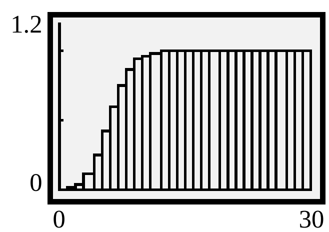
  $np = (120)(0.1)$
  $= 12 > 5$

  $n(1 - p) = (120)(1 - 0.1)$
  $= (120)(0.9)$
  $= 108 > 5$

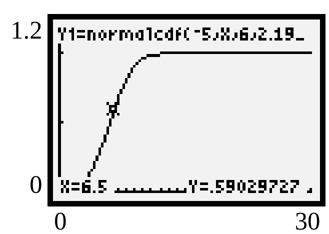
  From these two examples, we can see that a normal model is used.
- This implies that the number of repetitions is small, so an approximation will not work well.

10. **a)** For x small, the function has small values, which approach 1 as x increases.

b)



c)



Problem Solving, page 512

1. **a)** **i)** The graph is stretched horizontally by a factor of σ.

ii) For the area to remain the same, the graph must be compressed vertically by a factor of σ.

b) The graph is translated μ units right.

c) $y = \frac{1}{\sigma} f\left(\frac{x - \mu}{\sigma}\right)$

Selected Solutions — Chapter 8

2. b) Distribution 1: $n = 30, p = \frac{1}{6}$

$$\begin{aligned}\mu &= np \\ &= (30)\left(\frac{1}{6}\right) \\ &= 5 \\ \sigma &= \sqrt{np(1-p)} \\ &= \sqrt{(30)\left(\frac{1}{6}\right)\left(1-\frac{1}{6}\right)} \\ &= \sqrt{(5)\left(\frac{5}{6}\right)} \\ &= \frac{5\sqrt{6}}{6}\end{aligned}$$

Distribution 2: $n = 30, p = \frac{5}{6}$

$$\begin{aligned}\mu &= np \\ &= (30)\left(\frac{5}{6}\right) \\ &= 25 \\ \sigma &= \sqrt{np(1-p)} \\ &= \sqrt{(30)\left(\frac{5}{6}\right)\left(1-\frac{5}{6}\right)} \\ &= \sqrt{(25)\left(\frac{1}{6}\right)} \\ &= \frac{5\sqrt{6}}{6}\end{aligned}$$

c) The means are different because the probabilities are different. The standard deviations are the same because $p_1 = 1 - p_2$.

d) $y = f(x - 20)$; The graph is translated 20 units right. Since the standard deviation is the same, there is no compression or expansion.

3. The equation of the normal curve is $y = \frac{1}{2\pi}e^{-\frac{x^2}{2}}$.

a) $y = \frac{1}{2\pi\sigma}e^{-\frac{((x-\mu)/\sigma)^2}{2}}$

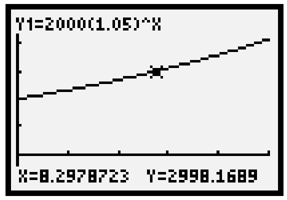
b)
$$\begin{aligned}y &= \frac{1}{2\pi\left(\frac{5}{\sqrt{6}}\right)}e^{-\frac{((x-5)/(5\sqrt{6}))^2}{2}} \\ &= \frac{\sqrt{6}}{10\pi}e^{-\frac{3(x-5)^2}{25}}\end{aligned}$$

$$\begin{aligned}y &= \frac{1}{2\pi\left(\frac{5}{\sqrt{6}}\right)}e^{-\frac{((x-25)/(5\sqrt{6}))^2}{2}} \\ &= \frac{\sqrt{6}}{10\pi}e^{-\frac{3(x-25)^2}{25}}\end{aligned}$$

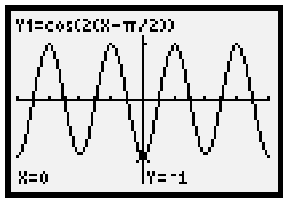
Selected Solutions — Chapter 8

8 Cumulative Review, page 514

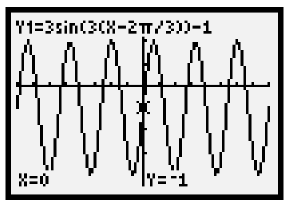
2. b)



9. a)



b)



20. a)