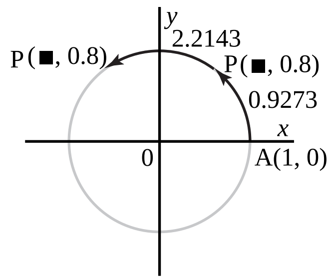
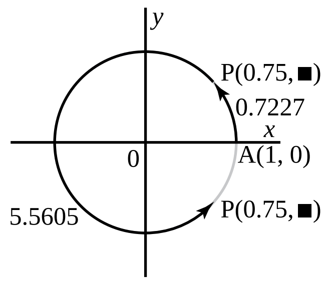
Selected Solutions — Chapter 5

5.1 Exercises, page 302

2. Diagrams and explanations may vary. For $\sin x = 0.8$:
Draw a unit circle.

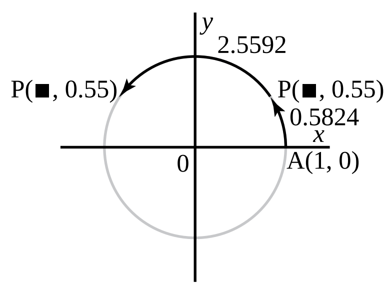


When the second coordinate of P is 0.8, the arc length from A(1, 0) could be 0.9273 units or 2.2143 units.
For $\cos x = 0.75$:
Draw a unit circle.



When the first coordinate of P is 0.75, the arc length from A(1, 0) could be 0.7227 units or 5.5605 units.

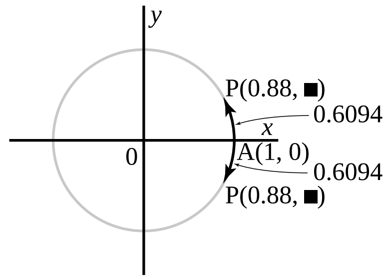
4. Diagrams and explanations may vary. For $\sin x = 0.55$:
Draw a unit circle.



When the second coordinate of P is 0.55, the arc length from A(1, 0) could be 0.5824 units or 2.5592 units.

Selected Solutions — Chapter 5

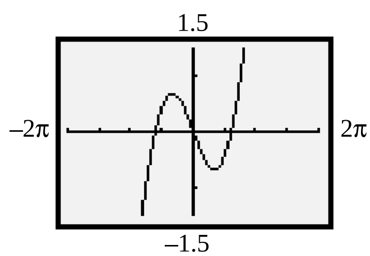
For $\cos x = 0.82$:
Draw a unit circle.



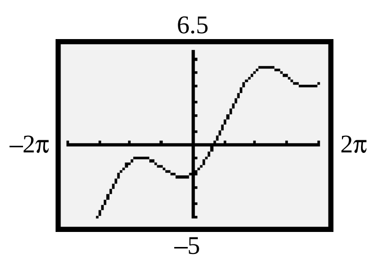
When the first coordinate of P is 0.82, the arc length from A(1, 0) could be 0.6094 units in the counterclockwise or in the clockwise direction.

9. The roots in exercises 8a and 8b have the same y-coordinates. This is because the equations are basically they same in each exercise. The only difference is in exercise 8b, the function is stretched by a factor of $\frac{\pi}{2}$. Therefore, the x-coordinates of the roots in 8a and 8b will differ, while the y-coordinates remain the same.

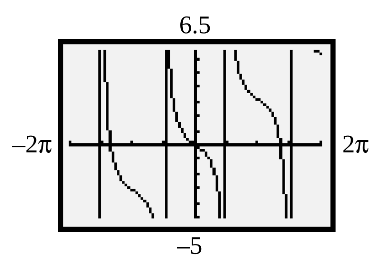
11. a)



12. a)



b)



13. a) There are 3 points where $y = \frac{1}{x}$ crosses $y = 2\cos x$, and one point where $y = \frac{1}{x}$ appears to just touch $y = 2\cos x$. Thus, there appears to be 4 roots in the interval $-2\pi \le x \le 2\pi$.

14. a) $y = 3\sin x$ appears to just touch $y = 3^x$. Thus, there appears to be one root.

15. When the graph is shifted to become $y = 3\cos x$, it intersects $y = 3^x$ an extra time. Therefore, there are two roots in the interval $-2\pi \le x \le 2\pi$.

Selected Solutions — Chapter 5

16. c) For $\left(-\frac{4\pi}{3}, -0.5\right)$:

$$\begin{aligned} y &= \cos 2x: \\ -0.5 &= \cos 2\left(-\frac{4\pi}{3}\right) \\ \text{R.S.} &= \cos\left(-\frac{8\pi}{3}\right) \\ &= -0.5 \\ &= \text{L.S.} \end{aligned}$$

$$\begin{aligned} y &= \cos x: \\ -0.5 &= \cos\left(-\frac{4\pi}{3}\right) \\ \text{R.S.} &= \cos\left(-\frac{4\pi}{3}\right) \\ &= -0.5 \\ &= \text{L.S.} \end{aligned}$$

$$\begin{aligned} y &= \cos\left(\frac{1}{2}x\right): \\ -0.5 &= \cos\left(\frac{1}{2}\left(-\frac{4\pi}{3}\right)\right) \\ \text{R.S.} &= \cos\left(-\frac{2\pi}{3}\right) \\ &= -0.5 \\ &= \text{L.S.} \end{aligned}$$

For $\left(\frac{4\pi}{3}, -0.5\right)$:

$$\begin{aligned} y &= \cos 2x: \\ -0.5 &= \cos 2\left(\frac{4\pi}{3}\right) \\ \text{R.S.} &= \cos\left(\frac{8\pi}{3}\right) \\ &= -0.5 \\ &= \text{L.S.} \end{aligned}$$

$$\begin{aligned} y &= \cos x: \\ -0.5 &= \cos\left(\frac{4\pi}{3}\right) \\ \text{R.S.} &= \cos\left(\frac{4\pi}{3}\right) \\ &= -0.5 \\ &= \text{L.S.} \end{aligned}$$

$$\begin{aligned} y &= \cos\left(\frac{1}{2}x\right): \\ -0.5 &= \cos\left(\frac{1}{2}\left(\frac{4\pi}{3}\right)\right) \\ \text{R.S.} &= \cos\left(\frac{2\pi}{3}\right) \\ &= -0.5 \\ &= \text{L.S.} \end{aligned}$$

For (0, 1):

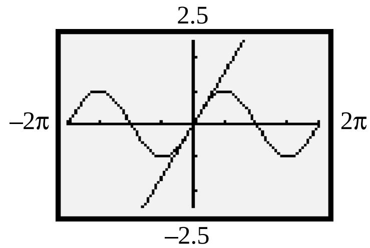
$$\begin{aligned} y &= \cos 2x: \\ 1 &= \cos 2(0) \\ \text{R.S.} &= \cos 0 \\ &= 1 \\ &= \text{L.S.} \end{aligned}$$

$$\begin{aligned} y &= \cos x: \\ 1 &= \cos 0 \\ \text{R.S.} &= \cos 0 \\ &= 1 \\ &= \text{L.S.} \end{aligned}$$

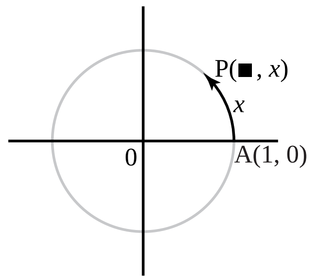
$$\begin{aligned} y &= \cos\left(\frac{1}{2}x\right): \\ 1 &= \cos\left(\frac{1}{2}(0)\right) \\ \text{R.S.} &= \cos\left(\frac{1}{2}(0)\right) \\ &= \cos 0 \\ &= 1 \\ &= \text{L.S.} \end{aligned}$$

Selected Solutions — Chapter 5

17. a)

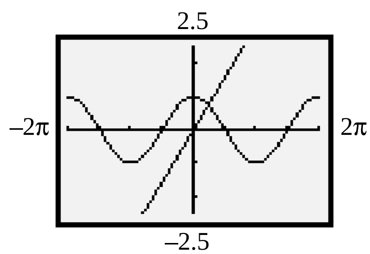


There is one point of intersection on the graph.

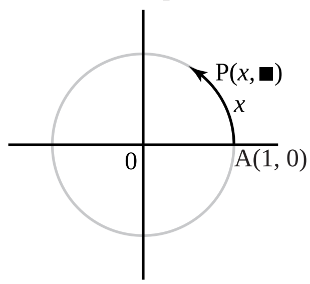


The only place where the second coordinate of P is equal to the arc length is when $x = 0$.

b)

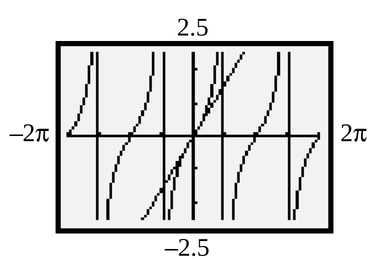


There is one point of intersection on the graph.



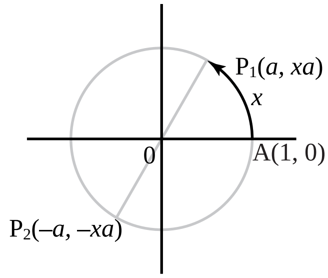
There is only one place where the first coordinate of P can be equal to the arc length.

c)



Selected Solutions — Chapter 5

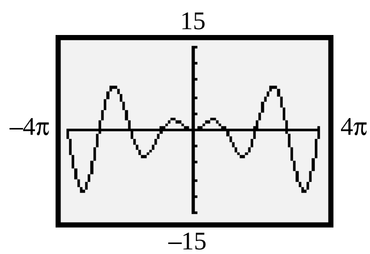
There is an infinite number of points of intersection on the graph.



The slope of OP, or $\tan x$, is zero when $x = 0$, thus $\tan x = x$ when $x = 0$. There are two other points on the circle where the slope of OP is equal to x. These two points represent an infinite number of points of intersection for $y = \tan x$ and $y = x$.

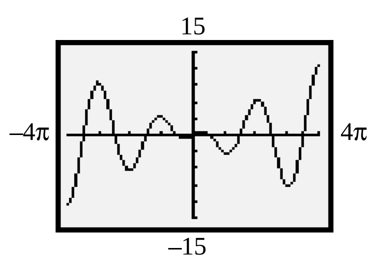
18. No. The rounding of the second term would accumulate, eventually making the solution far from correct.

19. a)



c) When $x = 0$ or $\sin x = 0$, $x \sin x = 0$. When x is small, $\sin x \geq 0$, so $x \sin x \geq 0$ and small. For $x > 0$, as x increases, $|x \sin x|$ increases, fluctuating between positive and negative as $\sin x$ fluctuates between positive and negative. For $x < 0$, as x decreases, $|x \sin x|$ increases, fluctuating between positive and negative as $\sin x$ fluctuates between negative and positive.

20. a)

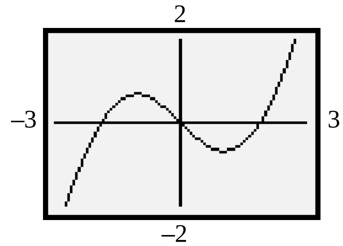


c) When $x = 0$ or $\cos x = 0$, $x \cos x = 0$. When x is small, $\cos x$ is close to 1, so $x \cos x$ is close to x. For $x > 0$, as x increases, $|x \cos x|$ increases, fluctuating between positive and negative as $\cos x$ fluctuates between positive and negative. For $x < 0$, as x decreases, $|x \cos x|$ increases, fluctuating between positive and negative as $\cos x$ fluctuates between negative and positive.

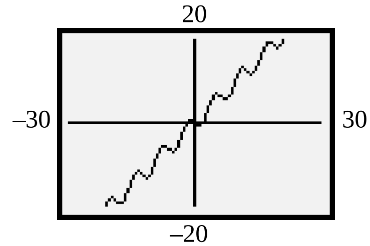
Selected Solutions — Chapter 5

Exploring with a Graphing Calculator, page 305

1. a)



b)



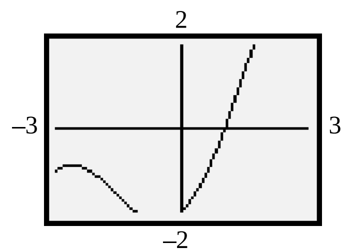
The graph decreases or increases from 0 to 1, which creates the bumps.

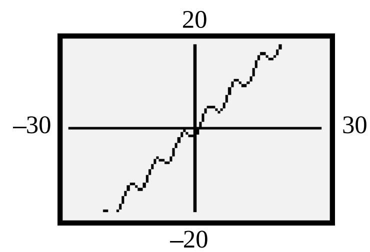
2. The bumps will be barely visible, because the increases and decreases are so small compared to the scale.

3. It will look like the graph of $y = -x$. This is because $\sin x$ is close to x for small values of x, so

$$\begin{aligned} y &= x - 2\sin x \\ &\doteq x - 2x \\ &= -x \end{aligned}$$

4.





The graph decreases or increases from 0 to 1, which creates the bumps.

Zoomed out again, the graph looks like $y = x$, because the increases and decreases are so small compared to the scale.

Zoomed in several times, the graph disappears. This is because $\cos x$ is close to 1 for small values of x, so

$$\begin{aligned} y &= x - 2\cos x \\ &\doteq x - 2 \end{aligned}$$

This doesn't show on the zoomed-in graph.

