HELP

ADDISON-WESLEY
Mathematics 12

Selected Solutions — Chapter 6

Investigate, page 354

1. c) Multiply the number of choices in each category by each other, that is, $2 \times 3 \times 2$.

6.1 Exercises, page 356

4. I assumed that they only wear the items once.

10. There are $10 \times 10 \times 10 = 1000$ ways to arrange 3 digits if the order is taken into account. Thus, the chances are 1 in 1000 of winning the Straight Play. Each set of 3 numbers can be arranged in $3 \times 2 \times 1 = 6$ ways. Divide 1000 by 6 to get the number of possibilities if the order does not matter. $1000 \div 6 \doteq 167$, so there is a 1 in 167 chance of winning the Box Play.

13. b) To find the number of combinations of licence plates that can be made with any letter A to Z or any numeral 0 to 9, I know that I multiply the choices of each digit by each other. In 13a, there are 26 letters or 10 numbers, which totals 36. The solution is $36 \times 36 \times 36 \times 36 \times 36 \times 36$. This is different from the number that would be produced because I would only have letters in three of the digits and numbers in the other 3 digits. I would calculate it to be $26 \times 10 \times 26 \times 26 \times 10 \times 10$.

16. There are 100 000 000 000 000 possible sonnets, which is probably more than the number of people that have ever lived, or even more than will ever live.

19. a) Answers may vary. “To support and stimulate the potential and prosperity of the environment department of Canada.” “To preserve and protect the competitiveness and creation of the trade department of Canada.”

20. a) For the first column: There are 15 choices for the first number, 14 for the second, 13 for the third, 12 for the fourth, and 11 for the fifth. Thus, there are $15 \times 14 \times 13 \times 12 \times 11 = 360\ 360$ different first columns. There are also 360 360 different second, fourth, and fifth columns. There are only four numbers to choose for the third column, so there are $15 \times 14 \times 13 \times 12 = 32\ 760$ different third columns. The total number of Bingo cards is the product of the numbers for the different columns: $360\ 360^4 \times 32\ 760 \doteq 5.52 \times 10^{26}$.

b) Divide the number of Bingo cards by a population of 6 billion, then by the number of seconds in an hour (3600), the number of hours in a day (24), and the number of days in a year (365), to get 2 919 660 482 years.

c) Divide the number of Bingo cards by 0.1 to get the number of millimetres, then convert to kilometres by dividing by 1 000 000, to get 5.52×10^{21} km.

Selected Solutions — Chapter 6

Investigate, page 360

1. a) List the permutations in an ordered fashion, to be sure you do not miss any, or count any possibility more than once.

b) List the permutations in an ordered fashion, to be sure you do not miss any, or count any possibility more than once.

2. a) There are 5 choices for the first person, 4 for the second, 3 for the third, 2 for the fourth, and 1 for the fifth. Thus, there are $5 \times 4 \times 3 \times 2 \times 1 = 120$ permutations.

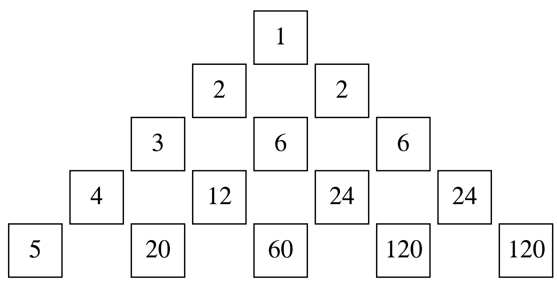
b) There are 5 choices for the first book, 4 for the second, 3 for the third, 2 for the fourth, and 1 for the fifth. Thus, there are $5 \times 4 \times 3 \times 2 \times 1 = 120$ permutations.

c) There are 5 choices for the first letter, 4 for the second, 3 for the third, 2 for the fourth, and 1 for the fifth. Thus, there are $5 \times 4 \times 3 \times 2 \times 1 = 120$ permutations.

5. c) ${}_nP_r$ is the number of permutations of n different objects taken r at a time.

6.2 Exercises, page 364

3.



The numbers along the outside on the left are 1, 2, 3, ….
The numbers along the outside on the right are 1!, 2!, 3!, ….
The numbers along the second diagonal from the right are 2!, 3!, 4!, ….
The numbers along the third diagonal from the right are $\frac{3!}{2!}$, $\frac{4!}{2!}$, $\frac{5!}{2!}$, ….
The numbers along the fourth diagonal from the right are $\frac{4!}{3!}$, $\frac{5!}{3!}$, ….

11. ${}_6P_9$ is not defined because $6 < 9$. ${}_{-6}P_3$ is not defined because –6 is not a natural number or 0 and $-6 < 9$. ${}_6P_{2.5}$ is not defined because 2.5 is not a natural number or 0.

12. If some books are the same, the same arrangement may be counted more than once.

20. Explanations may vary. For exercise 19b:
I set up the equation ${}_6P_n = 120$, which is $\frac{6!}{(6-n)!} = 120$, which simplifies to $\frac{720}{(6-n)!} = 120$, or $6 = (6-n)!$.

So $(6-n)! = 3!$
$6 - n = 3$
$n = 3$

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1. b) No, because there would be repetitions.

2. b) The number of permutations is half the number for 4 different letters.

3. b) The number of permutations is $\frac{1}{6}$ the number for 4 different letters.

4. b) The number of permutations is $\frac{1}{4}$ the number for 4 different letters.

5. The number of permutations of n objects taken n at a time, if there are a alike of one kind, b alike of another kind, c alike of another kind, and so on, is: $\frac{n!}{a!b!c!\ldots}$.

6.3 Exercises, page 368

7. 32 is the number of possible combinations of T and F answers for the test.
Explanations may vary. For exercise 8b on page 357, if there are 2 choices for each of the 5 questions, I would multiply 2 by 2 by 2 by 2 by 2 and the result would be 32 permutations. If I were to break down the question into different possibilities that the answer could be arranged, I would end up with 6 different parts, as in exercise 6. Hence, the results of exercise 6 are the same as exercise 8b.

9. I simplified $\frac{10!}{3!2!4!1!}$, since there are 10 questions, where 3 answers are A, 2 are B, 4 are C, and 1 is D.

10. b) There would be only 60 possible itineraries.

c) For part a, I simplified $\frac{7!}{2!3!}$, since there are 7 days, where 2 were spent in Edmonton and 3 in Yellowknife. For part b, the first and last days are fixed, so I only considered the 5 middle days, of which the pilot spent 1 in Winnipeg, 1 in Regina, 2 in Edmonton, and 1 in Yellowknife. Thus, there are $\frac{5!}{2!}$ or 60 possible itineraries.

11. To get from A to B, you must travel 3 blocks south and 3 blocks east for each possible path. Thus, find the number of permutations for SSSEEE. This is $\frac{6!}{3!3!}$ or 20.

