Selected Solutions — Chapter 7

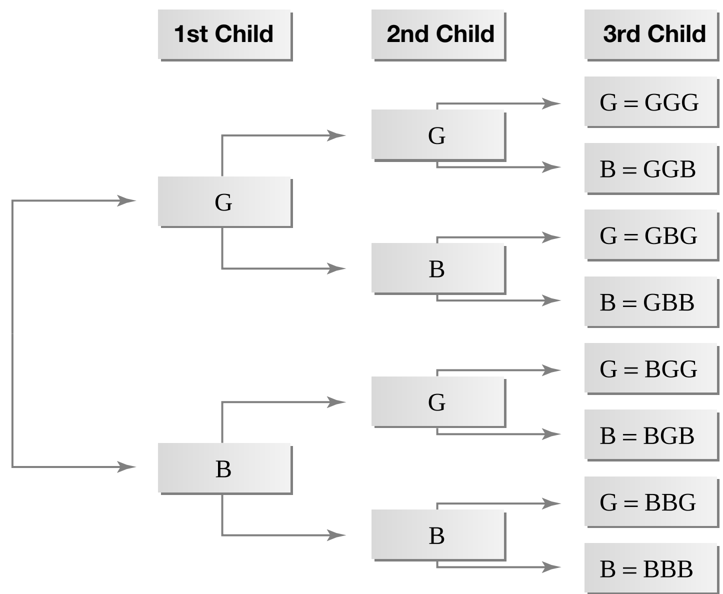
Investigate, page 402

Answers may vary.

In a family with 3 children, I guess the probability of having 2 girls is $\frac{1}{6}$.

1. Four students in my class have 2 other siblings.
   Only 1 of these families have exactly 2 girls.
   The probability from these results is $\frac{1}{4}$.
   This probability is larger than my guess.

2. The results of the coin tosses were HTH, HHT, THT, TTH, HHH, THH, TTH.
   The relative frequency of 2 girls was $\frac{3}{7}$.
   This is larger than my guess.

3. 

   The number of branches that contain girls are 3.
   The total number of branches are 8.
   The probability of 2 girls in a family of 3 children is $\frac{3}{8}$.
   The probability is larger than my guess, but smaller than the results in my class.
   You are assuming that there is an equal chance of having a boy or a girl each time a child is born.

7.1 Exercises, page 405

4. **c)** Answers may vary. The probabilities should be quite close.

5. **b)** It is just as likely. The probability of tails on the 5th toss is not affected by the results of the first 4 tosses.

Modelling the Probability of Heads or Tails, page 406

- The lighter side, since the heavier side will tend toward the bottom.
- There is another possible outcome: The coin could land on its edge.

Selected Solutions — Chapter 7

7. **a)** I would assign the numbers 1, 2, and 3 to the first outcome, and 4, 5, and 6 to the second outcome.

b) I would assign the numbers 1 and 2 to the first outcome, 3 and 4 to the second outcome, and 5 and 6 to the third outcome.

c) I would disregard the outcomes 5 and 6. 1, 2, 3, and 4 are the four outcomes.

d) I would disregard the outcome 6. 1, 2, 3, 4, and 5 are the five outcomes.

8. **b)** Answers may vary. A possible alternative could be to use two different coins. 0 heads represents the outcome 1. A head on the first coin and a tail on the second coin represents the outcome 2. A tail on the first coin and a head on the second coin represents the outcome 3. 2 heads represents the outcome 4.

10. Answers may vary. My experimental results were:

a) $\frac{146}{300}$, or 0.49 **b)** $\frac{72}{300}$, or 0.24 **c)** $\frac{80}{300}$, or 0.27 **d)** $\frac{22}{300}$, or 0.07

11. **b)** I assume that I am equally likely to draw each card.

13. **a)** The randBin instruction is randBin$\left(1, \frac{1}{6}, 4\right)$.
Let the 1 represent the number 6 on the die.
My experimental results for each set of rolls were:
0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 2, 2
The experimental probability of getting a 6 is $\frac{10}{60} = \frac{1}{6}$.

b) The randBin instruction is randBin$\left(1, \frac{1}{6}, 48\right)$.
Let the 1 represent the number 6 on the die.
Take the numbers in pairs, the first number being the first die, and the second number, the other die.
My experimental results for pairs of 6s in 24 rolls were: 1, 0, 2, 2, 1
The experimental probability of getting pairs of 6s is $\frac{6}{120} = \frac{1}{20}$

Linking Ideas, page 408

2. **a)** The pennies would decrease by about half after each toss, since P(heads) = $\frac{1}{2}$.

b) The graph would decrease exponentially.