5. The points of intersection occur when

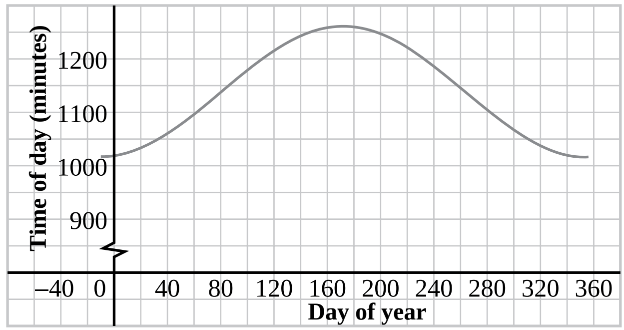
$$\begin{aligned} x - 2\sin x &= x - 2\cos x \\ \sin x &= \cos x \\ \tan x &= 1 \\ x &= \frac{\pi}{4} + n\pi \end{aligned}$$

Selected Solutions — Chapter 5

Mathematical Modelling, page 306

Represent the sunset times as ordered pairs of the form (day of the year, minute of the day). Thus, June 21 at 21:01 is represented by (172, 1261), and December 21 at 16:57 by (355, 1017).

1. Choose the cosine function. Draw a graph.



The value of a is half the difference between the values of the two second coordinates, or $\frac{1261 - 1017}{2} = 122$. The maximum occurs at (172, 1261), so the value of c is 172. The period is one year, so the value of p is 365. So far, we have

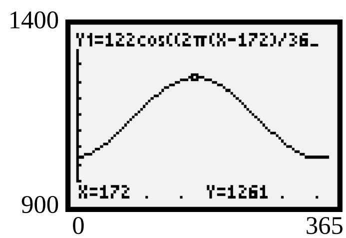
$s = 122\cos\frac{2\pi(n - 172)}{365} + d$

Substitute the point (172, 1261):

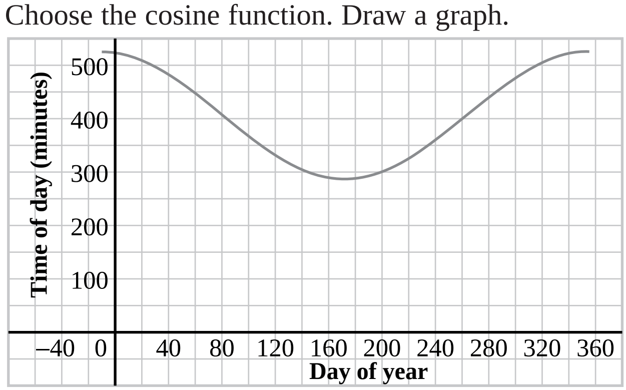
$1261 = 122 + d$

$d = 1139$

3.



4. a) Choose the cosine function. Draw a graph.



On June 21 the sun rises at 4:47 A.M. This is represented by (172, 287). On December 21 the sun rises at 8:45 A.M. This is represented by (355, 525).

Selected Solutions — Chapter 5

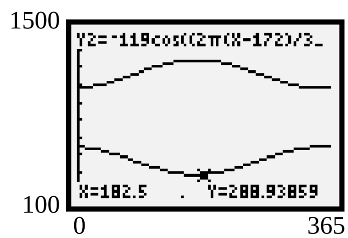
The value of a is negative, since the graph resembles a negative cosine function. Find half the difference between the values of the two second coordinates, or $\frac{525 - 287}{2} = 119$. Thus, $a = -119$. The minimum occurs at (172, 287), so the value of c is 172. The period is one year, so the value of p is 365. So far, we have

$$s = -119 \cos \frac{2\pi(n - 172)}{365} + d$$

Substitute the point (172, 287):

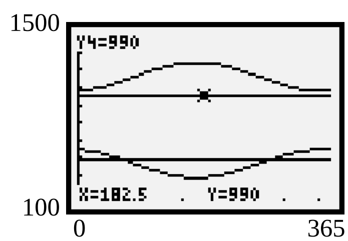
$$287 = -119 + d$$
$$d = 406$$

b)



5. a) The equations of the morning and afternoon pickup times are straight lines represented by $s = 435$ and $s = 990$.

b)

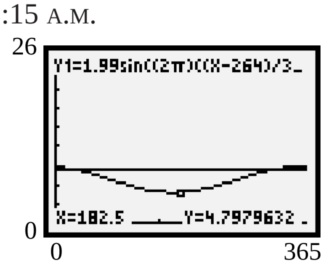


6. a) Use the graphing calculator to determine the points of intersection. They are (68, 435) and (276, 435).

b) Convert the first coordinates of these points of intersection back to dates. The school bus picks up students before sunrise between October 3 and March 9, or 157 days.

c) There are no dates when the school bus drops off students after sunset.

7. Have morning pick-up earlier. For example, morning pick-up at 8:15 A.M.



This changes the days of darkness to 84 days.

Selected Solutions — Chapter 5

5.2 Exercises, page 313

1. a) The period of $\sin x$ is 2π, so there are two solutions.

b) The period of $\sin 2x$ is π, so there are four solutions.

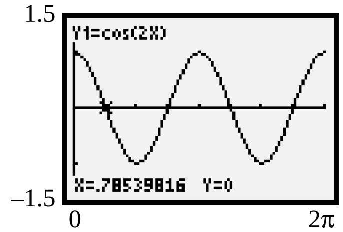
c) The period of $\sin 3x$ is $\frac{2\pi}{3}$, so there are six solutions.

d) The period of $\cos 2x$ is π, so there are four solutions.

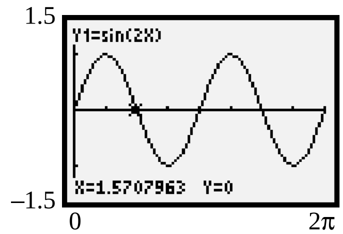
e) The period of $\cos 4x$ is $\frac{\pi}{2}$, so there are eight solutions.

f) The period of $\cos 8x$ is $\frac{\pi}{4}$, so there are sixteen solutions.

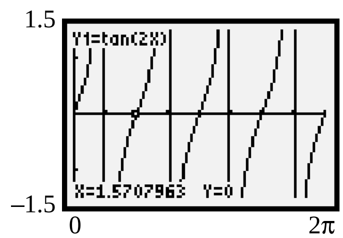
2. a)



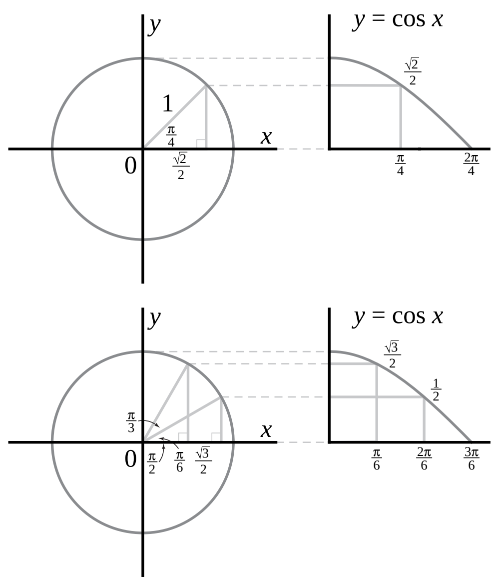
3. a)



4. a)



10. a)



Selected Solutions — Chapter 5

15. $4\cos^2 x - 2\cos x - 1 = 0$

$$4\cos^2 x - 2\cos x = 1$$
$$\cos^2 x - \frac{1}{2}\cos x = \frac{1}{4}$$
$$\cos^2 x - \frac{1}{2}\cos x + \frac{1}{16} = \frac{1}{4} + \frac{1}{16}$$
$$\left(\cos x - \frac{1}{4}\right)^2 = \frac{5}{16}$$
$$\cos x - \frac{1}{4} = \pm\frac{\sqrt{5}}{4}$$
$$\cos x = \frac{1}{4} \pm \frac{\sqrt{5}}{4}$$
$$\cos x = \frac{1 \pm \sqrt{5}}{4}$$

Using the information from exercise 11 in 3.5 Exercises on page 189, $x = \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$.

16. a)

$$2\sin x = 3 + 2\csc x$$
$$2\sin x = 3 + \frac{2}{\sin x}$$
$$2\sin^2 x = 3\sin x + 2$$
$$2\sin^2 x - 3\sin x - 2 = 0$$
$$(2\sin x + 1)(\sin x - 2) = 0$$

$2\sin x = -1$ or $\sin x = 2$

Only $2\sin x = -1$ has a solution.

$$2\sin x = -1$$
$$\sin x = -\frac{1}{2}$$

For $0 \le x < 2\pi$, $x = \frac{7\pi}{6}, \frac{11\pi}{6}$.

The general solution is $x = \frac{7\pi}{6} + 2n\pi$ or $x = \frac{11\pi}{6} + 2n\pi$.

b)

$$4\tan x + \cot x = 5$$
$$4\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = 5$$
$$4\sin^2 x + \cos^2 x = 5\sin x\ \cos x$$
$$4\sin^2 x - 5\sin x\ \cos x + \cos^2 x = 0$$
$$(4\sin x - \cos x)(\sin x - \cos x) = 0$$

$4\sin x = \cos x$ or $\sin x = \cos x$

$\tan x = \frac{1}{4}$

$\tan x = 1$

For $0 \le x < 2\pi$, $x = 0.2450, 3.3866, \frac{\pi}{4}, \frac{5\pi}{4}$.

The general solution is $x = 0.2450 + n\pi$ or $x = \frac{\pi}{4} + n\pi$.

5.3 Exercises, page 319

1. As x increases, AB gets longer, and goes down and up on the y-axis. This illustrates that $y = \cos x$ is periodic. As x decreases, AB gets shorter, and eventually becomes 0. The fact that the slope is always 0 illustrates that $\cos x = \cos(-x)$.

Selected Solutions — Chapter 5

2. As x increases, AB gets longer, and the slope fluctuates between -1 and 1. This illustrates that $y = \sin x$ is periodic and that $-\sin x = \sin(-x)$.

3. a) As x increases, $\sin x$ increases from 0 to 1, $\tan x$ increases from 0 to infinity, and $\cos x$ decreases from 1 to 0. This illustrates why the graphs look like they do for angles between 0 and $\frac{\pi}{2}$.

b) $\triangle$PON gets tall and narrow, until it is not a triangle any more. This also illustrates the behaviour of the graphs of $\sin x$, $\cos x$, and $\tan x$.

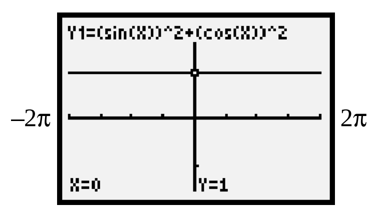
4. a)
$$\begin{aligned}\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{6} &= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\\ &= \frac{1}{4} + \frac{3}{4}\\ &= 1\end{aligned}$$

b)
$$\begin{aligned}\sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} &= \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2\\ &= \frac{2}{4} + \frac{2}{4}\\ &= 1\end{aligned}$$

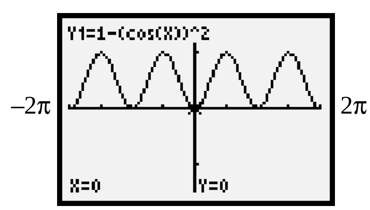
c)
$$\begin{aligned}\sin^2\frac{\pi}{2} + \cos^2\frac{\pi}{2} &= 1^2 + 0^2\\ &= 1\end{aligned}$$

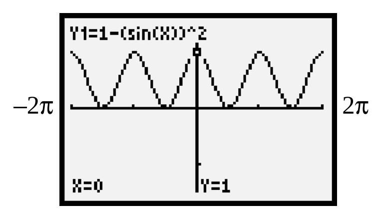
d)
$$\begin{aligned}\sin^2(2.4) + \cos^2(2.4) &= (0.6755)^2 + (-0.7374)^2\\ &= 0.4563 + 0.5437\\ &= 1\end{aligned}$$

5. a)



b)





c) The graph in part a shows $y = 1$. The graph in part b shows $y = \sin^2 x$ or $y = \cos^2 x$.

6. a)
$$\begin{aligned}\cot\frac{\pi}{3} &= \frac{\sqrt{3}}{3}\\ \frac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}} &= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\\ &= \frac{1}{\sqrt{3}}\\ &= \frac{\sqrt{3}}{3}\end{aligned}$$

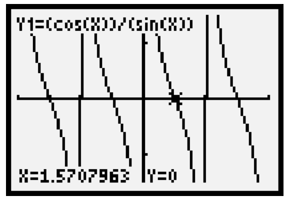
Thus, $\cot\frac{\pi}{3} = \dfrac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}}$.

$\cot 2 \doteq -0.4577$

$\frac{\cos 2}{\sin 2} \doteq -0.4577$

Thus, $\cot 2 = \frac{\cos 2}{\sin 2}$.

b)

-2π  2π

7. a) $\tan^2\frac{\pi}{4} + 1 = (1)^2 + 1$

$= 2$

$\sec^2\frac{\pi}{4} = (\sqrt{2})^2$

$= 2$

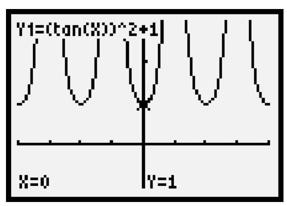
Thus, $\tan^2\frac{\pi}{4} + 1 = \sec^2\frac{\pi}{4}$.