12. a) To get from A to B, you must travel 4 blocks south and 4 blocks east for each possible path. Thus, find the number of permutations for SSSSEEEE. This is $\frac{8!}{4!4!}$, or 70.

b) To get from A to B, you must travel 3 blocks south and 5 blocks east for each possible path. Thus, find the number of permutations for SSSEEEEE. This is $\frac{8!}{3!5!}$, or 56.

Selected Solutions — Chapter 6

14. a) i) To get from A to B, you must travel one edge forward, one right, and one down. Thus, find the number of permutations of FRD. This is 3!, or 6.

ii) To get from A to B, you must travel one edge forward, two right, and two down. Thus, find the number of permutations of FRRDD. This is $\frac{5!}{2!2!}$, or 30.

iii) To get from A to B, you must travel two edges forward, two right, and two down. Thus, find the number of permutations of FFRRDD. This is $\frac{6!}{2!2!2!}$, or 90.

b) i) Find the number of permutations of 10 forward, 10 right, and 10 down. This is $\frac{30!}{10!10!10!}$, or about 5.55×10^{12}.

iii) Find the number of permutations of 12 forward, 8 right, and 10 down. This is $\frac{30!}{12!8!10!}$, or about 3.78×10^{12}.

15. There are 5 different sets of 7 letters, if order doesn't matter. They are KANAGAN, OANAGAN, OKNAGAN, OKAAGAN, and OKANAAN. Find the number of permutations of each set.

KANAGAN: $\frac{7!}{2!3!} = 420$

OANAGAN: $\frac{7!}{2!3!} = 420$

OKNAGAN: $\frac{7!}{2!2!} = 1260$

OKAAGAN: $\frac{7!}{3!} = 840$

OKANAAN: $\frac{7!}{2!3!} = 420$

Add the permutations for each set:
$420 + 420 + 1260 + 840 + 420 = 3360$. There are 3360 7-letter permutations of the letters in the word OKANAGAN.

6.4 Exercises, page 374

2.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

a) The numbers down the left and right sides are all 1. Each other number is the sum of the 2 numbers above and to the left and right of it. The numbers in each row read the same from left to right and from right to left.

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4. **c)** Choosing 4 students is the same as not choosing 6 students, which is the same as choosing 6 students. Also, both a and b use the same equation, namely, $\frac{10!}{6!4!}$.

15. **c)** Order matters in part b, but not in part a. For exercise 15a, I have 10 people to fill in 3 positions. To find this, I used the formula $\frac{n!}{(n-r)!r!}$, and substituted $n = 10$ and $r = 3$. I get an answer of 120 permutations. In exercise 15b, since three people have been used, I only have 7 people to fill the next three position. The equation is $\frac{7!}{4!7!}$, or 35. Therefore, the answers are different.

22. Choosing 6 numbers from 49, is less than choosing 5 numbers from 49 and 1 number from 42 because when you choose your 5 numbers and then one number there is an extra 42 times as many combinations from which to choose. This can be shown with the following calculation.

$$\begin{aligned} {}_{49}C_6 &= \frac{49 \times 48 \times 47 \times 46 \times 45 \times 44}{6 \times 5 \times 4 \times 3 \times 2} \\ &= \frac{49 \times 48 \times 47 \times 46 \times 45}{5 \times 4 \times 3 \times 2} \times \frac{44}{6} \\ &= {}_{49}C_5 \times \frac{22}{3} \end{aligned}$$

${}_{49}C_5 \times {}_{42}C_1 = {}_{49}C_5 \times 42$

Thus, ${}_{49}C_5 \times {}_{42}C_1 > {}_{49}C_6$, since $42 > \frac{22}{3}$.

23. **a)** ${}_{52}C_{26} > {}_{52}C_x$ for $x \neq 26$

b) The maximum is the number of different hands of 26 cards.

c) It is symmetrical because ${}_{52}C_x = {}_{52}C_{52-x}$.

d) A hand can contain from 0 to 52 cards.

27. There are ${}_7C_6$ or 7 ways to choose 6 numbers from 7 numbers. There are ${}_8C_6$ or $\frac{8 \times 7}{2} = 28$ ways to choose 6 numbers from 8 numbers. There are ${}_9C_6$ or $\frac{9 \times 8 \times 7}{3 \times 2} = 84$ ways to choose 6 numbers from 9 numbers.

28. **a)** Find the number of committees with 2 boys, 3 boys, 4 boys, and 5 boys, and then add them.

2 boys: $${}_8C_2 \times {}_{12}C_3 = 28 \times 220 = 6160$$

3 boys: $${}_8C_3 \times {}_{12}C_2 = 56 \times 66 = 3696$$

4 boys: $${}_8C_4 \times {}_{12}C_1 = 70 \times 12 = 840$$

5 boys: $${}_8C_5 \times {}_{12}C_0 = 56 \times 1 = 56$$

$6160 + 3696 + 840 + 56 = 10\ 752$

There are 10 752 possible committees.

b) Find the number of committees with 2 girls, 3 girls, 4 girls, and 5 girls, and then add them.

2 girls: ${}_{12}C_2 \times {}_8C_3 = 3696$ (from part a)

3 girls: ${}_{12}C_3 \times {}_8C_2 = 6160$ (from part a)

4 girls: ${}_{12}C_4 \times {}_8C_1 = 495 \times 8$
$= 3960$

5 girls: ${}_{12}C_5 \times {}_8C_0 = 792 \times 1$
$= 792$

$3696 + 6160 + 3960 + 792 = 14\ 608$

There are 14 608 possible committees.

c) There can be 3 boys and 2 girls or 2 boys and 3 girls.
3 boys and 2 girls: ${}_8C_3 \times {}_{12}C_2 = 3696$ (from part a)
2 boys and 3 girls: ${}_8C_2 \times {}_{12}C_3 = 6160$ (from part a)
$3696 + 6160 = 9856$
There are 9856 possible committees.

29. There are ${}_3C_1$ ways to vote for president, ${}_3C_1$ ways to vote for secretary, and ${}_2C_1$ ways to vote for treasurer. Find the product to find the number of ways a ballot can be marked.

$${}_3C_1 \times {}_3C_1 \times {}_2C_1 = 3 \times 3 \times 2 = 18$$

There are 18 ways a ballot can be marked.

30. If you choose the first number, there are 8 numbers that are not consecutive to it, and 7 that are not consecutive to the first or second number. Thus, there are 8×7 or 56 ways.

31. **a)** Because there are two identical letters, we must consider 2 cases.

- combinations with 2 Os
  Consider the 2 Os as one element that must be chosen. The other two letters must be chosen from the remaining 4. This can be done in ${}_4C_2 = \frac{4!}{2!2!}$, or 6 ways.
- combinations with at most 1 O
  This means that there are 5 different letters from which to choose 3. ${}_5C_3 = \frac{5!}{3!2!}$, or 10 ways.