3. **b)** It is a curve of best fit, so it does not necessarily pass through all the points.

4. **a)** a would always be about 100, and b would always be about 0.5.

b) The graph would look approximately like the graph of $y = 100x^{\frac{1}{2}}$.

5. The value of b would remain the same, but a would be approximately 100, 1000, and 10 000.

6. **a)** The graph would look like the graph of $y = ax^{\frac{1}{2}}$.

7. **b)** Each toss is a “half-life.”

10. They have an initial value and a fractional decay rate like the radioactive decay situations. They use tosses of coins or throws of dice instead of time.

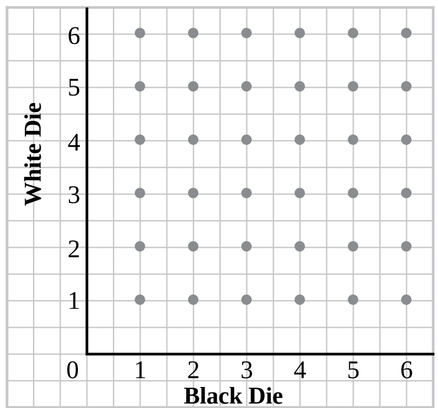
Selected Solutions — Chapter 7

Investigate, page 410

1.

		White die					
		1	2	3	4	5	6
Black die	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

2.



7.2 Exercises, page 414

3. The complement is the probability of no precipitation.

8. **b)** Tossing a coin has 2 possible outcomes H or T. Every time another coin is added to the toss, the number of possible outcomes doubles. A diagram may have been used to answer part a, for example:

		Coin 2: H	Coin 2: T
Coin 1	H	HH	HT
	T	TH	TT

17. **a)** (0, 1), (9, 0), (8, 9), (7, 8), (6, 7), (5, 6), (4, 5), (3, 4), (2, 3), (1, 2), (0, 1), (9, 0), (8, 9), (7, 8), (6, 7), (5, 6), (4, 5), (3, 4), (2, 3), (1, 2), (0, 1), (9, 0), (8, 9), (7, 8), (6, 7), (5, 6), (4, 5), (3, 4), (2, 3), (1, 2), (0, 1), (9, 0), (8, 9), (7, 8), (6, 7), (5, 6), (4, 5), (3, 4), (2, 3), (1, 2), (0, 1), (9, 0), (8, 9), (7, 8), (6, 7), (5, 6), (4, 5), (3, 4), (2, 3), (1, 2), (0, 1), (9, 0), (8, 9), (7, 8), (6, 7), (5, 6), (4, 5), (3, 4), (2, 3), (1, 2), (0, 1)

b) There are 10 different points in the sample space, one for each possible pair of lengths of time until the next trains.

c) If Anita eats with her mother, the westbound train arrives first, which means the first number in the ordered pair is greater than the second number. This occurs 6 out of 61 times. Thus, it is more likely that Anita will eat with her grandmother.

Selected Solutions — Chapter 7

18. If only A is present, the blood types are A+ and A–. If only B is present, the blood types are B+ and B–. If A and B are present, the blood types are AB+ and AB–. If neither A nor B is present, the blood types are O+ and O–. Thus, the sample space for the different blood types is A+, A–, B+, B–, AB+, AB–, O+, O–.