$\tan^2 5 + 1 \doteq 12.4279$

$\sec^2 5 \doteq 12.4279$

Thus, $\tan^2 5 + 1 = \sec^2 5$.

b)

-2π  2π

8. a) $\cos\left(\pi - \frac{\pi}{6}\right) = \cos\frac{5\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

$-\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

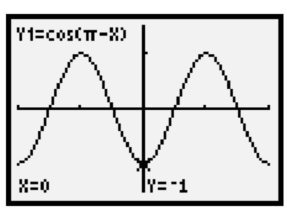
Thus, $\cos\left(\pi - \frac{\pi}{6}\right) = -\cos\frac{\pi}{6}$.

$\cos(\pi - 2.9) \doteq 0.9710$

$-\cos 2.9 \doteq 0.9710$

Thus, $\cos(\pi - 2.9) = -\cos 2.9$

b)

-2π  2π

Selected Solutions — Chapter 5

9. **a)** The graphs do not coincide. Not an identity.

b) The graphs coincide. An identity.

c) The graphs do not coincide. Not an identity.

d) The graphs do not coincide. Not an identity.

e) The graphs coincide. An identity.

f) The graphs coincide. An identity.

10. Explanations may vary. For part a, $\tan(-x) = \tan x$:

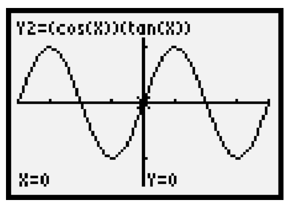
I graphed $y = \tan(-x)$ and $y = \tan x$. The graphs do not coincide, so $\tan(-x) = \tan x$ is not an identity.

For part b, $\tan(-x) = -\tan x$:

I graphed $y = \tan(-x)$ and $y = -\tan x$. The graphs coincide, so $\tan(-x) = -\tan x$ is an identity.

11. Answers may vary.

a)

-2π 

2π

c) In the example, the graphs are of $y = \tan x$. In my graph, the graphs are of $y = \sin x$.

12. $\sin^2 x + \cos^2 x = 1$

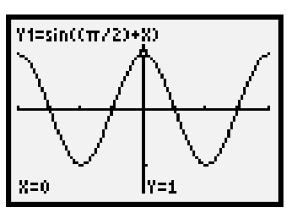
Divide each side by $\sin^2 x$.

$1 + \cot^2 x = \csc^2 x$

13. **a)** Let $x = \frac{\pi}{2}$.

$$\sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \sin \pi$$
$$= 0$$
$$\sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = \sin 0$$
$$= 0$$

b)

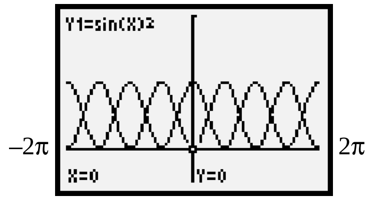
-2π 

2π

c) If the graph of $y = \sin x$ is translated $\frac{\pi}{2}$ units left, it becomes $y = \sin\left(\frac{\pi}{2} + x\right)$ and coincides with the graph of $y = \cos x$. If the graph of $y = \sin x$ is translated $\frac{\pi}{2}$ units right, then reflected in the y-axis, it becomes $y = \sin\left(x - \frac{\pi}{2}\right)$, then $y = \sin\left(\frac{\pi}{2} - x\right)$, and coincides with the graph of $y = \cos x$.

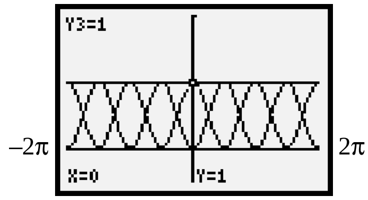
Selected Solutions — Chapter 5

14. a)

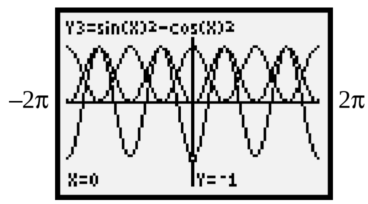


b) When $\sin^2 x = 0$, $\cos^2 x = 1$. When $\sin^2 x = \frac{1}{2}$, $\cos^2 x = \frac{1}{2}$. Everywhere else, the sum of the two functions also appears to be 1.

c)



16. a)



18.

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{1}{\cot x} = \frac{\frac{1}{\csc x}}{\frac{1}{\sec x}}$$

$$\frac{1}{\cot x \ \sec x} = \frac{1}{\csc x}$$

$$\csc x = \cot x \ \sec x$$

$$\sec x = \frac{\csc x}{\cot x}$$

$$\cot x = \frac{\csc x}{\sec x}$$

19. a) Yes, it is possible. You can use reciprical functions to replace $\sin x$, $\cos x$, $\tan x$, $\csc x$, $\cot x$, and $\sec x$. For example

$\sin x = \frac{\tan x}{\sec x}$ $\csc x = \frac{\cot x}{\cos x}$

$\cos x = \frac{\cot x}{\csc x}$ $\sec x = \frac{\csc x}{\cot x}$

$\tan x = \frac{\sec x}{\csc x}$ $\cot x = \frac{\csc x}{\sec x}$

b) Yes, it is possible. You can use reciprical functions to replace $\sin x$, $\cos x$, $\tan x$, $\csc x$, $\cot x$, and $\sec x$. For example

$\sin x = \cos x \ \tan x$ $\csc x = \cot x \ \sec x$

$\cos x = \sin x \ \cot x$ $\sec x = \tan x \ \csc x$

$\tan x = \sin x \ \sec x$ $\cot x = \csc x \ \cos x$

Selected Solutions — Chapter 5

5.4 Exercises, page 326

1. a) When $x = \frac{\pi}{4}$,

$$\begin{aligned}\tan x \ \cos x &= \tan\frac{\pi}{4}\ \cos\frac{\pi}{4}\\ &= (1)\left(\frac{\sqrt{2}}{2}\right)\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

$$\begin{aligned}\sin x &= \sin\frac{\pi}{4}\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

Use a calculator to verify the identity when $x = 2$.

$$\begin{aligned}\tan x \ \cos x &= \tan 2\ \cos 2\\ &\doteq 0.9093\end{aligned}$$

$$\begin{aligned}\sin x &= \sin 2\\ &\doteq 0.9093\end{aligned}$$

b) When $x = \frac{\pi}{4}$,

$$\begin{aligned}\cos x &= \cos\frac{\pi}{4}\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

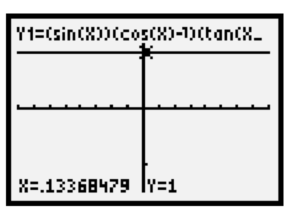
$$\begin{aligned}\frac{\sin x}{\tan x} &= \frac{\sin\frac{\pi}{4}}{\tan\frac{\pi}{4}}\\ &= \frac{\frac{\sqrt{2}}{2}}{1}\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

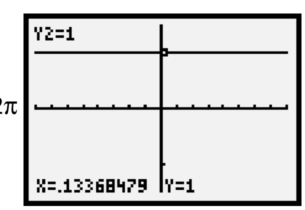
Use a calculator to verify the identity when $x = 2$.

$$\begin{aligned}\cos x &= \cos 2\\ &= -0.4161\end{aligned}$$

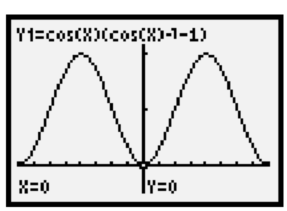
$$\begin{aligned}\frac{\sin x}{\tan x} &= \frac{\sin 2}{\tan 2}\\ &= -0.4161\end{aligned}$$

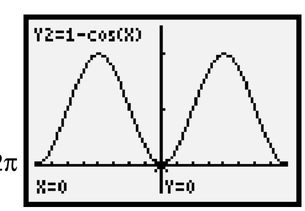
2. a)

-2π  2π

-2π  2π

b)

-2π  2π

-2π  2π

Selected Solutions — Chapter 5

3. a) Left side:

$$\begin{aligned}\sin x \ \cot x &= \sin x \times \frac{\cos x}{\sin x}\\ &= \cos x\\ &= \text{Right side}\end{aligned}$$

b) Left side:

$$\begin{aligned}\tan x \ \csc x &= \frac{\sin x}{\cos x} \times \frac{1}{\sin x}\\ &= \frac{1}{\cos x}\\ &= \sec x\\ &= \text{Right side}\end{aligned}$$

c) Right side:

$$\begin{aligned}\frac{\tan x}{\sec x} &= \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}}\\ &= \frac{\sin x}{\cos x} \times \cos x\\ &= \sin x\\ &= \text{Left side}\end{aligned}$$

d) Right side:

$$\begin{aligned}\frac{\cot x}{\csc x} &= \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}\\ &= \frac{\cos x}{\sin x} \times \sin x\\ &= \cos x\\ &= \text{Left side}\end{aligned}$$

4. a) Left side:

$$\begin{aligned}\csc x\,(1 + \sin x) &= \csc x + \csc x \ \sin x\\ &= \csc x + 1\\ &= 1 + \csc x\\ &= \text{Right side}\end{aligned}$$

b) Left side:

$$\begin{aligned}\sin x\,(1 + \csc x) &= \sin x + \sin x \ \csc x\\ &= \sin x + 1\\ &= 1 + \sin x\\ &= \text{Right side}\end{aligned}$$

c) Left side:

$$\begin{aligned}\frac{1 - \tan x}{1 - \cot x} &= \frac{1 - \frac{\sin x}{\cos x}}{1 - \frac{\cos x}{\sin x}}\\ &= \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}}\\ &= \left(\frac{\cos x - \sin x}{\cos x}\right)\left(\frac{\sin x}{\sin x - \cos x}\right)\\ &= -\frac{\sin x}{\cos x}\\ &= -\tan x\\ &= \text{Right side}\end{aligned}$$

Selected Solutions — Chapter 5

d) Left side:

$$\begin{aligned}\frac{1+\cot x}{1+\tan x} &= \frac{1+\dfrac{\cos x}{\sin x}}{1+\dfrac{\sin x}{\cos x}}\\ &= \frac{\dfrac{\sin x+\cos x}{\sin x}}{\dfrac{\cos x+\sin x}{\cos x}}\\ &= \left(\frac{\sin x+\cos x}{\sin x}\right)\left(\frac{\cos x}{\cos x+\sin x}\right)\\ &= \frac{\cos x}{\sin x}\\ &= \cot x\\ &= \text{Right side}\end{aligned}$$

5. Left side:

$$\begin{aligned}\frac{\cos x}{1+\sin x} &= \frac{\cos x}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}\\ &= \frac{\cos x\,(1-\sin x)}{1-\sin^2 x}\\ &= \frac{\cos x\,(1-\sin x)}{\cos^2 x}\\ &= \frac{1-\sin x}{\cos x}\\ &= \text{Right side}\end{aligned}$$

7. a) For $x=\frac{\pi}{6}$:

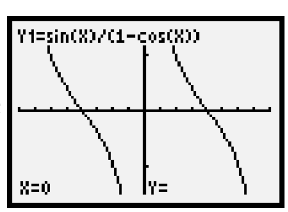
$$\begin{aligned}\frac{\sin x}{1-\cos x} &= \frac{\sin\frac{\pi}{6}}{1-\cos\frac{\pi}{6}}\\ &= \frac{\frac{1}{2}}{1-\frac{\sqrt{3}}{2}}\\ &= \frac{\frac{1}{2}}{\frac{2-\sqrt{3}}{2}}\\ &= \frac{1}{2-\sqrt{3}}\\ &= 2+\sqrt{3}\end{aligned}$$

$$\begin{aligned}\frac{1+\cos x}{\sin x} &= \frac{1+\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}\\ &= \frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}\\ &= \frac{\frac{2+\sqrt{3}}{2}}{\frac{1}{2}}\\ &= 2+\sqrt{3}\end{aligned}$$

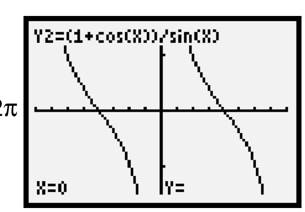
Use a calculator to verify the identity when $x = 1.5$.

$$\frac{\sin x}{1-\cos x} = \frac{\sin 1.5}{1-\cos 1.5} \doteq 1.0734 \qquad \frac{1+\cos x}{\sin x} = \frac{1+\cos 1.5}{\sin 1.5} \doteq 1.0734$$

b)

-2π 

2π

-2π 

2π

c) Left side:

$$\begin{aligned}\frac{\sin x}{1-\cos x} &= \frac{\sin x}{1-\cos x} \times \frac{1+\cos x}{1+\cos x}\\ &= \frac{\sin x\,(1+\cos x)}{1-\cos^2 x}\\ &= \frac{\sin x\,(1+\cos x)}{\sin^2 x}\\ &= \frac{1+\cos x}{\sin x}\\ &= \text{Right side}\end{aligned}$$

8. a) For $x = \frac{\pi}{3}$:

$$\begin{aligned}\frac{1+\sin x}{1+\csc x} &= \frac{1+\sin\frac{\pi}{3}}{1+\csc\frac{\pi}{3}}\\ &= \frac{1+\frac{\sqrt{3}}{2}}{1+\frac{2}{\sqrt{3}}}\\ &= \frac{\frac{2+\sqrt{3}}{2}}{\frac{\sqrt{3}+2}{\sqrt{3}}}\\ &= \left(\frac{2+\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{\sqrt{3}+2}\right)\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