Adding the results from both cases, there are 16 4-letter combinations of the letters of the word ONOWAY.

b) Because there are two identical letters, we must consider 2 cases.

- combinations with 2 Os
  Consider the 2 Os as one element that must be chosen. The other two letters must be chosen from the remaining 5 letters. This can be done in ${}_5C_2 = \frac{5!}{3!2!}$, or 10 ways.
- combinations with at most 1 O.
  This means that there are 6 different letters from which to choose 4. ${}_6C_4 = \frac{6!}{4!2!}$, or 15 ways.

Adding the results from both cases, there are 25 4-letter combinations of the letters of the word OSBOURNE.

c) Because there are three identical letters, we must consider 3 cases.

- combinations with 3 Os

Selected Solutions — Chapter 6

Consider the 3 Os as one element that must be chosen. The other 1 letter must be chosen from the remaining 4 letters. This can be done in ${}_4C_1 = \frac{4!}{3!}$, or 4 ways.

- combinations with 2 Os
  Consider the 2 Os as one element that must be chosen. The other 2 letters must be chosen from the remaining 4 letters. This can be done in ${}_4C_2 = \frac{4!}{2!2!}$, or 6 ways.
- combinations with at most 1 O
  This means that there are 5 different letters from which to choose 4 letters. ${}_5C_4 = \frac{5!}{4!}$, or 5 ways.

Adding the results from the three cases, there are 15 4-letter combinations of the letters in the word OUTLOOK.

32. **a)** Find the prime factors of 36. They are 2, 2, 3, 3. To find how many factors there are, determine the number of ways to select 1 number, 2 numbers, 3 numbers, and 4 numbers from the prime factors.
There are 2 ways to select 1 number: 2 or 3
There are 3 ways to select 2 numbers: 2×2, 2×3, 3×3
There are 2 ways to select 3 numbers: $2 \times 2 \times 3$, $3 \times 3 \times 2$
There is 1 way to select 4 numbers: $2 \times 2 \times 3 \times 3$
The factors are 2, 3, 4, 6, 9, 12, 18, 36, and of course, 1. There are 9 factors of 36.

b) Find the prime factors of 360. They are 2, 2, 2, 3, 3, 5. To find how many factors there are, determine the numbers of ways to select 1 number, 2 numbers, 3 numbers, 4 numbers, 5 numbers, and 6 numbers from the prime factors.
There are 3 ways to select 1 number: 2, 3, 5
There are 5 ways to select 2 numbers: 2×2, 2×3, 2×5, 3×3, 3×5
There are 6 ways to select 3 numbers: $2 \times 2 \times 2$, $2 \times 2 \times 3$, $2 \times 2 \times 5$, $2 \times 3 \times 3$, $2 \times 3 \times 5$, $3 \times 3 \times 5$
There are 5 ways to select 4 numbers: $2 \times 2 \times 2 \times 3$, $2 \times 2 \times 2 \times 5$, $2 \times 2 \times 3 \times 3$, $2 \times 2 \times 3 \times 5$, $2 \times 3 \times 3 \times 5$
There are 3 ways to select 5 numbers: $2 \times 2 \times 2 \times 3 \times 3$, $2 \times 2 \times 2 \times 3 \times 5$, $2 \times 2 \times 3 \times 3 \times 5$
There is 1 way to select 6 numbers: $2 \times 2 \times 2 \times 3 \times 3 \times 5$
The factors are 2, 3, 5, 4, 6, 10, 9, 15, 8, 12, 20, 18, 30, 45, 24, 40, 36, 60, 90, 72, 120, 180, 360, and of course, 1. There are 24 factors of 360.

c) Find the prime factors of 3600. There are 2, 2, 2, 2, 3, 3, 5, 5. To find how many factors there are, determine the numbers of ways to select 1 number, 2 numbers, 3 numbers, 4 numbers, 5 numbers, 6 numbers, 7 numbers, and 8 numbers.
There are 3 ways to select 1 number: 2, 3, 5
There are 6 ways to select 2 numbers: 2×2, 2×3, 2×5, 3×3, 3×5, 5×5
There are 8 ways to select 3 numbers: $2 \times 2 \times 2$, $2 \times 2 \times 3$, $2 \times 2 \times 5$, $2 \times 3 \times 3$, $2 \times 3 \times 5$, $2 \times 5 \times 5$, $3 \times 3 \times 5$, $3 \times 5 \times 5$

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There are 9 ways to select 4 numbers: $2 \times 2 \times 2 \times 2$, $2 \times 2 \times 2 \times 3$, $2 \times 2 \times 2 \times 5$, $2 \times 2 \times 3 \times 3$, $2 \times 2 \times 3 \times 5$, $2 \times 2 \times 5 \times 5$, $2 \times 3 \times 3 \times 5$, $2 \times 3 \times 5 \times 5$, $3 \times 3 \times 5 \times 5$

There are 8 ways to select 5 numbers: $2 \times 2 \times 2 \times 2 \times 3$, $2 \times 2 \times 2 \times 2 \times 5$, $2 \times 2 \times 2 \times 3 \times 3$, $2 \times 2 \times 2 \times 3 \times 5$, $2 \times 2 \times 2 \times 5 \times 5$, $2 \times 2 \times 3 \times 3 \times 5$, $2 \times 2 \times 3 \times 5 \times 5$, $2 \times 3 \times 3 \times 5 \times 5$

There are 6 ways to select 6 numbers: $2 \times 2 \times 2 \times 2 \times 3 \times 3$, $2 \times 2 \times 2 \times 2 \times 3 \times 5$, $2 \times 2 \times 2 \times 2 \times 5 \times 5$, $2 \times 2 \times 2 \times 3 \times 3 \times 5$, $2 \times 2 \times 2 \times 3 \times 5 \times 5$, $2 \times 2 \times 3 \times 3 \times 5 \times 5$

There are 3 ways to select 7 numbers: $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$, $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5$, $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

There is 1 way to select 8 numbers: $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

The factors are 2, 3, 5, 4, 6, 10, 9, 15, 25, 8, 12, 20, 18, 30, 50, 45, 75, 16, 24, 40, 36, 60, 100, 90, 150, 225, 48, 80, 72, 120, 200, 180, 300, 450, 144, 240, 400, 360, 600, 900, 720, 1200, 1800, 3600, and of course, 1. There are 45 factors of 3600.

33. a) Multiply the probability of scratching the correct 3 spots on the card (from exercise 8) by the probability of selecting the prize-winning card.