Mathematics File, page 417

1. a)

		Die B					
		2	2	2	2	6	6
Die A	1	W	W	W	W	W	W
	1	W	W	W	W	W	W
	1	W	W	W	W	W	W
	5	L	L	L	L	W	W
	5	L	L	L	L	W	W
	5	L	L	L	L	W	W

2. a)

		Die C					
		3	3	3	3	3	3
Die B	2	W	W	W	W	W	W
	2	W	W	W	W	W	W
	2	W	W	W	W	W	W
	2	W	W	W	W	W	W
	6	L	L	L	L	L	L
	6	L	L	L	L	L	L

3. a)

		Die D					
		4	4	4	4	0	0
Die C	3	W	W	W	W	L	L
	3	W	W	W	W	L	L
	3	W	W	W	W	L	L
	3	W	W	W	W	L	L
	3	W	W	W	W	L	L
	3	W	W	W	W	L	L

4. a)

		Die A					
		1	1	1	5	5	5
Die D	4	L	L	L	W	W	W
	4	L	L	L	W	W	W
	4	L	L	L	W	W	W
	4	L	L	L	W	W	W
	0	W	W	W	W	W	W
	0	W	W	W	W	W	W

Selected Solutions — Chapter 7

5. If A > B and B > C, then A > C.

b) If $x = 2y$ and $y = 2z$, then $x \neq y$.

c) i) If the area of a triangle $= \frac{1}{2}bh$ and $b = 2$, then $A = h$.

ii) If the area of a triangle $= \frac{1}{2}bh$ and $b = 3$, then $A \neq h$.

Problem Solving, page 418

1. Answers may vary. She should choose m-t-m, because she would play the better player only once.

2. b) t-m-t; no

3. Answers may vary.

c) t-m-t; no

4. a) win, win, lose; win, win, win; lose, win, win

b) Answers may vary. P(m) = 0.7, P(t) = 0.6

For m-t-m:
P(at least 2 consecutive wins)
= (0.7)(0.6)(0.3) + (0.7)(0.6)(0.7) + (0.3)(0.6)(0.7)
= 0.546

For t-m-t:
P(at least 2 consecutive wins)
= (0.6)(0.7)(0.4) + (0.6)(0.7)(0.6) + (0.4)(0.7)(0.6)
= 0.588

c) She should choose t-m-t. This does not agree with my guess.

5. If you divide both equations by (0.6)(0.7), you are left with 0.3 + 0.7 + 0.3 or 1.3 for the first calculation, and 0.4 + 0.6 + 0.4 or 1.4 for the second calculation. Thus, the second calculation results in a greater probability.

6. a) For m-t-m:

$$\begin{aligned}\text{P(at least 2 consecutive wins)} &= mt(1-m) + mtm + (1-m)tm \\ &= mt - m^2t + m^2t + mt - tm^2 \\ &= 2mt - m^2t \\ &= mt(2-m)\end{aligned}$$

For t-m-t:

$$\begin{aligned}\text{P(at least 2 consecutive wins)} &= tm(1-t) + tmt + (1-t)mt \\ &= tm - t^2m + t^2m + mt - mt^2 \\ &= 2mt - mt^2 \\ &= mt(2-t)\end{aligned}$$

b)

$$\begin{aligned} m &> t \\ -m &< -t \\ 2-m &< 2-t \\ mt(2-m) &< mt(2-t)\end{aligned}$$

Thus, Suk-Yee is more likely to win the second series: t-m-t.

8. Answers may vary. I'd have to include another probability of drawing against each player.

Selected Solutions — Chapter 7

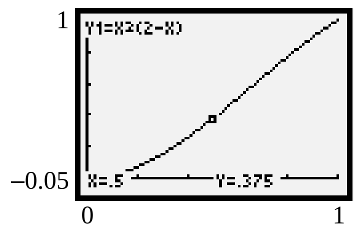
9. For m-m-m:

$$\begin{aligned}\text{P(at least 2 consecutive wins)} &= mm(1-m) + mmm + (1-m)mm \\ &= m^2 - m^3 + m^3 + m^2 - m^3 \\ &= 2m^2 - m^3 \\ &= m^2(2-m)\end{aligned}$$

For t-t-t:

$$\begin{aligned}\text{P(at least 2 consecutive wins)} &= tt(1-t) + ttt + (1-t)tt \\ &= t^2 - t^3 + t^3 + t^2 - t^3 \\ &= 2t^2 - t^3 \\ &= t^2(2-t)\end{aligned}$$

Draw a graph of $f(x) = x^2(2-x)$ for appropriate values of x.



Note that $f(m) > f(t)$ for all values of m and t such that $m > t$ and m and t are between 0 and 1.