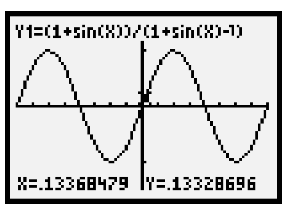
$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

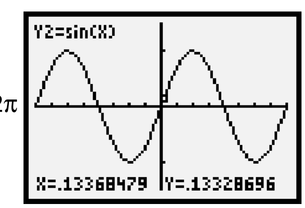
Use a calculator to verify the identity when $x = 1.5$.

$$\begin{aligned}\frac{1+\sin x}{1+\csc x} &= \frac{1+\sin 1.5}{1+\csc 1.5}\\ &\doteq 0.9975\end{aligned}$$

$\sin 1.5 \doteq 0.9975$

b)

-2π  2π

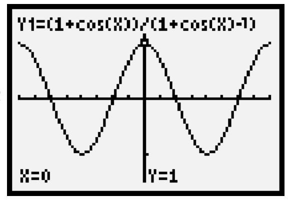
-2π  2π

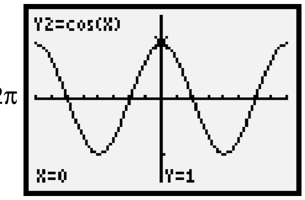
c) Left side:

$$\begin{aligned}\frac{1+\sin x}{1+\csc x} &= \frac{1+\sin x}{1+\frac{1}{\sin x}}\\ &= \frac{1+\sin x}{\frac{\sin x+1}{\sin x}}\\ &= (1+\sin x)\left(\frac{\sin x}{\sin x+1}\right)\\ &= \sin x\\ &= \text{Right side}\end{aligned}$$

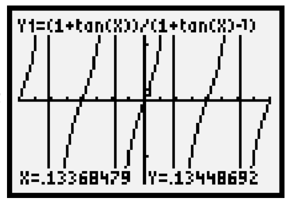
Selected Solutions — Chapter 5

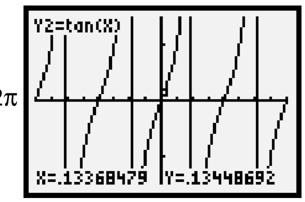
9. a)

-2π  2π

-2π  2π

b)

-2π  2π

-2π  2π

10. a) Left side:

$$\begin{aligned}
\frac{1+\cos x}{1-\cos x}+\frac{1+\sec x}{1-\sec x} &= \frac{1+\cos x}{1-\cos x}+\frac{1+\dfrac{1}{\cos x}}{1-\dfrac{1}{\cos x}}\\
&= \frac{1+\cos x}{1-\cos x}+\frac{\dfrac{\cos x+1}{\cos x}}{\dfrac{\cos x-1}{\cos x}}\\
&= \frac{1+\cos x}{1-\cos x}+\frac{\cos x+1}{\cos x-1}\\
&= \frac{1+\cos x}{1-\cos x}-\frac{1+\cos x}{1-\cos x}\\
&= 0\\
&= \text{Right side}
\end{aligned}$$

b) Left side:

$$\begin{aligned}
&\frac{1+\sin x}{1-\sin x}+\frac{1+\csc x}{1-\csc x}\\
&= \frac{1+\sin x}{1-\sin x}+\frac{1+\dfrac{1}{\sin x}}{1-\dfrac{1}{\sin x}}\\
&= \frac{1+\sin x}{1-\sin x}+\frac{\dfrac{\sin x+1}{\sin x}}{\dfrac{\sin x-1}{\sin x}}\\
&= \frac{1+\sin x}{1-\sin x}+\frac{\sin x+1}{\sin x-1}\\
&= \frac{1+\sin x}{1-\sin x}-\frac{1+\sin x}{1-\sin x}\\
&= 0\\
&= \text{Right side}
\end{aligned}$$

$$\begin{aligned}
&\frac{1+\tan x}{1-\tan x}+\frac{1+\cot x}{1-\cot x}\\
&= \frac{1+\tan x}{1-\tan x}+\frac{1+\dfrac{1}{\tan x}}{1-\dfrac{1}{\tan x}}\\
&= \frac{1+\tan x}{1-\tan x}+\frac{\tan x+\dfrac{1}{\tan x}}{\tan x-\dfrac{1}{\tan x}}\\
&= \frac{1+\tan x}{1-\tan x}+\frac{\tan x+1}{\tan x-1}\\
&= \frac{1+\tan x}{1-\tan x}-\frac{1+\tan x}{1-\tan x}\\
&= 0\\
&= \text{Right side}
\end{aligned}$$

Selected Solutions — Chapter 5

11. a) Left side:

$$\begin{aligned}\frac{\cos x}{1+\sin x}+\frac{\cos x}{1-\sin x}&=\frac{\cos x\,(1-\sin x+1+\sin x)}{1-\sin^2 x}\\&=\frac{2\cos x}{\cos^2 x}\\&=\frac{2}{\cos x}\\&=2\sec x\\&=\text{Right side}\end{aligned}$$

b) $\dfrac{\sin x}{1+\cos x}+\dfrac{\sin x}{1-\cos x}=2\csc x$

Left side:

$$\begin{aligned}\frac{\sin x}{1+\cos x}+\frac{\sin x}{1-\cos x}&=\frac{\sin x\,(1-\cos x+1+\cos x)}{1-\cos^2 x}\\&=\frac{2\sin x}{\sin^2 x}\\&=\frac{2}{\sin x}\\&=2\csc x\\&=\text{Right side}\end{aligned}$$

$\dfrac{\tan x}{1+\cos x}+\dfrac{\tan x}{1-\cos x}=2\sec x\ \csc x$

Left side:

$$\begin{aligned}\frac{\tan x}{1+\cos x}+\frac{\tan x}{1-\cos x}&=\frac{\tan x\,(1-\cos x+1+\cos x)}{1-\cos^2 x}\\&=\frac{2\tan x}{\sin^2 x}\\&=\frac{2\dfrac{\sin x}{\cos x}}{\sin^2 x}\\&=2\frac{\sin x}{\cos x}\times\frac{1}{\sin^2 x}\\&=\frac{2}{\cos x\ \sin x}\\&=2\sec x\ \csc x\\&=\text{Right side}\end{aligned}$$

12. a) Left side:

$$\begin{aligned}\sin x\ \tan x+\sec x&=\frac{\sin^2 x}{\cos x}+\frac{1}{\cos x}\\&=\frac{\sin^2 x+1}{\cos x}\\&=\text{Right side}\end{aligned}$$

b) Left side:

$$\begin{aligned}\frac{\sin x+\tan x}{\cos x+1}&=\frac{\sin x+\dfrac{\sin x}{\cos x}}{\cos x+1}\\&=\frac{\dfrac{\sin x\,(\cos x+1)}{\cos x}}{\cos x+1}\\&=\frac{\sin x}{\cos x}\\&=\tan x\\&=\text{Right side}\end{aligned}$$

Selected Solutions — Chapter 5

c) Left side:

$$\begin{aligned}\sin^2 x\ \cot^2 x &= \sin^2 x\frac{\cos^2 x}{\sin^2 x}\\ &= \cos^2 x\\ &= 1-\sin^2 x\\ &= \text{Right side}\end{aligned}$$

d) Left side:

$$\begin{aligned}\csc^2 x - 1 &= \frac{1}{\sin^2 x} - 1\\ &= \frac{1-\sin^2 x}{\sin^2 x}\\ &= \frac{\cos^2 x}{\sin^2 x}\\ &= \csc^2 x\ \cos^2 x\\ &= \text{Right side}\end{aligned}$$

13. a) Left side:

$$\begin{aligned}\tan x + \cot x &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\\ &= \frac{\sin^2 x + \cos^2 x}{\cos x\ \sin x}\\ &= \frac{1}{\cos x\ \sin x}\\ &= \sec x\ \csc x\\ &= \text{Right side}\end{aligned}$$

b) Left side:

$$\begin{aligned}\sec^2 x + \csc^2 x &= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\\ &= \frac{\sin^2 x + \cos^2 x}{\cos 2x\ \sin 2x}\\ &= \frac{1}{\cos^2 x\ \sin^2 x}\\ &= \sec^2 x\ \csc^2 x\end{aligned}$$

c) The graphs of $y = \sec^2 x - \csc^2 x$ and $y = \frac{\sec^2 x}{\csc^2 x}$ do not coincide.

d) Left side:

$$\begin{aligned}\sec^2 x + \csc^2 x &= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\\ &= \frac{\sin^2 x + \cos^2 x}{\cos^2 x\ \sin^2 x}\\ &= \frac{1}{\cos^2 x\ \sin^2 x}\end{aligned}$$

Right side:

$$\begin{aligned}(\tan x + \cot x)^2 &= \tan^2 x + 2\tan x \cot x + \cot^2 x\\ &= \frac{\sin^2 x}{\cos^2 x} + 2 + \frac{\cos^2 x}{\sin^2 x}\\ &= \frac{\sin^4 x + 2\cos^2 x\ \sin^2 x + \cos^4 x}{\cos^2 x\ \sin^2 x}\\ &= \frac{(\sin^2 x + \cos^2 x)^2}{\cos^2 x\ \sin^2 x}\\ &= \frac{1}{\cos^2 x\ \sin^2 x}\end{aligned}$$

Selected Solutions — Chapter 5

e) The graphs of $y = \cos^2 x$ and $y = \sin x\,(\csc x + \sin x)$ do not coincide.

f) Right side:

$$\begin{aligned}\cos x\,(\sec x - \cos x) &= \cos x\left(\frac{1}{\cos x} - \cos x\right)\\ &= 1 - \cos^2 x\\ &= \sin^2 x\\ &= \text{Left side}\end{aligned}$$

14. a) Left side:

$$\begin{aligned}\frac{\sin x + \cos x}{\csc x + \sec x} &= \frac{\sin x + \cos x}{\dfrac{1}{\sin x} + \dfrac{1}{\cos x}}\\ &= \frac{\sin x + \cos x}{\dfrac{\cos x + \sin x}{\sin x\ \cos x}}\\ &= \sin x\ \cos x\\ &= \text{Right side}\end{aligned}$$

b) Left side:

$$\begin{aligned}\frac{\sin x + \tan x}{\csc x + \cot x} &= \frac{\sin x + \dfrac{\sin x}{\cos x}}{\dfrac{1}{\sin x} + \dfrac{\cos x}{\sin x}}\\ &= \frac{\dfrac{\sin x\,(\cos x + 1)}{\cos x}}{\dfrac{1 + \cos x}{\sin x}}\\ &= \frac{\sin x\,(\cos x + 1)}{\cos x} \times \frac{\sin x}{1 + \cos x}\\ &= \frac{\sin^2 x}{\cos x}\\ &= \sin x \tan x\\ &= \text{Right side}\end{aligned}$$

15. a) Left side:

$$\begin{aligned}\frac{1}{1 + \sin x} + \frac{1}{1 - \sin x} &= \frac{1 - \sin x + 1 + \sin x}{1 - \sin^2 x}\\ &= \frac{2}{\cos^2 x}\\ &= 2\sec^2 x\\ &= \text{Right side}\end{aligned}$$

b) Left side:

$$\begin{aligned}\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} &= \frac{1 - \cos x + 1 + \cos x}{1 - \cos^2 x}\\ &= \frac{2}{\sin^2 x}\\ &= 2\csc^2 x\\ &= \text{Right side}\end{aligned}$$

Selected Solutions — Chapter 5

16. a) Left side:

$$\frac{\tan x}{\sec x + 1} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + 1}$$

$$= \frac{\frac{\sin x}{\cos x}}{\frac{1 + \cos x}{\cos x}}$$

$$= \frac{\sin x}{1 + \cos x}$$

$$= \frac{\sin x}{1 + \cos x} \times \frac{1 - \cos x}{1 - \cos x}$$

$$= \frac{\sin x\,(1 - \cos x)}{1 - \cos^2 x}$$

$$= \frac{\sin x\,(1 - \cos x)}{\sin^2 x}$$

$$= \frac{1 - \cos x}{\sin x}$$

Right side:

$$\frac{\sec x - 1}{\tan x} = \frac{\frac{1}{\cos x} - 1}{\frac{\sin x}{\cos x}}$$

$$= \frac{\frac{1 - \cos x}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$= \frac{1 - \cos x}{\sin x}$$

b) $\dfrac{\cot x}{\csc x + 1} = \dfrac{\csc x - 1}{\cot x}$

Left side:

$$\frac{\cot x}{\csc x + 1} = \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} + 1}$$

$$= \frac{\frac{\cos x}{\sin x}}{\frac{1 + \sin x}{\sin x}}$$

$$= \frac{\cos x}{1 + \sin x}$$

$$= \frac{\cos x}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x}$$

$$= \frac{\cos x\,(1 - \sin x)}{1 - \sin^2 x}$$

$$= \frac{\cos x\,(1 - \sin x)}{\cos^2 x}$$

$$= \frac{1 - \sin x}{\cos x}$$

Right side:

$$\frac{\csc x - 1}{\cot x} = \frac{\frac{1}{\sin x} - 1}{\frac{\cos x}{\sin x}}$$

$$= \frac{\frac{1 - \sin x}{\sin x}}{\frac{\cos x}{\sin x}}$$

$$= \frac{1 - \sin x}{\cos x}$$

17. a) $y = \sin x^2$
The function is even, $y = 0$ when $x = 0$, and the function has negative and positive values.

b) $y = \sin^2 x$
The function is even, $y = 0$ when $x = 0$, and $y \geq 0$.

c) $y = \cos x^2$
The function is even, $y = 1$ when $x = 0$, and the function has negative and positive values.

d) $y = \cos^2 x$
The function is even, $y = 1$ when $x = 0$, and $y \geq 0$.