$\frac{1}{84} \times \frac{2}{12\ 718\ 300} = \frac{1}{534\ 168\ 600}$

b) Assume that all the cards are distributed. This means that the two \$100 000 cards have been distributed. The probability that a person with one of the prize-winning cards doesn't scratch the correct 3 spots is $\frac{83}{84}$. The probability that the two people with the prize-winning cards don't scratch the correct 3 spots is $\frac{83}{84} \times \frac{83}{84}$. This is the probability that the contest organizers will not have to award a \$100 000 prize. Thus, the probability that they will have to award a prize is $1 - \frac{83}{84} \times \frac{83}{84}$, or $\frac{167}{7056}$ (about 0.0237).

Mathematical Modelling, page 379

1. a) i) ${}_{10}C_9 = 10$

ii) ${}_{70}C_{11} \doteq 2.164 \times 10^{12}$

b) $10 \times 2.164 \times 10^{12} = 2.164 \times 10^{13}$

c) ${}_{80}C_{20} \doteq 3.535 \times 10^{18}$

d) $P(\text{pick 10, match 9}) = \frac{2.164 \times 10^{13}}{3.535 \times 10^{18}}$

$\doteq 0.000\ 006\ 122$

2. a) i) ${}_{7}C_7 = 1$

ii) ${}_{73}C_{13} \doteq 8.627 \times 10^{13}$

b) $1 \times 8.627 \times 10^{13} = 8.627 \times 10^{13}$

c) $P(\text{pick 7, match 7}) = \frac{8.627 \times 10^{13}}{{}_{80}C_{20}}$

$\doteq 0.000\ 244$

3. Since the probability in exercise 2 is greater, picking 7 and matching 7 is more likely.

Selected Solutions — Chapter 6

4. P(pick x, match y) $= \dfrac{{}_{x}C_{y} \times {}_{80-x}C_{20-y}}{{}_{80}C_{20}}$

5. a) P(pick 10, match 8) $= \dfrac{{}_{10}C_{8} \times {}_{70}C_{12}}{{}_{80}C_{20}}$
$\doteq 0.000\ 135\ 4$

b) P(pick 8, match 5) $= \dfrac{{}_{8}C_{5} \times {}_{72}C_{15}}{{}_{80}C_{20}}$
$\doteq 0.018$

c) P(pick 4, match 2) $= \dfrac{{}_{4}C_{2} \times {}_{76}C_{18}}{{}_{80}C_{20}}$
$\doteq 0.213$

d) P(pick 2, match 2) $= \dfrac{{}_{2}C_{2} \times {}_{78}C_{18}}{{}_{80}C_{20}}$
$\doteq 0.60$

6. a) Expectation = probability of winning × prize money
$= 0.60 \times \$10$
$= \$6$

b) i) P(pick 1, match 1) $= \dfrac{{}_{1}C_{1} \times {}_{79}C_{19}}{{}_{80}C_{20}}$
$= 0.25$

Expectation $= 0.25 \times \$2$
$= \$0.50$

ii) P(pick 3, match 3) $= \dfrac{{}_{3}C_{3} \times {}_{77}C_{17}}{{}_{80}C_{20}}$
$\doteq 0.014$

P(pick 3, match 2) $= \dfrac{{}_{3}C_{2} \times {}_{77}C_{18}}{{}_{80}C_{20}}$
$\doteq 0.139$

Expectation $= 0.014 \times \$20 + 0.139 \times \$2$
$= \$0.558$

iii) P(pick 4, match 4) $= \dfrac{{}_{4}C_{4} \times {}_{76}C_{16}}{{}_{80}C_{20}}$
$\doteq 0.003$

P(pick 4, match 3) $= \dfrac{{}_{4}C_{3} \times {}_{76}C_{17}}{{}_{80}C_{20}}$
$\doteq 0.043$

P(pick 4, match 2) $= \dfrac{{}_{4}C_{2} \times {}_{76}C_{18}}{{}_{80}C_{20}}$
$\doteq 0.213$

Expectation $= 0.003 \times \$50 + 0.043 \times \$2 + 0.213 \times \$1$
$= \$0.578$

9. a) The expectations would be higher because the probabilities would be higher.

b) Change the 80s in the formula to 70s.

10. P(pick 20, match 19) $= \dfrac{{}_{20}C_{19} \times {}_{60}C_{1}}{{}_{80}C_{20}}$
$\doteq 3.394 \times 10^{-16}$

To find the probability of this happening three times in a row, find P(pick 20, match 19) × P(pick 20, match 19) × P(pick 20, match 19).
[P(pick 20, match 19)]$^3 \doteq 3.91 \times 10^{-44}$

Selected Solutions — Chapter 6

11. a) In both lotteries you pick your own numbers. But in Club Keno, there are many more ways to pick your numbers, and still more ways to match your numbers.

b) i) $P(\text{pick 6, match 5}) = \dfrac{{}_6C_5 \times {}_{49-6}C_{6-5}}{{}_{49}C_6}$

$= \dfrac{{}_6C_5 \times {}_{43}C_1}{{}_{49}C_6}$

$\doteq 0.000\ 0184$

ii) $P(\text{pick 6, match 4}) = \dfrac{{}_6C_4 \times {}_{49-6}C_{6-4}}{{}_{49}C_6}$

$= \dfrac{{}_6C_4 \times {}_{43}C_2}{{}_{49}C_6}$

$\doteq 0.000\ 969$

iii) $P(\text{pick 6, match 3}) = \dfrac{{}_6C_3 \times {}_{49-6}C_{6-3}}{{}_{49}C_6}$

$= \dfrac{{}_6C_3 \times {}_{43}C_3}{{}_{49}C_6}$

$\doteq 0.0177$

6.5 Exercises, page 384

1. a) Left side $= {}_7C_5$

$= \dfrac{7!}{5!2!}$

Right side $= {}_7C_2$

$= \dfrac{7!}{2!5!}$

$= \dfrac{7!}{5!2!}$

$=$ Left side

b) ${}_7C_5$ represents the number of committees of 5 people that can be selected from 7 people. Visualize 7 people in a line: ABCDEFG. Denote each person selected for the committee by Y and each person not selected by N. For example: YNYYNYY. The number of committees is the number of ways of arranging 5 Ys and 2 Ns. ${}_7C_2$ represents the number of committees of 2 people that can be selected from 7 people. For example: YNYNNNN. The number of committees is the number of ways of arranging 2 Ys and 5 Ns. Since the number of ways of arranging 5 Ys and 2 Ns is the same as the number of ways of arranging 2 Ys and 5 Ns, ${}_7C_5 = {}_7C_2$.