7.3 Exercises, page 423

11. **a)** A and B refers to the regions that are in both A and B. Regions 3 and 5 are in both A and B.

b) Regions 6 and 5 are in both A and C.

c) Regions 5 and 7 are in both B and C.

d) A or B refers to the regions that are in A, the regions that are in B, and the regions that are in both. Regions 2, 3, 4, 5, 6, and 7 are included in A or B.

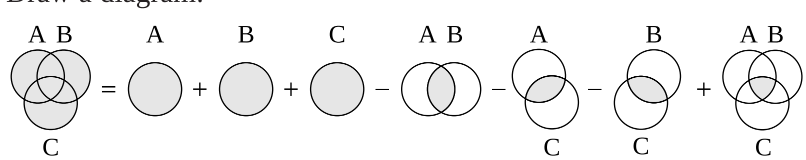
e) Regions 3, 4, 5, 6, 7, and 8 are included in B or C.

f) Regions 2, 3, 5, 6, 7, and 8 are included in A or C.

g) A and B and C refers to the regions that belong to all three of A, B, and C. Region 5 is included in A and B and C.

h) A or B or C refers to the regions that are in A, the regions that are in B, and the regions that are in C. Regions 2, 3, 4, 5, 6, 7, and 8 are included in A or B or C.

12. Draw a diagram:



From the diagram,

P(A or B or C)

= P(A) + P(B) + P(C) – P(A and B) – P(A and C)

– P(B and C) + P(A and B and C)

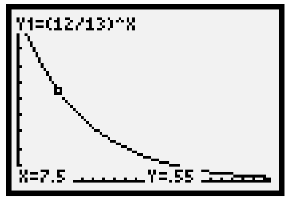
Selected Solutions — Chapter 7

7.4 Exercises, page 429

7. **b)** The sum is 1 because these are all the possible outcomes.

15. **b)** The sum is 1 because these are all the possible outcomes.

21. **b)** 1.0



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0 47

The graph has the same general shape, but decreases more slowly.

29. **a)** Let P(A) be the probability of at least one pair of sixes. Then $P(\overline{A})$ is the probability of no pairs of sixes. The probability of not getting a pair of sixes in one roll is $\frac{35}{36}$.

$$\begin{aligned} P(A) &= 1 - P(\overline{A}) \\ &= 1 - \left(\frac{35}{36}\right)^{24} \\ &\doteq 0.49 \end{aligned}$$

b) Obtaining at least 1 six in 4 rolls is more likely than obtaining at least 1 pair of sixes in 24 rolls.

30. **a)** P(Drawing 1 white ball from Bag 1) = $\frac{2}{5}$

P(Drawing 1 white ball from Bag 2) = $\frac{4}{9}$

P(Drawing 1 white ball from each) = $\left(\frac{2}{5}\right)\left(\frac{4}{9}\right)$

$= \frac{8}{45}$

b) P(Drawing 1 red ball from Bag 1) = $\frac{3}{5}$

P(Drawing 1 red ball from Bag 2) = $\frac{5}{9}$

P(Drawing 1 red ball from each) = $\left(\frac{3}{5}\right)\left(\frac{5}{9}\right)$

$= \frac{15}{45}$

$= \frac{1}{3}$

c) You have to consider two possibilities: drawing a white ball from the first bag and a red ball from the second bag, and a red ball from the first bag and a white ball from the second bag.

P(Drawing 1 white ball from Bag 1 and 1 red ball from Bag 2)

$= \left(\frac{2}{5}\right)\left(\frac{5}{9}\right) = \frac{10}{45}$

P(Drawing 1 red ball from Bag 1 and 1 white ball from Bag 2)

$= \left(\frac{3}{5}\right)\left(\frac{4}{9}\right) = \frac{12}{45}$

P(Drawing 1 red, 1 white ball) = $\frac{10}{45} + \frac{12}{45}$

$= \frac{22}{45}$

Selected Solutions — Chapter 7

Mathematical Modelling, page 434

1. **a)** People with cancer = 0.5% of 1 000 000
= 0.005 × 1 000 000
= 5000

b) People without cancer = 1 000 000 – 5000
= 995 000

2. **a)** positive test = 98% of 5000
= 0.98 × 5000
= 4900

b) negative test = 5000 – 4900
= 100

3. **a)** negative test = 98% of 995 000
= 0.98 × 995 000
= 975 100

b) positive test = 995 000 – 975 100
= 19 900

4. **a)** positive test total = 4900 – 19 900
= 24 800

b) 4900, from exercise 2a

c) P(cancer having tested positive) $= \frac{4900}{24\ 800}$
$\doteq 0.1976$
$= 19.76\%$