Selected Solutions — Chapter 5

19. If $f(x) = \sec x$,

then $\frac{1}{1+\sec x} + \frac{1}{1-\sec x} = \frac{1-\sec x + 1 + \sec x}{(1+\sec x)(1-\sec x)}$

$= \frac{2}{1-\sec^2 x}$

$= \frac{2}{-\tan^2 x}$

$= -2\cot^2 x$

If $f(x) = \csc x$,

then $\frac{1}{1+\csc x} + \frac{1}{1-\csc x} = \frac{1-\csc x + 1 + \csc x}{(1+\csc x)(1-\csc x)}$

$= \frac{2}{1-\csc^2 x}$

$= \frac{2}{-\cot^2 x}$

$= -2\tan^2 x$

If $f(x) = \tan x$,

then $\frac{1}{1+\tan x} + \frac{1}{1-\tan x} = \frac{1-\tan x + 1 + \tan x}{(1+\tan x)(1-\tan x)}$

$= \frac{2}{1-\tan^2 x}$

If $f(x) = \cot x$,

then $\frac{1}{1+\cot x} + \frac{1}{1-\cot x} = \frac{1-\cot x + 1 + \cot x}{(1+\cot x)(1-\cot x)}$

$= \frac{2}{1-\cot^2 x}$

20. Answers may vary.

$(\sin x)(\cot x)(\sec x) = (\cos x)(\tan x)(\csc x)$

a) When $x = \frac{\pi}{4}$

$\text{LS} = \left(\sin\frac{\pi}{4}\right)\left(\cot\frac{\pi}{4}\right)\left(\sec\frac{\pi}{4}\right)$
$= \left(\frac{1}{\sqrt{2}}\right)(1)(\sqrt{2})$
$= 1$

$\text{RS} = \left(\cos\frac{\pi}{4}\right)\left(\tan\frac{\pi}{4}\right)\left(\csc\frac{\pi}{4}\right)$
$= \left(\frac{1}{\sqrt{2}}\right)(1)(\sqrt{2})$
$= 1$

When $x = 2$

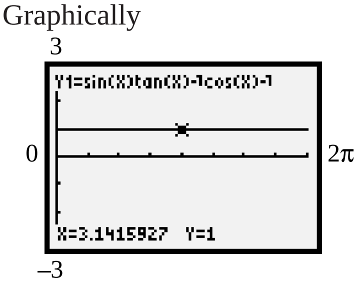
$\text{LS} = (\sin 2)(\cot 2)(\sec 2)$

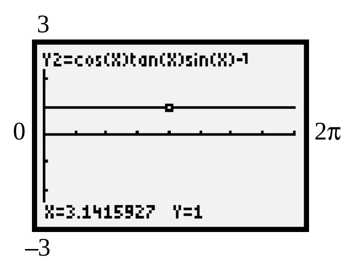
$\text{RS} = (\cos 2)(\tan 2)(\csc 2)$; $= (0.9093)(-0.4577)(-2.4030)$

$= (-0.4161)(-2.1850)(1.0998)$; $= 1.0000$

$= 1.0000$

Graphically





b) $\text{LS} = (\sin x)(\cot x)(\sec x)$
$= (\cos x)(\tan x)\frac{(\csc x)}{(\sec x)}(\sec x)$
$= (\cos x)(\tan x)(\csc x)$

$\text{RS} = (\cos x)(\tan x)(\csc x)$

Selected Solutions — Chapter 5

21. To find an identity that involved all six trigonometric functions I used all the different definitions of each function. I used trial and error to get the left side of the equation to equal the right side of the equation. I used $\sin x = \cos x \ \tan x$, $\cot x = \frac{\csc x}{\sec x}$ and came up with $\sin x \ \cot x \ \sec x = \cos x \ \tan x \ \csc x$.

Investigate, page 329

1. a) $3(x + y) = 3x + 3y$
This is the distributive property of multiplication over addition.

b) This can be explained by giving counter-examples for each pair.

$$(1 + 1)^2 \neq 1^2 + 1^2$$
$$4 \neq 2$$
$$\sqrt{1 + 1} \neq \sqrt{1} + \sqrt{1}$$
$$\sqrt{2} \neq 2$$
$$\frac{1}{1 + 1} \neq \frac{1}{1} + \frac{1}{1}$$
$$\frac{1}{2} \neq 2$$

2. No.
$\sin (1 + 1) \doteq 0.9093$
$\sin (1) + \sin (1) \doteq 1.6829$

3. No.
$\cos (1 + 1) \doteq -0.4161$
$\cos (1) + \sin (1) \doteq 1.0806$

4. Only the expressions $3(x + y)$ and $3x + 3y$ have the same values.
This is the distributive property of multiplication over subtraction.

5.5 Exercises, page 333

1. a) For $x = 2.2$ and $y = 1.4$:

$$\sin (x + y) = \sin (2.2 + 1.4)$$
$$= \sin 3.6$$
$$\doteq -0.4425$$
$$\sin x \ \cos y + \cos x \ \cos y = \sin 2.2 \ \cos 1.4 + \cos 2.2 \ \sin 1.4$$
$$\doteq -0.4425$$

b) For $x = 1.8$ and $y = -3.2$:

$$\sin (x + y) = \sin (1.8 + (-3.2))$$
$$= \sin (-1.4)$$
$$\doteq -0.9854$$
$$\sin x \ \cos y + \cos x \ \cos y = \sin 1.8 \ \cos (-3.2) + \cos 1.8 \ \sin (-3.2)$$
$$\doteq -0.9854$$

Selected Solutions — Chapter 5

2. a) For $x = 3.5$ and $y = 4.8$:

$$\begin{aligned}\cos(x+y) &= \cos(3.5+4.8)\\ &= \cos 8.3\\ &\doteq -0.4314\end{aligned}$$

$$\begin{aligned}\cos x\ \cos y - \sin x\ \sin y &= \cos 3.5\ \cos 4.8 - \sin 3.5\ \sin 4.8\\ &\doteq -0.4314\end{aligned}$$

b) For $x = 5.7$ and $y = -2.4$:

$$\begin{aligned}\cos(x+y) &= \cos(5.7+(-2.4))\\ &= \cos 3.3\\ &\doteq -0.9875\end{aligned}$$

$$\begin{aligned}\cos x\ \cos y - \sin x\ \sin y &= \cos 5.7\ \cos(-2.4) - \sin 5.7\ \sin(-2.4)\\ &\doteq -0.9875\end{aligned}$$

3. a)

$$\begin{aligned}\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right) &= \sin\frac{\pi}{4}\cos\frac{\pi}{4} + \cos\frac{\pi}{4}\sin\frac{\pi}{4}\\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\\ &= \frac{2}{4}+\frac{2}{4}\\ &= 1\end{aligned}$$

$$\begin{aligned}\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right) &= \sin\left(\frac{2\pi}{4}\right)\\ &= \sin\left(\frac{\pi}{2}\right)\\ &= 1\end{aligned}$$

b)

$$\begin{aligned}\cos\left(\frac{\pi}{4}+\frac{\pi}{4}\right) &= \cos\frac{\pi}{4}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\sin\frac{\pi}{4}\\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\\ &= \frac{2}{4}-\frac{2}{4}\\ &= 0\end{aligned}$$

$$\begin{aligned}\cos\left(\frac{\pi}{4}+\frac{\pi}{4}\right) &= \cos\left(\frac{2\pi}{4}\right)\\ &= \cos\left(\frac{\pi}{2}\right)\\ &= 0\end{aligned}$$

6. There is an infinite number of sums and differences that result in a given angle.

8. For part a:
The first coordinate of P is the cosine of ∠POA. Thus, Graph 4, which is the cosine graph, represents the first coordinate of P.
For part b:
The second coordinate of P is the sine of ∠POA. Thus, Graph 1, which is the sine graph, represents the second coordinate of P.
The arc length from P to Q is 1, so ∠POQ = 1 radian. Thus, ∠QOA = ∠POA + 1.
For part c:
The second coordinate of Q is the sine of ∠QOA. Thus, Graph 2, which is the sine graph with a horizontal shift of –1, represents the second coordinate of Q.

Selected Solutions — Chapter 5

10. a) When $x = \frac{\pi}{4}$:

$$\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right) = \sin\frac{3\pi}{4}$$
$$= \frac{\sqrt{2}}{2}$$

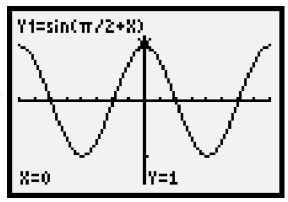
$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

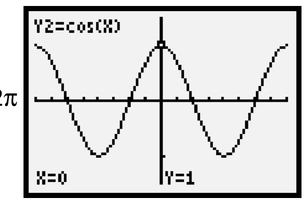
When $x = 2.7$:

$$\sin\left(\frac{\pi}{2} + 2.7\right) \doteq -0.9041$$

$$\cos 2.7 \doteq -0.9041$$

b)

-2π  2π

-2π  2π

c) Left side:

$$\sin\left(\frac{\pi}{2} + x\right) = \sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x$$
$$= 1\cos x + 0\sin x$$
$$= \cos x$$
$$= \text{Right side}$$

11. a) When $x = \frac{\pi}{6}$:

$$\cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \cos\frac{\pi}{3}$$
$$= \frac{1}{2}$$

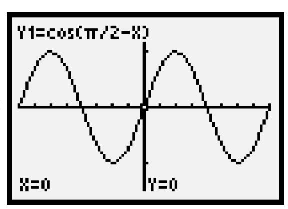
$$\sin\frac{\pi}{6} = \frac{1}{2}$$

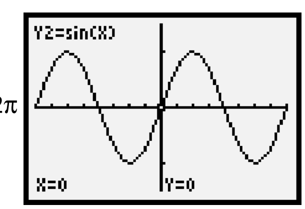
When $x = 1.2$:

$$\cos\left(\frac{\pi}{2} - 1.2\right) \doteq 0.9320$$

$$\sin 1.2 \doteq 0.9320$$

b)

-2π  2π

-2π  2π

c) Left side:

$$\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$$
$$= 0\cos x + 1\sin x$$
$$= \sin x$$
$$= \text{Right side}$$

12. a) Left side:

$$\sin\left(\frac{\pi}{2} - x\right) = \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$$
$$= 1\cos x - 0\sin x$$
$$= \cos x$$
$$= \text{Right side}$$

b) Left side:

$$\begin{aligned}\cos(\pi + x) &= \cos\pi\ \cos x - \sin\pi\ \sin x\\ &= -1\cos x - 0\sin x\\ &= -\cos x\end{aligned}$$

c) Left side:

$$\begin{aligned}\cos\left(\frac{3\pi}{2} + x\right) &= \cos\frac{3\pi}{2}\ \cos x - \sin\frac{3\pi}{2}\ \sin x\\ &= 0\cos x - (-1)\sin x\\ &= \sin x\\ &= \text{Right side}\end{aligned}$$

d) Left side:

$$\begin{aligned}\sin\left(\frac{3\pi}{2} - x\right) &= \sin\frac{3\pi}{2}\ \cos x - \cos\frac{3\pi}{2}\ \sin x\\ &= -1\cos x - 0\sin x\\ &= -\cos x\\ &= \text{Right side}\end{aligned}$$

16. $$\begin{aligned}\cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) &= \cos\frac{\pi}{4}\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

$$\begin{aligned}\cos\frac{\pi}{2} - \cos\frac{\pi}{4} &= 0 - \frac{\sqrt{2}}{2}\\ &= -\frac{\sqrt{2}}{2}\end{aligned}$$

$$\begin{aligned}\sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right) &= \sin\frac{\pi}{4}\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

$$\sin\frac{\pi}{2} - \sin\frac{\pi}{4} = 1 - \frac{\sqrt{2}}{2}$$

17. a) Left side:

$$\begin{aligned}\sin\left(x + \frac{0\pi}{6}\right) &= \sin x\ \cos\frac{0\pi}{6} + \cos x\ \sin\frac{0\pi}{6}\\ &= 1\sin x + 0\cos x\\ &= \text{Right side}\end{aligned}$$

Left side:

$$\begin{aligned}\sin\left(x + \frac{\pi}{6}\right) &= \sin x\ \cos\frac{\pi}{6} + \cos x\ \sin\frac{\pi}{6}\\ &= \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\\ &= \text{Right side}\end{aligned}$$

Left side:

$$\begin{aligned}\sin\left(x + \frac{2\pi}{6}\right) &= \sin x\ \cos\frac{2\pi}{6} + \cos x\ \sin\frac{2\pi}{6}\\ &= \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\\ &= \text{Right side}\end{aligned}$$

Left side:

$$\begin{aligned}\sin\left(x + \frac{3\pi}{6}\right) &= \sin x\ \cos\frac{3\pi}{6} + \cos x\ \sin\frac{3\pi}{6}\\ &= 0\sin x + 1\cos x\\ &= \text{Right side}\end{aligned}$$

b) $\sin\left(x + \frac{4\pi}{6}\right) = -\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$