2. a) Left side $= {}_nC_r$

$= \dfrac{n!}{r!(n-r)!}$

Right side $= {}_nC_{n-r}$

$= \dfrac{n!}{(n-r)!(n-(n-r))!}$

$= \dfrac{n!}{(n-r)!r!}$

$= \dfrac{n!}{r!(n-r)!}$

$=$ Left side

b) ${}_nC_r$ represents the number of committees of r people that can be selected from n people. The number of committees is the number of ways of arranging r Ys and $(n - r)$ Ns. ${}_nC_{n-r}$ represents the number of committees of $(n - r)$ people that can be selected from n people. The number of committees is the number of ways of arranging $(n - r)$ Ys and r Ns. Since the number of ways of arranging r Ys and $(n - r)$ Ns is the same as the number of ways of arranging $(n - r)$ Ys and r Ns, ${}_nC_r = {}_nC_{n-r}$.

3. a)

$$\begin{aligned}\text{Left side} &= {}_7C_5\\ &= \frac{7!}{5!2!}\\ &= 21\end{aligned}$$

$$\begin{aligned}\text{Right side} &= {}_6C_4 + {}_6C_5\\ &= \frac{6!}{4!2!} + \frac{6!}{5!1!}\\ &= 15 + 6\\ &= 21\\ &= \text{Left side}\end{aligned}$$

b) ${}_7C_5$ represents the number of committees of 5 people that can be selected from 7 people: ABCDEFG. Either person A is on the committee or person A is not on the committee. If A is on the committee, the other 4 committee members must be chosen from the other 6 people; there are ${}_6C_4$ ways to do this. If A is not on the committee, all 5 committee members must be chosen from the other 6 people; there are ${}_6C_5$ ways to do this. Since there are no other possibilities, the number of ways to choose the committee is ${}_6C_4 + {}_6C_5$. Therefore, ${}_7C_5 = {}_6C_4 + {}_6C_5$.

4. a)

$$\begin{aligned}\text{Left side} &= {}_nC_r\\ &= \frac{n!}{r!(n-r)!}\end{aligned}$$

$$\begin{aligned}\text{Right side} &= {}_{n-1}C_{r-1} + {}_{n-1}C_r\\ &= \frac{(n-1)!}{(r-1)!(n-1-(r-1))!} + \frac{(n-1)!}{r!(n-1-r)!}\\ &= \frac{(n-1)!}{(r-1)!(n-r)!} + \frac{(n-1)!}{r!(n-1-r)!}\\ &= \frac{(n-1)!}{(r-1)!(n-r)!} \times \frac{r}{r} + \frac{(n-1)!}{r!(n-1-r)!} \times \frac{n-r}{n-r}\\ &= \frac{r(n-1)!}{r(r-1)!(n-r)!} + \frac{(n-r)(n-1)!}{r!(n-r)(n-r-1)!}\\ &= \frac{r(n-1)!}{r!(n-r)!} + \frac{(n-r)(n-1)!}{r!(n-r)!}\\ &= \frac{r(n-1)! + n(n-1)! - r(n-1)!}{r!(n-r)!}\\ &= \frac{n(n-1)!}{r!(n-r)!}\\ &= \frac{n!}{r!(n-r)!}\\ &= \text{Left side}\end{aligned}$$

b) ${}_nC_r$ represents the number of committees of r people that can be selected from n people: $x_1, x_2, x_3, \ldots, x_n$. Either person x_1 is on the committee or person x_1 is not on the committee. If x_1 is on the committee, the other $(r - 1)$ committee members must be chosen from the other $(n - 1)$ people; there are ${}_{n-1}C_{r-1}$ ways to do this.

Selected Solutions — Chapter 6

If x_1 is not on the committee, all r committee members must be chosen from the other $(n - 1)$ people; there are ${}_{n-1}C_r$ ways to do this. Since there are no other possibilities, the number of ways to choose the committee is ${}_{n-1}C_{r-1} + {}_{n-1}C_r$. Therefore, ${}_nC_r = {}_{n-1}C_{r-1} + {}_{n-1}C_r$.

5. b) The $(r + 1)$th number in the $(n + 1)$th row is ${}_nC_r = \frac{n!}{r!(n-r)!}$.
For the second number, $r = 1$. Thus, the second number is

$$\begin{aligned} {}_nC_1 &= \frac{n!}{1!(n-1)!} \\ &= n \end{aligned}$$

The second number in the $(n + 1)$th row is n.

6. b) The $(r + 1)$th number in the $(n + 1)$th row is ${}_nC_r = \frac{n!}{r!(n-r)!}$.
For the third number, $r = 2$. Thus, the third number is

$$\begin{aligned} {}_nC_2 &= \frac{n!}{2!(n-2)!} \\ &= \frac{n(n-1)}{2} \end{aligned}$$

The third number in the $(n + 1)$th row is $\frac{n(n-1)}{2}$.

7. a) The numbers in the second diagonal are the second numbers in each row. From exercise 5b, the second number in the $(n + 1)$th row is n. Thus, the numbers are the natural numbers.

b) The numbers in the second diagonal are the number of ways of selecting 1 person from n people. This is n for row $n + 1$. Thus, the numbers are the natural numbers.

8. a) Answers may vary. $1 + 2 + 3 + 4 = 10$
This is equivalent to ${}_1C_1 + {}_2C_1 + {}_3C_1 + {}_4C_1 = {}_5C_2$.
${}_5C_2 = {}_4C_1 + {}_4C_2$ by the recursive pattern.
${}_4C_2 = {}_3C_1 + {}_3C_2$ by the recursive pattern.
Thus, ${}_5C_2 = {}_4C_1 + {}_3C_1 + {}_3C_2$.
${}_3C_2 = {}_2C_1 + {}_2C_2$ by the recursive pattern.
Thus, ${}_5C_2 = {}_4C_1 + {}_3C_1 + {}_2C_3 + {}_2C_2$.

$$\begin{aligned} {}_2C_2 &= 1 \\ &= {}_1C_1 \end{aligned}$$

Thus, ${}_5C_2 = {}_4C_1 + {}_3C_1 + {}_2C_1 + {}_1C_1$.

9. a) These represent the number of ways of selecting 2 people from n people.

b) Answers may vary. $1 + 3 + 6 + 10 = 20$. This is equivalent to ${}_2C_2 + {}_3C_2 + {}_4C_2 + {}_5C_2 = {}_6C_3$.
${}_6C_3 = {}_5C_2 + {}_5C_3$ by the recursive pattern.
${}_5C_3 = {}_4C_2 + {}_4C_3$ by the recursive pattern.
Thus, ${}_6C_3 = {}_5C_2 + {}_4C_2 + {}_4C_3$.
${}_4C_3 = {}_3C_2 + {}_3C_3$ by the recursive pattern.
Thus, ${}_6C_3 = {}_5C_2 + {}_4C_2 + {}_3C_2 + {}_3C_3$.

$$\begin{aligned} {}_3C_3 &= 1 \\ &= {}_2C_2 \end{aligned}$$

Thus, ${}_6C_3 = {}_5C_2 + {}_4C_2 + {}_3C_2 + {}_2C_2$.