5. **b)**

Cell	Formula	Amount
C2	=B1∗1000000	5 000
C3	=1000000–C2	995 000
D5	=D1∗C2	4 900
D6	=C2–D5	100
D7	=D1∗C3	975 100
D8	=C3–D7	19 900
D10	=D5+D8	24 800
D11	=D6+D7	975 200
D13	=D5/D10	0.1976
D14	=1–D13	0.802
D15	=D6/D11	0.0001
D16	=1–D15	0.9999

c) **i)** The probabilities in cells D14 and D15 would increase, and those in cells D13 and D16 would decrease.

ii) The probabilities in cells D14 and D15 would decrease, and those in cells D13 and D16 would increase. At 100% accuracy, D14 and D15 would be equal to 0, and D13 and D16 would be equal to 1.

6. This is the result from cell D15, or 0.0001.

Selected Solutions — Chapter 7

7.5 Exercises, page 439

4. a) The probability of a defective chip would decrease, because the production at the more reliable factory (factory 2) would increase. The probability of a defective chip coming from factory 1 would decrease, for the same reason.

b) The probability of a defective chip would increase, and it would be even more likely to come from factory 1, since it would be producing even more defective chips.

c) The probability of a defective chip would increase, and the probability of it coming from factory 2 would increase, since it would be producing more defective chips.

11. Represent these events:
A: A person has cancer.
$\overline{A}$: A person does not have cancer.
B: A person tests positive for cancer.

$$P(A) = 0.005$$
$$P(\overline{A}) = 0.995$$
$$\begin{aligned} P(B) &= P(A) \times P(B|A) + P(\overline{A}) \times P(B|\overline{A}) \\ &= 0.005 \times 0.98 + 0.995 \times 0.02 \\ &= 0.0248 \end{aligned}$$
$$\begin{aligned} P(A|B) &= \frac{P(A) \times P(B|A)}{P(B)} \\ &= \frac{0.005 \times 0.98}{0.0248} \\ &\doteq 0.1976 \\ &= 19.76\% \end{aligned}$$

The probability that a person who tests positive for cancer has cancer is about 20%.

12. Represent these events:
A: A person has glaucoma.
$\overline{A}$: A person does not have glaucoma.
B: A person tests negative for glaucoma.
C: A person tests positive for glaucoma.

$$P(A) = 0.008$$
$$P(\overline{A}) = 0.992$$

a) $$\begin{aligned} P(B) &= P(A) \times P(B|A) + P(\overline{A}) \times P(B|\overline{A}) \\ &= 0.008 \times 0.05 + 0.992 \times 0.95 \\ &= 0.9428 \end{aligned}$$

The probability that a randomly selected person will test negative is about 94%.

b) $$\begin{aligned} P(A|B) &= \frac{P(A) \times P(B|A)}{P(B)} \\ &= \frac{0.008 \times 0.05}{0.9428} \\ &\doteq 0.0004 \\ &= 0.04\% \end{aligned}$$

The probability that a person who tests negative has glaucoma is about 0.04%.

Selected Solutions — Chapter 7

c) $$\begin{aligned} P(C) &= P(A) \times P(C|A) + P(\overline{A}) \times P(C|\overline{A}) \\ &= 0.008 \times 0.95 + 0.992 \times 0.05 \\ &= 0.0572 \end{aligned}$$

$$\begin{aligned} P(\overline{A}|C) &= \frac{P(\overline{A}) \times P(C|\overline{A})}{P(C)} \\ &= \frac{0.992 \times 0.05}{0.0572} \\ &\doteq 0.8671 \\ &= 86.71\% \end{aligned}$$

The probability that a person who tests positive does not have glaucoma is about 87%.

7.6 Exercises, page 444

24. P(at least 2 balls are yellow)
= P(2 balls are yellow) + P(3 balls are yellow)
For 2 yellow balls and 1 white ball, there are ${}_3C_2$ or 3 combinations.
For 3 yellow balls, there is 1 combination.
P(at least 2 balls are yellow)
= 3(0.3)(0.3)(0.7) + (0.3)(0.3)(0.3)
= 0.216
The probability is 21.6%.