Left side:

$$\begin{aligned}\sin\left(x + \frac{4\pi}{6}\right) &= \sin x \cos\frac{4\pi}{6} + \cos x \sin\frac{4\pi}{6} \\ &= -\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x \\ &= \text{Right side}\end{aligned}$$

$\sin\left(x + \frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$

Left side:

$$\begin{aligned}\sin\left(x + \frac{5\pi}{6}\right) &= \sin x \cos\frac{5\pi}{6} + \cos x \sin\frac{5\pi}{6} \\ &= -\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x \\ &= \text{Right side}\end{aligned}$$

$\sin\left(x + \frac{6\pi}{6}\right) = -1\sin x + 0\cos x$

Left side:

$$\begin{aligned}\sin\left(x + \frac{6\pi}{6}\right) &= \sin x \cos\frac{6\pi}{6} + \cos x \sin\frac{6\pi}{6} \\ &= -1\sin x + 0\cos x \\ &= \text{Right side}\end{aligned}$$

18. a) $\sin\left(x + \frac{0\pi}{4}\right) = 1\sin x + 0\cos x$

$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$

$\sin\left(x + \frac{2\pi}{4}\right) = 0\sin x + 1\cos x$

$\sin\left(x + \frac{3\pi}{4}\right) = \left(-\frac{1}{\sqrt{2}}\right)\sin x + \left(\frac{1}{\sqrt{2}}\right)\cos x$

b) $\sin\left(x + \frac{0\pi}{2}\right) = 1\sin x + 0\cos x$

$\sin\left(x + \frac{\pi}{2}\right) = 0\sin x + 1\cos x$

$\sin\left(x + \frac{2\pi}{2}\right) = (-1)\sin x + 0\cos x$

$\sin\left(x + \frac{3\pi}{2}\right) = 0\sin x + (-1)\cos x$

The graphs alternate $\sin x$, $\cos x$, $-\sin x$, $-\cos x$

c) $\sin(x + 0\pi) = 1\sin x + 0\cos x$

$\sin(x + \pi) = (-1)\sin x + 0\cos x$

$\sin(x + 2\pi) = 1\sin x + 0\cos x$

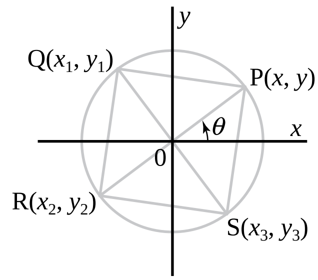
$\sin(x + 3\pi) = (-1)\sin x + 0\cos x$

The graphs alternate $\sin x$, $-\sin x$, $\sin x$, $-\sin x$

19. b) Yes. The coordinates of P and Q do not depend on what quadrant they are in.

Selected Solutions — Chapter 5

21. Explanations may vary. For part b:
First, I drew a diagram:



I let θ represent ∠POA. Thus, the coordinates of P are ($\cos\theta$, $\sin\theta$).
Thus, $x = \cos\theta$ and $y = \sin\theta$.

Since PQRS is a square, $\angle POQ = \frac{\pi}{2}$. Thus, $\angle AOQ = \theta + \frac{\pi}{2}$

The coordinates of Q are thus $\left(\cos\left(\theta + \frac{\pi}{2}\right), \sin\left(\theta + \frac{\pi}{2}\right)\right)$.

$$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta \qquad\qquad \sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$$
$$= -y \qquad\qquad = x$$

Thus, the coordinates of Q are ($-y$, x).

I can see by symmetry that the coordinates of R are ($-x$, $-y$), and the coordinates of S are (y, $-x$).

22. **a)** Left side:

$$\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right)$$
$$= \sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x$$
$$= 2\sin\frac{\pi}{4}\cos x$$
$$= 2\left(\frac{\sqrt{2}}{2}\right)\cos x$$
$$= \sqrt{2}\cos x$$
$$= \text{Right side}$$

d) Left side:

$$\sin(\theta + x) + \sin(\theta - x)$$
$$= \sin\theta\cos x + \cos\theta\sin x + \sin\theta\cos x - \cos\theta\sin x$$
$$= 2\sin\theta\cos x$$
$$= \text{Right side}$$

24. **a)** $\sin(x + y) = \sin x + \sin y$
$\sin x\cos y + \cos x\sin y = \sin x + \sin y$
This is true if $\cos y = 1$ and $\cos x = 1$. Choose x and y from the values $2n\pi$ for n any integer.

Selected Solutions — Chapter 5

b) $\cos(x+y) - \cos x + \cos y$
$\cos x \cos y - \sin x \sin y = \cos x + \cos y$
This is true for $\cos y = 1$ and $-\sin x \sin y = \cos y$.
Since $\cos y = 1$,
$-\sin x \sin y = 1$
$\sin x \sin y = -1$
Thus $\sin x = 1$ and $\sin y = -1$ or $\sin x = -1$ and $\sin y = 1$.
In the first case, $x = \frac{\pi}{2} + 2n\pi$ and $y = \frac{3\pi}{2} + 2n\pi$. But if $y = \frac{3\pi}{2} + 2n\pi$, then $\cos y = 0$. This is impossible.
In the second case, $x = \frac{3\pi}{2} + 2n\pi$ and $y = \frac{\pi}{2} + 2n\pi$. But if $y = \frac{\pi}{2} + 2n\pi$, then $\cos y = 0$. This is impossible. Thus, there are no values of x and y for which $\cos(x+y) = \cos x + \cos y$.

c)
$$\tan(x+y) = \tan x + \tan y$$
$$\frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}$$
$$\frac{\sin(x+y)}{\cos x \cos y - \sin x \sin y} = \frac{\cos y \sin x + \cos x \sin y}{\cos x \cos y}$$
$$\frac{\sin(x+y)}{\cos x \cos y - \sin x \sin y} = \frac{\sin(x+y)}{\cos x \cos y}$$
Thus, either $\sin(x+y) = 0$, or $\frac{1}{\cos x \cos y - \sin x \sin y} = \frac{1}{\cos x \cos y}$.
In the first case, $x + y = 2n\pi$ for any integer n.
In the second case, $\sin x \sin y = 0$. Thus $x = n\pi$, $y = n\pi$, or both.

25. Explanations may vary. For part a:
I expanded the left side of the equation:
$\sin x \cos y + \cos x \sin y = \sin x + \sin y$
I saw that this is true if $\cos y = 1$ and $\cos x = 1$. I can choose x and y from the values $2n\pi$ for n any integer.

26. a) $\sin(x+k) = \sin x + \sin k$
$\sin x \cos k + \cos x \sin k = \sin x + \sin k$
This appears to be true if $\cos k = 1$ and $\cos x = 1$. But, if $\cos k = 1$, $k = 2n\pi$, in which case $\sin k = 0$. Thus, when $k = 2n\pi$,
Left side:
$$\begin{aligned}\sin x \cos k + \cos x \sin k &= \sin x \cos 2n\pi + \cos x \sin 2n\pi \\ &= \sin x\end{aligned}$$
Right side:
$$\begin{aligned}\sin x + \sin k &= \sin x + \sin 2n\pi \\ &= \sin x\end{aligned}$$
The equation is true for all values of x if $k = 2n\pi$.

b) $\cos(x+k) = \cos x + \cos k$
$\cos x \cos k - \sin x \sin k = \cos x + \cos k$
This appears to be true if $\cos k = 1$ and $-\sin x \sin k = \cos k$.
If $\cos k = 1$, $k = 2n\pi$, which implies that $-\sin x \sin k = 0$. But this is impossible, since $-\sin x \sin k = \cos k$. Thus, there are no values of k for which $\cos(x+k) = \cos x + \cos k$.

Selected Solutions — Chapter 5

27.

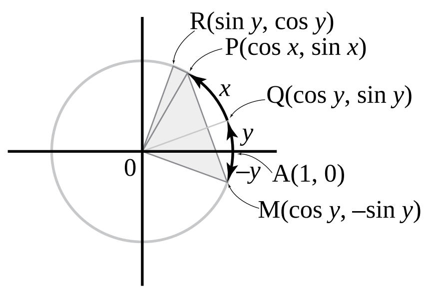
$$\begin{aligned}
PR &= SB \\
PR^2 &= SB^2 \\
(\cos x + \sin y)^2 + (\sin x - \cos y)^2 &= \cos^2 (x - y) + [\sin (x - y) - 1]^2 \\
\cos^2 x + 2\cos x \ \sin y + \sin^2 y + \sin^2 x - 2\sin x \ \cos y + \cos^2 y &= \cos^2 (x - y) + \sin^2 (x - y) - 2\sin (x - y) + 1 \\
\cos^2 x + \sin^2 x + \sin^2 y + \cos^2 y + 2\cos x \ \sin y - 2\sin x \ \cos y &= 1 - 2\sin (x - y) + 1 \\
1 + 1 - 2(\sin x \ \cos y - \cos x \ \sin y) &= 2 - 2\sin (x - y) \\
2(\sin x \ \cos y - \cos x \ \sin y) &= 2\sin (x - y) \\
\sin x \ \cos y - \cos x \ \sin y &= \sin (x - y)
\end{aligned}$$

28. Deriving identities for $\cos (x + y)$ and $\sin (x + y)$.

Step 1

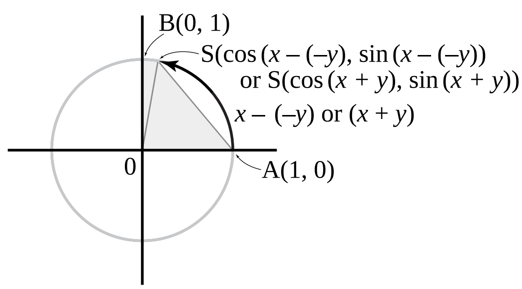
Let $P(\cos x, \ \sin x)$ and $Q(\cos y, \ \sin y)$ be points on a unit circle, where x and y represent the corresponding arc lengths from A(1, 0). Let M be on the unit circle such that the arc length from A to M is $-y$. Since M is the reflection of Q in the x-axis, the coordinates of M are represented by $(\cos y, \ -\sin y)$.

Let R be on the circle such that OR is perpendicular to OM and the coordinates of R may be represented by $(\sin y, \ \cos y)$



Step 2

Rotate the quadrilateral OMPR counterclockwise about the origin through $\angle$MOA. Then M coincides with A, R coincides with B(0, 1) and P coincides with S. The arc length from A to S is $x - (-y)$ or $x + y$. Hence, the coordinates of S are $(\cos (x + y), \ \sin (x + y))$.



Selected Solutions — Chapter 5

Step 3
Since $\angle POM = \angle SOA$, $\triangle POM \cong \triangle SOA$
Since the triangles are congruent,

$$PM = SA$$

Use the distance formula.

$$\sqrt{(\cos x - \cos y)^2 + (\sin x + \sin y)^2} = \sqrt{(\cos (x + y) - 1)^2 + (\sin (x + y) - 0)^2}$$

Square each side to eliminate the radicals.

$$(\cos x - \cos y)^2 + (\sin x + \sin y)^2 = (\cos (x + y) - 1)^2 + (\sin (x + y) - 0)^2$$

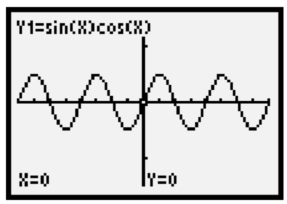
Expand.

$$\cos^2 x - 2\cos x \cos y + \cos^2 y + \sin^2 x + 2\sin x \sin y + \sin^2 y = \cos^2 (x + y) - 2\cos (x + y) + 1 + \sin^2 (x + y)$$
$$(\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2\cos x \cos y + 2\sin x \sin y = [\cos^2 (x + y) + \sin^2 (x + y)] + 1 - 2\cos (x + y)$$
$$2 - 2(\cos x \cos y - \sin x \sin y) = 2 - 2\cos (x + y)$$
$$\cos (x + y) = \cos x \cos y - \sin x \sin y$$

Similarly, using PR = SB, we can prove that
$\sin (x + y) = \sin x \cos y + \cos x \sin y$

Exploring with a Graphing Calculator, page 337

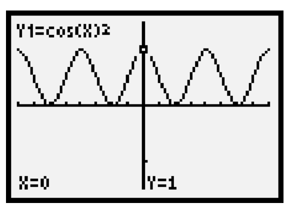
1. a)

-2π  2π

2. b) Left side:

$$\begin{aligned}\sin (2x) &= \sin (x + x)\\ &= \sin x \cos x + \cos x \sin x\\ &= 2\sin x \cos x\\ &= \text{Right side}\end{aligned}$$

3. a)

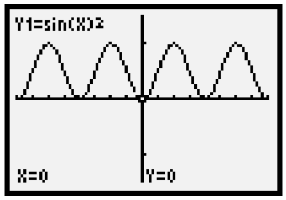
-2π  2π

4. b) Left side:

$$\begin{aligned}\cos (2x) &= \cos (x + x)\\ &= \cos x \cos x - \sin x \sin x\\ &= \cos^2 x - \sin^2 x\\ &= \cos^2 x - (1 - \cos^2 x)\\ &= 2\cos^2 x - 1\\ &= \text{Right side}\end{aligned}$$

Selected Solutions — Chapter 5

5. a)

-2π  2π

6. b) Left side:

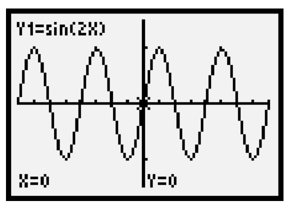
$$\begin{aligned}\cos(2x) &= \cos(x + x)\\ &= \cos x \cos x - \sin x \sin x\\ &= \cos^2 x - \sin^2 x\\ &= 1 - \sin^2 x - \sin^2 x\\ &= 1 - 2\sin^2 x\\ &= \text{Right side}\end{aligned}$$

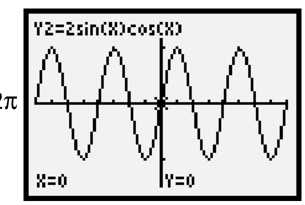
Investigate, page 338

1. $\sin(x + y) = \sin x \cos y + \cos x \cos y$

$$\begin{aligned}\sin(2x) &= \sin(x + x)\\ &= \sin x \cos x + \cos x \sin x\\ &= 2\sin x \cos x\end{aligned}$$

2. b)

-2π  2π

-2π  2π

3. a) Yes. This proves the identity for all values of x.