Selected Solutions — Chapter 6

10. b) Assume that it is equally likely for the ball to take each path.

11. At each obstacle, there are 2 choices. To get the number of paths for n rows, find the product of n 2s, or 2^n.

12. a) Place the numbers of Pascal's triangle in the diagram as shown:

```
            •
         1  •  1
      1  •  2  •  1
   1  •  3  •  3  •  1
1  •  4  •  6  •  4  •  1
   5  • 10  • 10  •  5
     15  • 20  • 15
        35  • 35
           70
```

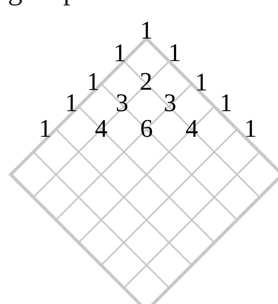
The number of ways is the number at the bottom, or 70.

b) Place the numbers of Pascal's triangle in the diagram as shown:

```
            •
         1  •  1
      1  •  2  •  1
   1  •  3  •  3  •  1
1  •  4  •  6  •  4  •  1
   5  • 10  • 10  •  5  •  1
     15  • 20  • 15  •  6  •  1
        35  • 35  • 21  •  7
           70  • 56  • 28
             126 • 84
                210
```

The number of ways is the number at the bottom, or 210.

13. b) This is like the pinball situation above. If you turn the grid around so the lower right corner is at the top, as shown, the number of ways to get to a grid point is the sum of the ways to get to the two grid points above and to the left and right of it.



14. To get from A to B, you must travel 3 blocks south and 3 blocks east for each possible path. Thus, find the number of permutations for SSSEEE. This is $\frac{6!}{3!3!}$ or 20.

15. a) There are 12 points. Each pair is connected with a line segment. The number of line segments is the number of ways to select 2 points from 12 points, or ${}_{12}C_2 = 66$.

Selected Solutions — Chapter 6

16. The number of diagonals is the number of line segments minus the number of sides: ${}_{20}C_2 - 20 = 170$

17. The number of diagonals is the number of line segments minus the number of sides:

$$\begin{aligned} {}_nC_2 - n &= \frac{n(n-1)}{2} - n \\ &= \frac{n^2 - n - 2n}{2} \\ &= \frac{n^2 - 3n}{2} \\ &= \frac{n(n-3)}{2} \end{aligned}$$

18. a) Row 1: 1; Row 2: 2; Row 3: 4; Row 4: 8
The sums are powers of 2.

b) It is the number of ways any number of items can be chosen from a set of 5 items.

c) Each item is either chosen or not chosen. Thus, there are two possibilities for each of 5 items, or $2 \times 2 \times 2 \times 2 \times 2 = 2^5$ possibilities altogether.

d) The numbers in the $(n + 1)$th diagonal represent choosing items from a set of n items. Each item is either chosen or not chosen. Thus, there are two possibilities for each of n items, or $2 \times 2 \times 2 \times \cdots \times 2 = 2^n$ possibilities altogether.

19. b) This is the 5th row of Pascal's triangle.
Part i is the number of ways of selecting 0 items from 4 items, or ${}_4C_0 = 1$.
Part ii is the number of ways of selecting 1 item from 4 items, or ${}_4C_1 = 4$.
Part iii is the number of ways of selecting 2 items from 4 items, or ${}_4C_2 = 6$.
Part iv is the number of ways of selecting 3 items from 4 items, or ${}_4C_3 = 4$.
Part v is the number of ways of selecting 4 items from 4 items, or ${}_4C_4 = 1$.

c) From exercise 18d, the sum of the numbers in the 5th row of Pascal's triangle is $2^4 = 16$.

20. Each item can be chosen or not chosen for the subset. There are $2 \times 2 \times 2 \times \cdots \times 2 = 2^n$ ways of doing this.

21. The number of line segments is found by determining the number of pairs of points. The points do not have to be on a circle for this to be done.

22. If n is even, the greatest number is ${}_{n-1}C_{\frac{n}{2}}$. If n is odd, the greatest number is ${}_{n-1}C_{\frac{(n-1)}{2}}$.

Selected Solutions — Chapter 6

Problem Solving, page 389

1. pigeons: students; pigeonholes: months

a) There are 2 students in one month, so $k = 1$. There are 12 months, so $n = 12$.

$$\begin{aligned} kn + 1 &= 1 \times 12 + 1 \\ &= 13 \end{aligned}$$

There must be 13 students in my class.

b) There are 3 students in one month, so $k = 2$. As in part a, $n = 12$.

$$\begin{aligned} kn + 1 &= 2 \times 12 + 1 \\ &= 25 \end{aligned}$$

There must be 25 students in my class.

c) There are 4 students in one month, so $k = 3$. As in part a, $n = 12$.

$$\begin{aligned} kn + 1 &= 3 \times 12 + 1 \\ &= 37 \end{aligned}$$

There must be 37 students in my class.

2. pigeons: students; pigeonholes: days in a year

There are 2 students in one day, so $k = 1$. There are 366 possible days in a year, so $n = 366$.

$$\begin{aligned} kn + 1 &= 366 + 1 \\ &= 367 \end{aligned}$$

There must be 367 students in my school.

3. pigeons: Canadians; pigeonholes: days in a year

There are 366 possible days in a year, so $n = 366$.

$$\begin{aligned} kn + 1 &= 30\ 000\ 000 \\ 366k + 1 &= 30\ 000\ 000 \\ 366k &= 29\ 999\ 999 \\ k &\doteq 81\ 967 \\ k + 1 &\doteq 81\ 968 \end{aligned}$$

Thus, at least 81 968 Canadians have birthdays on the same day.

4. a) pigeons: cards; pigeonholes: colours

There are 2 pigeonholes (red and black), so $n = 2$. I want 2 red or black cards, so $k + 1 = 2$, or $k = 1$.

$$\begin{aligned} kn + 1 &= 1 \times 2 + 1 \\ &= 3 \end{aligned}$$

I need to draw 3 cards.

b) pigeons: cards; pigeonholes: suits

There are 4 pigeonholes (hearts, diamonds, spades, and clubs), so $n = 4$. I want 2 cards from the same suit, so $k + 1 = 2$, or $k = 1$.

$$\begin{aligned} kn + 1 &= 1 \times 4 + 1 \\ &= 5 \end{aligned}$$

I need to draw 5 cards.

c) pigeons: cards; pigeonholes: face value

There are 13 pigeonholes, so $n = 13$. I want 2 cards with the same face value, so $k + 1 = 2$, or $k = 1$.

$$\begin{aligned} kn + 1 &= 1 \times 13 + 1 \\ &= 14 \end{aligned}$$

I need to draw 14 cards.