25. Yes. Since the outcomes are independent, their probabilities can be multiplied.

Exploring with a Graphing Calculator, page 448

1. a) The numbers 4 and 5 seem to appear most frequently. This is because the probability of success is 50%. The numbers 2 and 7 appear less frequently.

b) The same results are likely. Since the probability of heads is 50%, heads are likely to appear about half the time.

7.7 Exercises, page 457

2. The sum is 1. This is true because $\left(\frac{1}{6} + \frac{5}{6}\right)^6 = 1^6 = 1$.

18. For exercise 15, the individual results would be greater if P(heads) = 0.6, but the probabilities would still decrease as the number of tosses increased.
For exercise 16, the graph would look similar, but would have a greater initial value.
For exercise 17, the individual results would be greater.

Selected Solutions — Chapter 7

Modelling Answering Multiple-Choice Tests, page 459

- It assumes random guessing.
- No. A person cannot guess totally randomly.
- Answers may vary. It depends on how well you know the subject being tested.

20. To win the playoffs in 5 games, the Cougars must win 4 of the games and lose 1. The possible scenarios are: LWWWW, WLWWW, WWLWW, WWWLW.

$$P(\text{winning in 5 games}) = 4(0.32)(0.68)^4 \doteq 0.274 = 27.4\%$$

The probability of winning in 5 games is about 27%.

21. To lose the playoffs, the Bulldogs must lose 4 of the games.
The probability of losing the first 4 games is ${}_4C_4\,(0.32)^4$.
For the series to go to 5 games, they must lose the last game. Thus there will be 3 losses and 1 win in the first 4 games, for ${}_4C_3$ combinations. The probability of losing in 5 games is ${}_4C_3(0.32)^4(0.68)$.
For the series to go to 6 games, they must lose the last game. Thus there will be 3 losses and 2 wins in the first 5 games, for ${}_5C_3$ combinations. The probability of losing in 6 games is ${}_5C_3(0.32)^4(0.68)^2$.
For the series to go to 7 games, they must lose the last game. Thus there will be 3 losses and 3 wins in the first 6 games, for ${}_6C_3$ combinations. The probability of losing in 7 games is ${}_6C_3(0.32)^4(0.68)^3$.

P(losing)

$$= {}_4C_4(0.32)^4 + {}_4C_3(0.32)^4(0.68) + {}_5C_3(0.32)^4(0.68)^2 + {}_6C_3(0.32)^4(0.68)^3$$

$$\doteq 0.153$$

The probability that the Bulldogs lose the playoffs is about 15%.

22. a) Yes, you are more likely to pass a test with few questions. For example, the probability of passing a test with 4 questions is

$$({}_4C_2 + {}_4C_3 + {}_4C_4)\left(\frac{1}{2}\right)^4 = 0.6875.$$

The probability of passing a test with 10 questions is

$$({}_{10}C_5 + {}_{10}C_6 + {}_{10}C_7 + {}_{10}C_8 + {}_{10}C_9 + {}_{10}C_{10})\left(\frac{1}{2}\right)^{10} \doteq 0.6230.$$

b) Yes. For example, if the test has 5 questions, the probability of passing is

$$({}_5C_3 + {}_5C_4 + {}_5C_5)\left(\frac{1}{2}\right)^5 = 0.5.$$ If the test has 6 questions, the probability of passing is

$$({}_6C_3 + {}_6C_4 + {}_6C_5 + {}_6C_6)\left(\frac{1}{2}\right)^6 = 0.656\ 25.$$

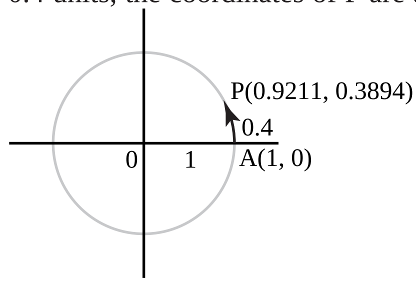
The probability increases when the number of questions increases from 5 to 6.