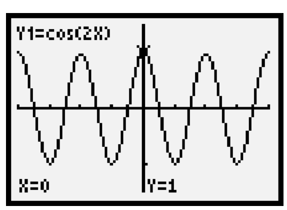
b) No. This proves the identity for some values of x.

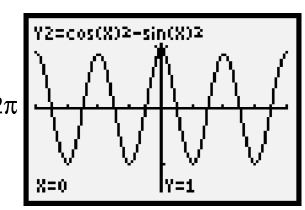
c) Yes. This proves the identity for all values of x.

4. $\cos(x + y) = \cos x \cos y - \sin x \sin y$

$$\begin{aligned}\cos(2x) &= \cos(x + x)\\ &= \cos x \cos x - \sin x \sin x\\ &= \cos^2 x - \sin^2 x\end{aligned}$$

5. b)

-2π  2π

-2π  2π

6. a) Yes. This proves the identity for all values of x.

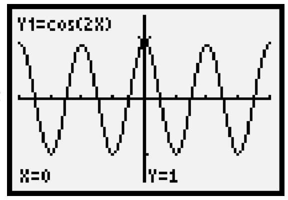
b) No. This proves the identity for some values of x.

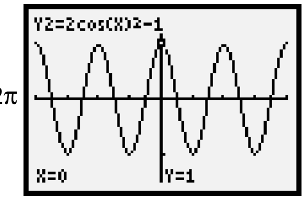
c) Yes. This proves the identity for all values of x.

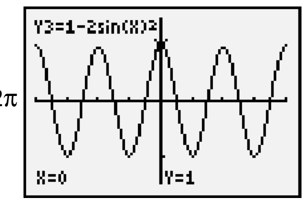
Selected Solutions — Chapter 5

7. $\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - \sin^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 1 - 2(1 - \cos^2 x)$
$= 2\cos^2 x - 1$

8. b)

–2π  2π

–2π  2π

–2π  2π

5.6 Exercises, page 342

1. a) Evaluate each side of the equation. The results are both approximately 0.783 326 9.

b) Evaluate each side of the equation. The results are both approximately –0.756 802 5.

c) Evaluate each side of the equation. The results are both approximately –0.977 864 6.

d) Evaluate each side of the equation. The results are both approximately 0.544 021 1.

e) Evaluate each side of the equation. The results are both approximately –0.873 297 3.

2. a) Evaluate each side of the equation. The results are both approximately 0.189 640 8.

b) Evaluate each side of the equation. The results are both approximately 0.960 170 3.

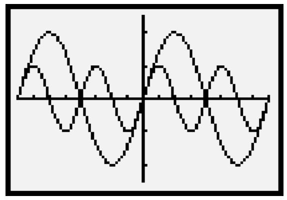
c) Evaluate each side of the equation. The results are both approximately 0.667 462 8.

d) Evaluate each side of the equation. The results are both approximately –0.957 659 5.

e) Evaluate each side of the equation. The results are both approximately –0.525 296 3.

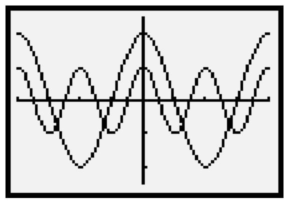
Selected Solutions — Chapter 5

3. a)



b) The graphs do not coincide for all values of x.

4. a)



b) The graphs do not coincide for all values of x.

8. a) The first coordinate of P is the cosine of ∠POA. Graph 4 represents a cosine function.

b) The second coordinate of P is the sine of ∠POA. Graph 1 represents a sine function.

c) The first coordinate of Q is the cosine of ∠QOA, which is twice the size of ∠POA. Graph 3 represents the cosine function of an angle twice the size of ∠POA.

d) The second coordinate of Q is the sine of ∠QOA, which is twice the size of ∠POA. Graph 2 represents the sine function of an angle twice the size of ∠POA.

10. a) For $x = \frac{\pi}{6}$:

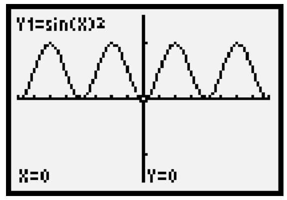
$$\sin^2 x = \sin^2 \frac{\pi}{6} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

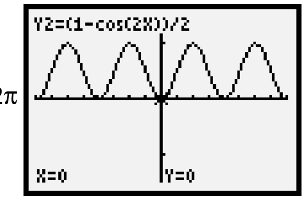
$$\frac{1 - \cos 2x}{2} = \frac{1 - \cos 2\left(\frac{\pi}{6}\right)}{2} = \frac{1 - \cos \frac{\pi}{3}}{2} = \frac{1 - \frac{1}{2}}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$

For $x = 2.3$ evaluate each side of the equation. The results are both approximately 0.556 076 3.

Selected Solutions — Chapter 5

b)

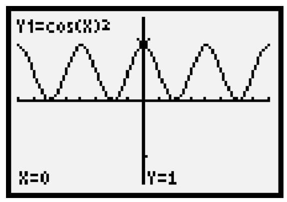
-2π  2π

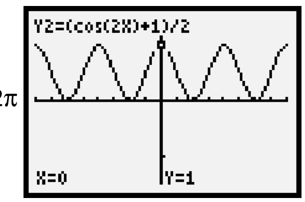
-2π  2π

c) Right side:

$$\frac{1-\cos 2x}{2} = \frac{1-(1-2\sin^2 x)}{2}$$
$$= \frac{2\sin^2 x}{2}$$
$$= \sin^2 x$$
$$= \text{Left side}$$

11. a)

-2π  2π

-2π  2π

b) Right side:

$$\frac{\cos 2x+1}{2} = \frac{2\cos^2 x-1+1}{2}$$
$$= \frac{2\cos^2 x}{2}$$
$$= \cos^2 x$$
$$= \text{Left side}$$

12. Explanations may vary. I used the equation $\cos 2x = 2\cos^2 x - 1$ and solved for $\cos^2 x$ to get $\cos^2 x = \frac{\cos 2x + 1}{2}$.

13. a) Right side:

$$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \, \cos x + \cos^2 x$$
$$= 1 + \sin 2x$$
$$= \text{Left side}$$

b) Right side:

$$2\cot x \, \sin^2 x = 2\frac{\cos x}{\sin x}\sin^2 x$$
$$= 2\cos x \, \sin x$$
$$= \sin 2x$$
$$= \text{Left side}$$

c) Right side:

$$\frac{1-\tan^2 x}{1+\tan^2 x} = \frac{1-\frac{\sin^2 x}{\cos^2 x}}{\sec^2 x}$$
$$= \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$$
$$= \cos 2x$$
$$= \text{Left side}$$

Selected Solutions — Chapter 5

d) Right side:

$$\frac{2}{1+\cos 2x} = \frac{2}{1+2\cos^2 x - 1}$$
$$= \frac{2}{2\cos^2 x}$$
$$= \sec^2 x$$
$$= \text{Left side}$$

16. a) $\sin\frac{\pi}{3} = \sin\frac{2\pi}{6}$

$$= 2\sin\frac{\pi}{6}\cos\frac{\pi}{6}$$
$$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{3}}{2}$$

$\cos\frac{\pi}{3} = \cos\frac{2\pi}{6}$

$$= 1 - 2\sin^2\frac{\pi}{6}$$
$$= 1 - 2\left(\frac{1}{2}\right)^2$$
$$= 1 - 2\left(\frac{1}{4}\right)$$
$$= \frac{1}{2}$$

b) $\sin\frac{2\pi}{3} = 2\sin\frac{\pi}{3}\cos\frac{\pi}{3}$

$$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3}}{2}$$

$\cos\frac{2\pi}{3} = 1 - 2\sin^2\frac{\pi}{3}$

$$= 1 - 2\left(\frac{\sqrt{3}}{2}\right)^2$$
$$= 1 - 2\left(\frac{3}{4}\right)$$
$$= -\frac{1}{2}$$

17. a) $\sin\frac{\pi}{2} = \sin\frac{2\pi}{4}$

$$= 2\sin\frac{\pi}{4}\cos\frac{\pi}{4}$$
$$= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= 1$$

$\cos\frac{\pi}{2} = \cos\frac{2\pi}{4}$

$$= 2\cos^2\frac{\pi}{4} - 1$$
$$= 2\left(\frac{\sqrt{2}}{2}\right)^2 - 1$$
$$= 2\left(\frac{2}{4}\right) - 1$$
$$= 0$$

Selected Solutions — Chapter 5

b) $\sin\pi = \sin\frac{2\pi}{2}$

$= 2\sin\frac{\pi}{2}\cos\frac{\pi}{2}$

$= 2(1)(0)$

$= 0$

$\cos\pi = \cos\frac{2\pi}{2}$

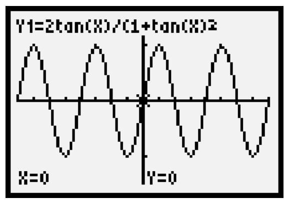
$= 2\cos^2\frac{\pi}{2} - 1$

$= 2(0)^2 - 1$

$= -1$

18. Explanations may vary. For exercise 17a:
I wrote $\sin\frac{\pi}{2}$ as $\sin\frac{2\pi}{4}$, then used the identity for $\sin 2x$ to write $\sin\frac{2\pi}{4}$ as $2\sin\frac{\pi}{4}\cos\frac{\pi}{4}$, which is equal to $2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$, or 1.
I wrote $\cos\frac{\pi}{2}$ as $\cos\frac{2\pi}{4}$, then used the identity $\cos 2x = 2\cos^2 x - 1$ to write $\cos\frac{2\pi}{4}$ as $2\cos^2\frac{\pi}{4} - 1$, which is equal to $2\left(\frac{\sqrt{2}}{2}\right)^2 - 1$, or 0.

19. a)

-2π  2π

c) Left side:

$$\frac{2\tan x}{1+\tan^2 x} = \frac{2\tan x}{\sec^2 x}$$

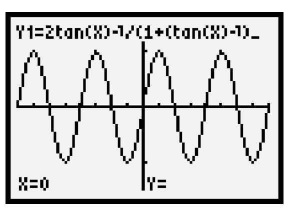
$$= \frac{2 \times \frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}}$$

$$= 2\sin x\cos x$$

$$= \sin 2x$$

= Right side

20. a)

-2π  2π

c) Left side:

$$\frac{2\cot x}{1+\cot^2 x} = \frac{2\cot x}{\csc^2 x}$$

$$= \frac{2\frac{\cos x}{\sin x}}{\frac{1}{\sin^2 x}}$$

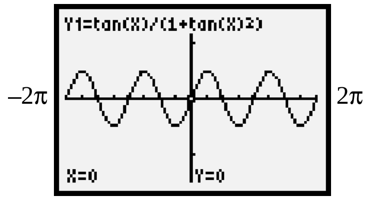
$$= 2\sin x\cos x$$

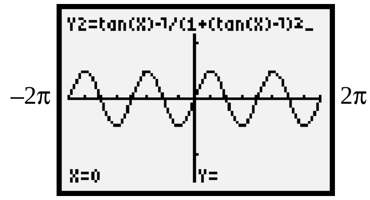
$$= \sin 2x$$

= Right side

Selected Solutions — Chapter 5

21. b)





22. b) Yes. The coordinates of P and Q do not depend on what quadrant they are in.

24. a) Left side:

$$\begin{aligned}\sin^2 2x + \cos^2 2x &= (2\sin x\ \cos x)^2 + (\cos^2 x - \sin^2 x)^2\\ &= 4\sin^2 x\ \cos^2 x + \cos^4 x - 2\sin^2 x\ \cos^2 x + \sin^4 x\\ &= \cos^4 x + 2\sin^2 x\ \cos^2 x + \sin^4 x\\ &= (\cos^2 x + \sin^2 x)^2\\ &= 1^2\\ &= 1\\ &= \text{Right side}\end{aligned}$$

b) I chose $\cos 2x = \cos^2 x - \sin^2 x$ so the expression would simplify to a perfect square trinomial.