Selected Solutions — Chapter 6

5. pigeons: socks; pigeonholes: colours
There are 2 pigeonholes (black and white), so $n = 2$. I want 2 socks of the same colour, so $k + 1 = 2$, or $k = 1$.

$$\begin{aligned} kn + 1 &= 1 \times 2 + 1 \\ &= 3 \end{aligned}$$

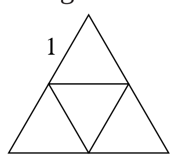
I must remove 3 socks.

6. pigeons: numbers; pigeonholes: different remainders when dividing by 4
There are 5 numbers, so $kn + 1 = 5$.
There are 4 remainders, 0, 1, 2, and 3, so $n = 4$.

$$\begin{aligned} kn + 1 &= 5 \\ 4k + 1 &= 5 \\ 4k &= 4 \\ k &= 1 \\ k + 1 &= 2 \end{aligned}$$

Thus, at least 2 of the numbers have the same remainder.

7. pigeons: points; pigeonholes: Divide the triangle into 4 equilateral triangles with side length 1, as shown.

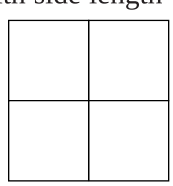


In each small triangle, no two points can be more than 1 unit apart.
There are 5 points, so $kn + 1 = 5$.
There are 4 small triangles, so $n = 4$.

$$\begin{aligned} kn + 1 &= 5 \\ 4k + 1 &= 5 \\ 4k &= 4 \\ k &= 1 \\ k + 1 &= 2 \end{aligned}$$

Thus, at least 2 of the points are in one of the small triangles, and no more than 1 unit apart.

8. pigeons: points; pigeonholes: Divide the square into 4 small squares with side length 1, as shown.



The longest distance in each small square is the diagonal, which has length $\sqrt{2}$. Thus, two points in one small square cannot be more than $\sqrt{2}$ units apart.

There are 5 points, so $kn + 1 = 5$.
There are 4 small squares, so $n = 4$.

Selected Solutions — Chapter 6

$kn + 1 = 5$
$4k + 1 = 5$
$4k = 4$
$k = 1$
$k + 1 = 2$

Thus, at least 2 of the points are in one of the small squares, and no more than $\sqrt{2}$ units apart.

9. a) pigeons: 6 numbers that are chosen; pigeonholes: five categories of numbers: 1, 2; 3, 4; 5, 6; 7, 8; or 9, 10
Six numbers are chosen, so $kn + 1 = 6$.
There are 5 categories, so $n = 5$.

$kn + 1 = 6$
$5k + 1 = 6$
$5k = 5$
$k = 1$
$k + 1 = 2$

Two numbers must be in one of the categories. In each category, the numbers differ by 1. Thus, at least 2 numbers have a difference of 1.

b) pigeons: 6 numbers that are chosen; pigeonholes: five categories of numbers: 1, 7; 2, 6; 3, 9; 4, 8; and 5, 10
Six numbers are chosen, so $kn + 1 = 6$.
There are 5 categories, so $n = 5$.

$kn + 1 = 6$
$5k + 1 = 6$
$5k = 5$
$k = 1$
$k + 1 = 2$

Two numbers must be in one of the categories. In each category, one number is a factor of the other. Thus, at least one number is a factor of another.

10. Take one person, say person A, away. The pigeons are the other 5 people. The pigeonholes are "knowing person A" and "not knowing person A." By the pigeonhole theorem, there must be at least 3 people in one of the pigeonholes.

Say the 3 people know person A. Of these 3 people, by the pigeonhole theorem, either at least 2 of them know each other, in which case, these two and person A are mutual acquaintances, or none of them know each other, in which case they are 3 mutual strangers.

Say the 3 people do not know person A. Of these 3 people, by the pigeonhole theorem, either at least 2 of them do not know each other, in which case, these 2 and person A are 3 mutual strangers, or they all know each other, in which case they are 3 mutual acquaintances.

11. Let the 6 numbers be a, b, c, d, e, f (in ascending order).
The pigeons are the sums a, a + b, a + b + c, a + b + c + d, a + b + c + d + e, a + b + c + d + e + f. If any of these sums is divisible by 6, it meets the conditions of the question.

Selected Solutions — Chapter 6

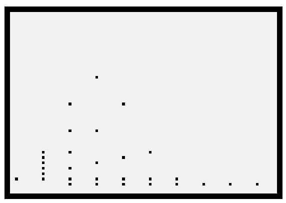
If none of the sums is divisible by 6, then there are 5 possibilities: remainders of 1, 2, 3, 4, and 5.

In this case, there are 6 pigeons (the sums) and 5 pigeonholes (the remainders), so there must be at least 2 sums with the same remainder. Consider these 2 sums, and subtract the smaller from the larger. The difference is a number divisible by 6, and it is also the sum of the numbers included in the larger number but not in the smaller number (for example, a + b + c + d − (a + b) = c + d). This is a sum of adjacent numbers, which also meets the required conditions.

Exploring with a Graphing Calculator, page 390

1. a) Answers may vary.

b)



Window should be set at

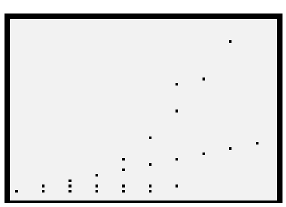
Xmin = 0 Ymin = 0
Xmax = 9.4 Ymax = 31
Xscl = 1 Yscl = 10
Xres = 1

c) $y_1 = {}_1C_x$: 0, 1; x must be between 0 and 1, and a whole number.
$y_2 = {}_2C_x$: 0, 1, 2; x must be between 0 and 2, and a whole number.
$y_3 = {}_3C_x$: 0, 1, 2, 3; x must be between 0 and 3, and a whole number.
$y_4 = {}_4C_x$: 0, 1, 2, 3, 4; x must be between 0 and 4, and a whole number.
$y_5 = {}_5C_x$: 0, 1, 2, 3, 4, 5; x must be between 0 and 5, and a whole number.
$y_6 = {}_6C_x$: 0, 1, 2, 3, 4, 5, 6; x must be between 0 and 6, and a whole number.

2. $y = {}_nC_x$ represents the number of ways of selecting x items from n items. The range is the entries of Pascal's triangle. As n increases, the maximum point shifts up and to the right.

3. a) Answers may vary.

b)



Window should be set at

Xmin = 0 Ymin = 0
Xmax = 9.4 Ymax = 10
Xscl = 1 Yscl = 1
Xres = 1

c) The domain for $y = {}_xC_r$ is $x \geq r$, where x is a whole number.

Selected Solutions — Chapter 6

4. $y = {}_xC_r$ represents the number of ways of selecting r items from x items, with x varying. As r increases, the graph of the function gets steeper. The range is the entries of Pascal's triangle from top to bottom.