Selected Solutions — Chapter 7

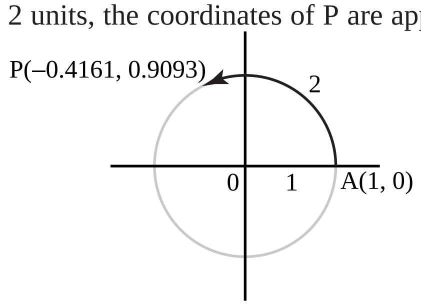
7 Cumulative Review, page 461

6. Ensure your calculator is in radian mode.

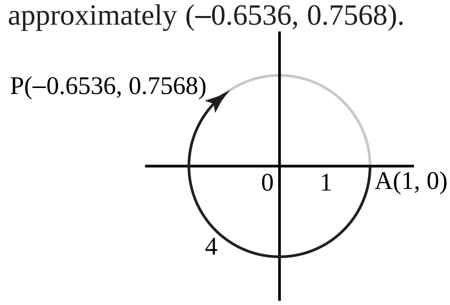
a) $\sin 0.4 \doteq 0.3894$ and $\cos 0.4 \doteq 0.9211$
This means that when the length of arc AP in a unit circle is 0.4 units, the coordinates of P are approximately (0.9211, 0.3894).



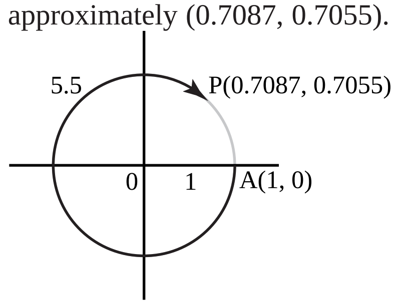
b) $\sin 2 \doteq 0.9093$ and $\cos 2 \doteq -0.4161$
This means that when the length of arc AP in a unit circle is 2 units, the coordinates of P are approximately (–0.4161, 0.9093).



c) $\sin(-4) \doteq 0.7568$ and $\cos(-4) \doteq -0.6536$
This means that when the length of arc AP in a unit circle is 4 units in the clockwise direction, the coordinates of P are approximately (–0.6536, 0.7568).



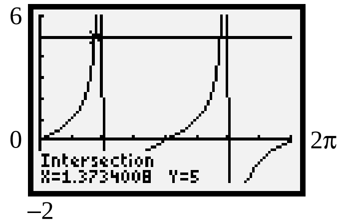
d) $\sin(-5.5) \doteq 0.7055$ and $\cos(-5.5) \doteq 0.7087$
This means that when the length of arc AP in a unit circle is 5.5 units in the clockwise direction, the coordinates of P are approximately (0.7087, 0.7055).



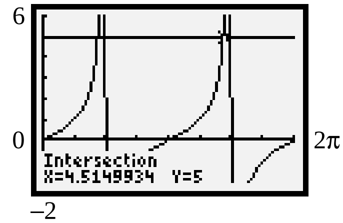
Selected Solutions — Chapter 7

7. a) Graph the functions $y = \tan x$ and $y = 5$ on the same screen. There are two roots in the interval $0 \le x \le 2\pi$. Each root of the equation $\tan x = 4$ is the x-coordinate of the point of intersection of the graphs of the two functions. Activate the intersect feature.

One root is $x \doteq 1.3734$.



Another root is $x \doteq 4.5150$.



Check.

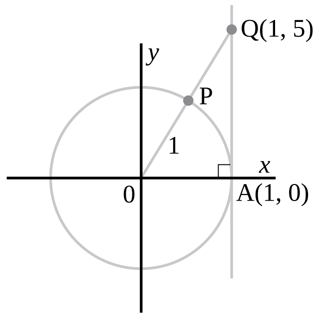
$\tan x = 5$

When $x \doteq 1.3734$, L.S. $= \tan 1.3734$ $\doteq 5$ R.S. $= 5$

When $x \doteq 4.4674$, L.S. $= \tan 4.5150$ $\doteq 5$ R.S. $= 5$

The roots are correct.

b) Draw a unit circle. When the length of arc AP in a unit circle is 1.3734 units or 4.5150 units, the line through 0 and P intersects the tangent lines at A(1, 0) at the point with coordinates (1, 5).



c) The tan function has a period of π. The expression representing all the roots of the equation is $x \doteq 1.3734 + n\pi$, where n is any integer.