26. a)

$$\begin{aligned}2\sin x\ \cos x - 1 &= 0\\ \sin 2x - 1 &= 0\\ \sin 2x &= 1\\ 2x &= \frac{\pi}{2} + 2n\pi\\ x &= \frac{\pi}{4} + n\pi\end{aligned}$$

b)

$$\begin{aligned}4\sin x\ \cos x - 1 &= 0\\ 2\sin 2x - 1 &= 0\\ 2\sin 2x &= 1\\ \sin 2x &= \frac{1}{2}\end{aligned}$$

$$2x = \frac{\pi}{6} + 2n\pi \text{ or } 2x = \frac{5\pi}{6} + 2n\pi$$

$$x = \frac{\pi}{12} + n\pi \text{ or } x = \frac{5\pi}{12} + n\pi$$

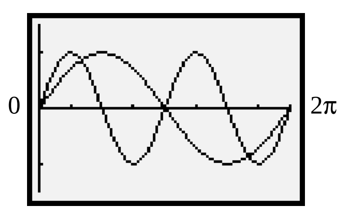
27. a)

$$\begin{aligned}\cos^2 x - \sin^2 x &= 1\\ \cos 2x &= 1\\ 2x &= 2n\pi\\ x &= n\pi\end{aligned}$$

b)

$$\begin{aligned}\sin^2 x - \cos^2 x &= 1\\ -\cos 2x &= 1\\ \cos 2x &= -1\\ 2x &= (2n-1)\pi\\ x &= (2n-1)\frac{\pi}{2}\end{aligned}$$

28. a)



b)

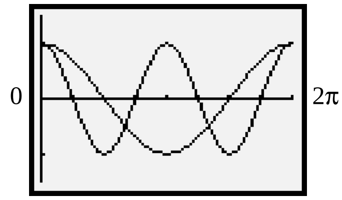
$$\begin{aligned}\sin x &= \sin 2x\\ \sin x &= 2\sin x\ \cos x\\ \sin x(1 - 2\cos x) &= 0\end{aligned}$$

$$\sin x = 0 \text{ or } \cos x = \frac{1}{2}$$

$$x = 0,\ \pi \text{ or } x = \frac{\pi}{3},\ \frac{5\pi}{3}$$

Selected Solutions — Chapter 5

29. **a)**



b)

$$\cos x = \cos 2x$$
$$\cos x = 2\cos^2 x - 1$$
$$2\cos^2 x - \cos x - 1 = 0$$
$$(2\cos x + 1)(\cos x - 1) = 0$$
$$\cos x = -\frac{1}{2} \text{ or } \cos x = 1$$
$$x = \frac{2\pi}{3}, \frac{4\pi}{3} \text{ or } x = 0$$

30. **a)** The right side looks almost like a perfect square trinomial.
b) The last term is negative instead of positive.

Problem Solving, page 346

1. **a)** $y = x$

b) $|x - \sin x| < 0.01$
Graph $y = x - \sin x$ and $y = \pm 0.01$. Find points on $y = x - \sin x$ that are between the lines $y = 0.01$ and $y = -0.01$. Use the intersect tool to find the points of intersection. These are (0.3925, 0.01) and (–0.3925, –0.01). So, $|x - \sin x| < 0.01$ for $-0.3925 < x < 0.3925$.

2. **a)** a is negative.

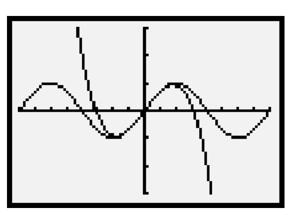
b) From the graph, a appears to be about –0.5. Thus, the equation of the parabola is $y = -0.5x^2 + 1$.

c) Find values for which $|-0.5x^2 + 1| < 0.01$. Graph $y = -0.5x^2 + 1$ and $y = \pm 0.01$. Find points on $y = -0.5x^2 + 1$ that are between the lines $y = 0.01$ and $y = -0.01$. Use the intersect tool to find the points of intersection. These are –1.4213 and 1.4213. So, $|-0.5x^2 + 1| < 0.01$ for $-1.4213 < x < 1.4213$

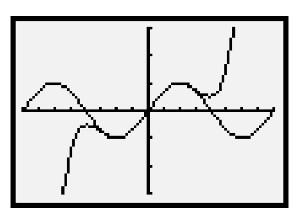
3. $\sin x = x$ when x is very close to 0 and $\cos x = 1 - 0.5x^2$ when x is very close to 0.

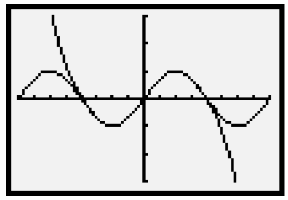
6. **a)** Answers may vary.

b) **i)**

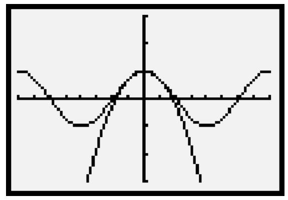


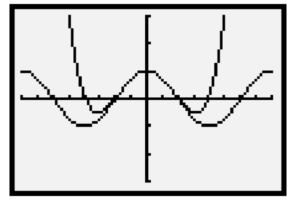
ii)

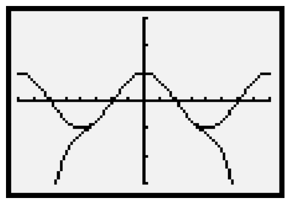


iii) 

7. a) Answers may vary.

b) i) 

ii) 

iii) 

8. $\sin x$ is an odd function, and $\cos x$ is an even function.

9. Graph the difference between the exact function and the approximate function for increasing numbers of terms. When the difference approaches 0.000 001, the approximation is accurate to 5 decimal places. This happens for 13 terms.

10. Graph the difference between the exact function and the approximate function for increasing numbers of terms. When the difference approaches 0.000 001, the approximation is accurate to 5 decimal places. This happens for 14 terms.

11. $k = \sqrt{-1}$

5 Review, 349

4. a) $\sec^2 x - 1 = \sin^2 x \sec^2 x$

$$\begin{aligned}\text{Left side} &= \sec^2 x - 1\\ &= \tan^2 x\end{aligned}$$

$$\begin{aligned}\text{Right side} &= \sin^2 x \sec^2 x\\ &= \sin^2 x\left(\frac{1}{\cos^2 x}\right)\\ &= \frac{\sin^2 x}{\cos^2 x}\\ &= \tan^2 x\\ &= \text{Left Side}\end{aligned}$$

Since Left side = Right side, the identity is proven.

Selected Solutions — Chapter 5

b) $\csc x - \cos x \tan x = \frac{\cos x}{\tan x}$

Left side $= \csc x - \cos x \tan x$

$= \frac{1}{\sin x} - \cos x \frac{\sin x}{\cos x}$

$= \frac{1}{\sin x} - \sin x$

$= \frac{\cos^2 x}{\sin x}$

Right side $= \frac{\cos x}{\tan x}$

$= \frac{\cos x}{\frac{\sin x}{\cos x}}$

$= \cos x \times \frac{\cos x}{\sin x}$

$= \frac{\cos^2 x}{\sin x}$

$=$ Left side

Since Left side = Right side, the identity is proven.

c) $\frac{\cos x + \cot x}{1 + \sin x} = \cot x$

Left side $= \frac{\cos x + \cot x}{1 + \sin x}$

$= \frac{\cos x + \frac{\cos x}{\sin x}}{1 + \sin x}$

$= \frac{\frac{\cos x \sin x + \cos x}{\sin x}}{1 + \sin x}$

$= \frac{\cos x (\sin x + 1)}{\sin x} \times \frac{1}{1 + \sin x}$

$= \cot x$

$=$ Right side

Right side $= \cot x$

Since Left side = Right side, the identity is proven.

d) $\cos^2 x \tan^2 x = 1 - \cos^2 x$

Left side $= \cos^2 x \tan^2 x$

$= \cos^2 x \frac{\sin^2 x}{\cos^2 x}$

$= \sin^2 x$

Right side $= 1 - \cos^2 x$

$= \sin^2 x$

$=$ Left side

Since Left side = Right side, the identity is proven.

5. Explanations may vary. For part a:
For the Left side, I used the Pythagorean Identity after it is divided by $\cos^2 x$, i.e., divide both sides of $\sin^2 x + \cos^2 x = 1$ by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$. Rearrange this equation to get $\sec^2 x - 1 = \tan^2 x$.
For the Right side, I used the reciprocal identity, i. e., $\sec^2 x = \frac{1}{\cos^2 x}$ and the quotient identity, i.e., $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$.

9. Explanation may vary. For part a:
I used the fact that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ and then I used the difference identity, i.e., $\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3} \cos\frac{\pi}{4} - \cos\frac{\pi}{3} \sin\frac{\pi}{4}$ to get the exact answer of $\frac{\sqrt{6} - \sqrt{2}}{2}$ or $\frac{\sqrt{2 - \sqrt{3}}}{2}$.

Selected Solutions — Chapter 5

12. a) $\sin 3x = \sin(2x + x)$

$= \sin 2x \cos x + \cos 2x \sin x$

$= 2 \sin x \cos x \cos x + \cos 2x \sin x$

$= 2 \sin x \cos^2 x + \cos 2x \sin x$

$= 2 \sin x \cos^2 x + (\cos^2 x - \sin^2 x) \sin x$

$= 2 \sin x \cos^2 x + \cos^2 x \sin x - \sin^3 x$

$= 3 \cos^2 x \sin x - \sin^3 x$

$\cos 3x = \cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$

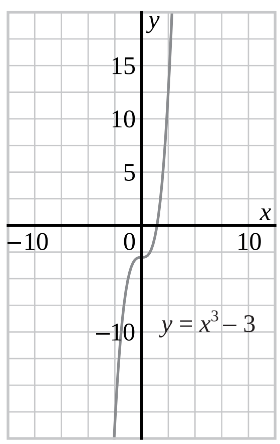
$= (\cos^2 x - \sin^2 x) \cos x - 2 \sin x \cos x \sin x$

$= \cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x$

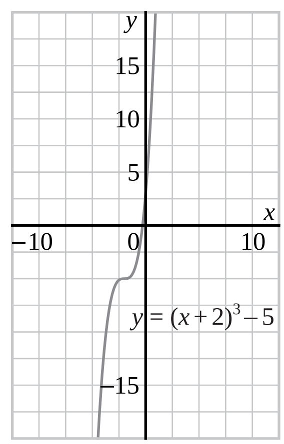
$= \cos^3 x - 2 \sin^2 x \cos x$

5 Cumulative Review, 350

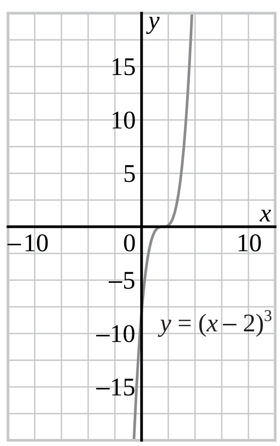
4. a)



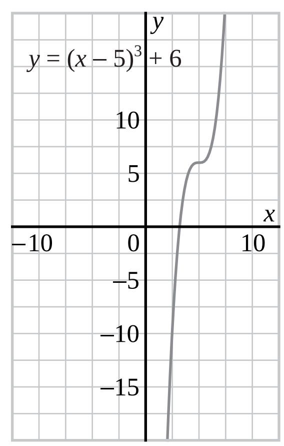
b)



c)



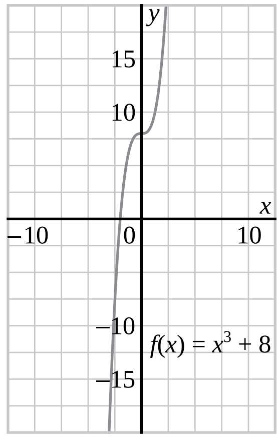
d)

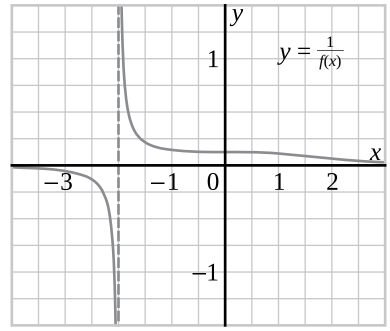


5. Explanations may vary. For part a:
I sketched the graph of $y = x^3$ and then I vertically translated it 3 units down.

Selected Solutions — Chapter 5

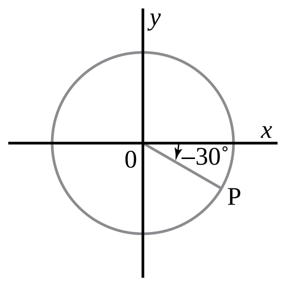
7. a)



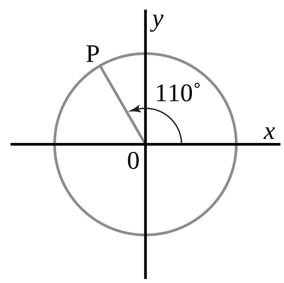


b) The asymptote is $x = -2$.

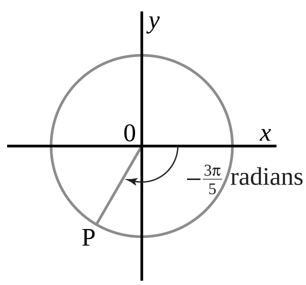
15. a)



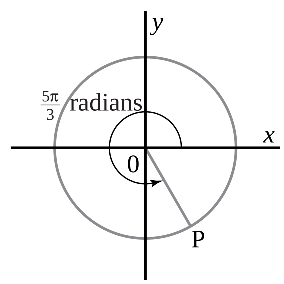
b)



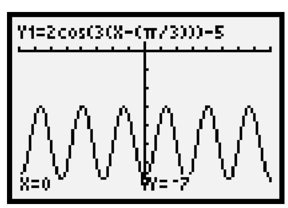
c)



e)



18. a)



b)