5. a) They are polynomial functions, since they include terms of the form ax^n.

Investigate, page 391

1. The coefficients are rows of Pascal's triangle. The coefficients of the expansion of $(x + 1)^n$ are the $(n + 1)$th row of Pascal's triangle.

5. For example, look at $(a + b)^5$.
$(a + b)^5 = (a + b)(a + b)(a + b)(a + b)(a + b)$

Each term of the expansion is the product of 5 factors. In each term, an a or a b is taken from each binomial factor.

The first term is a^5. It is formed by choosing the a from each of the 5 binomial factors. There is only one way to do this.

The second term contains a^4b. It is formed by choosing the b from any binomial factor and the 4 as from the other 4 binomial factors. The b can be chosen in ${}_5C_1$ ways, and for each way the 4 as can be chosen in only one way. Hence, the coefficient of a^4b is 5, and the second term is $5a^4b$.

The third term contains a^3b^2. There are ${}_5C_2$ ways to choose 2 bs from the 5 binomial factors and only one way to choose the as from the remaining factors. Hence, the coefficient of a^3b^2 is 10, and the third term is $10a^3b^2$.

Similarly, the fourth term is $10a^2b^3$, the fifth term is $5ab^4$, and the sixth term is b^5.

Therefore, $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$.
The coefficients are the numbers in the 5th row of Pascal's triangle.

6.6 Exercises, page 396

1. $(a + b)^3 = (a + b)(a + b)(a + b)$
Each term of the expansion is the product of 3 factors. In each term, an a or a b is taken from each binomial factor.

The first term is a^3. It is formed by choosing the a from each of the 3 binomial factors. There is only one way to do this.

The second term contains a^2b. It is formed by choosing the b from any binomial factor and the 2 as from the other 2 binomial factors. The b can be chosen in ${}_3C_1$ ways, and for each way the 2 as can be chosen in only one way. Hence, the coefficient of a^2b is 3, and the second term is $3a^2b$.

The third term contains ab^2. There are ${}_3C_2$ ways to choose 2 bs from the 3 binomial factors and only one way to choose the a from the remaining factors. Hence, the coefficient of ab^2 is 3, and the third term is $3ab^2$.

Similarly, the fourth term is b^3.

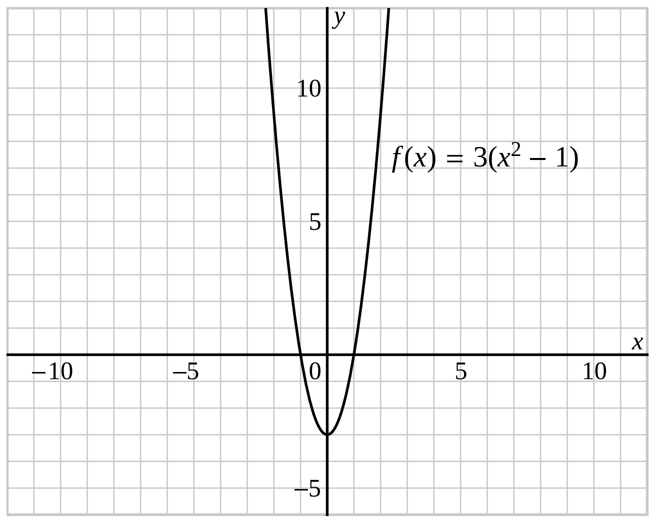
Therefore, $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. The coefficients are the numbers in the 3rd row of Pascal's triangle.

8. $(a + b)^n = {}_nC_0a^nb^0 + {}_nC_1a^{n-1}b^1 + {}_nC_2a^{n-2}b^2 + \cdots + {}_nC_na^{n-n}b^n$

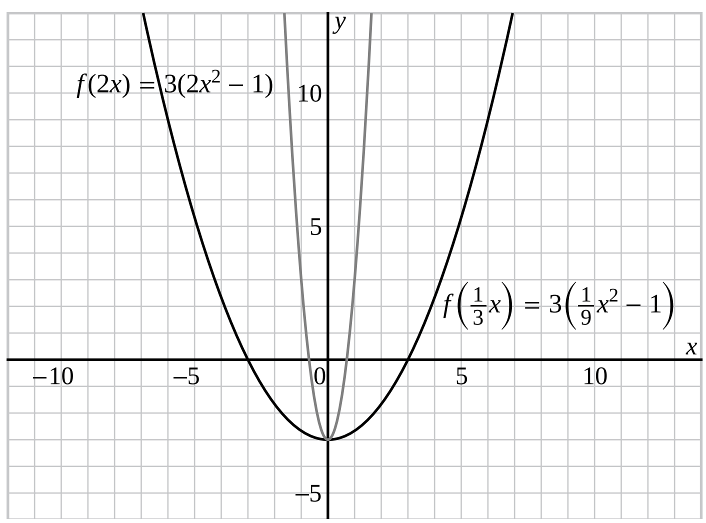
$$= \sum_{i=o}^{n} {}_nC_ia^{n-i}b^i$$

6 Cumulative Review page 398

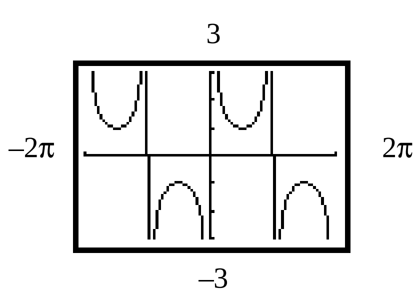
2. a)



b)

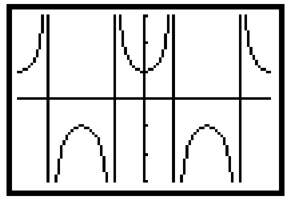


11. a)



b)

3

–2π  2π

–3

13. a) $\sin x \tan x + \sec x = \frac{\sin^2 x + 1}{\cos x}$

Left side	Reason
$\sin x \tan x + \sec x$	
$= \sin x\left(\frac{\sin x}{\cos x}\right) + \frac{1}{\cos x}$	Definition of tan x and sec x
$= \frac{\sin^2 x + 1}{\cos x}$	Simplifying fractions
Right Side	

b) $\frac{1 + \cos x}{1 - \cos x} = \frac{1 + \sec x}{\sec x - 1}$

Right side	Reason
$\frac{1 + \sec x}{\sec x - 1}$	
$= \frac{1 + \frac{1}{\cos x}}{\frac{1}{\cos x} - 1}$	Definition of sec x
$= \frac{\cos x + 1}{\cos x} \times \frac{\cos x}{1 - \cos x}$	Simplifying fractions
$= \frac{1 + \cos x}{1 - \cos x}$	Simplifying fractions
= Left Side	