Selected Solutions — Chapter 4

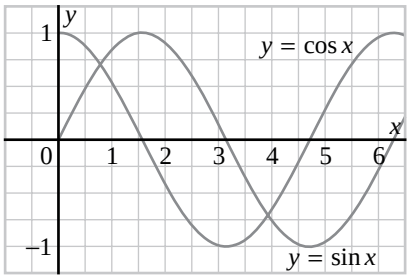
Investigate, page 226

1. When the counterclockwise distance from A(1, 0) to P is 2.6, the coordinates of P are about (–0.86, 0.52). Thus, $\cos 2.6 \doteq -0.86$ and $\sin 2.6 \doteq 0.52$.

2.

x	$\cos x$	$\sin x$
0	1.00	0
0.5	0.88	0.48
1.0	0.54	0.84
1.5	0.07	1.00
2.0	–0.42	0.91
2.5	–0.80	0.60
3.0	–0.99	0.14
3.5	–0.94	–0.35
4.0	–0.65	–0.76
4.5	–0.21	–0.98
5.0	0.28	–0.96
5.5	0.71	–0.71
6.0	0.96	–0.28

3.



4. **a)** You can go around the circle in either direction, as many times as you like. Thus, the domain is all real numbers. The range is the first coordinates of all possible points P, or $-1 \le x \le 1$.

b) Since the point P goes around a circle, the period is 2π.

c) The x-intercepts occur when $y = 0$. Thus, $\cos x = 0$ when the first coordinate of P is 0, which occurs when $x \doteq 1.57$ and $x \doteq 4.71$.

5. **a)** You can go around the circle in either direction, as many times as you like. Thus, the domain is all real numbers. The range is the second coordinates of all possible points P, or $-1 \le x \le 1$.

b) Since the point P goes around a circle, the period is 2π.

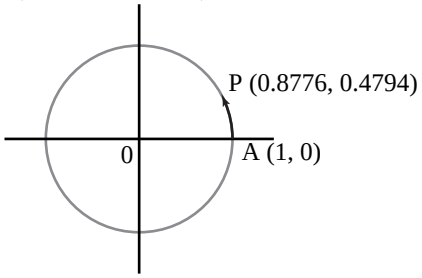
c) The x-intercepts occur when $y = 0$. Thus, $\sin x = 0$ when the second coordinate of P is 0, which occurs when $x = 0$ and $x \doteq 3.14$.

Selected Solutions — Chapter 4

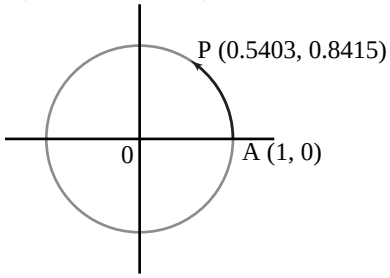
4.1 Exercises, page 233

4. **b)** The two graphs are two different ways of representing the same functions, so the values should be the same.

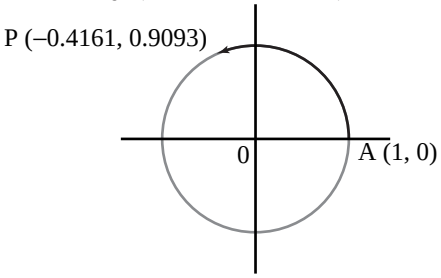
5. **a)** $\sin 0.5 \doteq 0.4794$ and $\cos 0.5 \doteq 0.8776$ mean that when the length of arc AP in a unit circle is 0.5 units, the coordinates of P are approximately (0.8776, 0.4794).



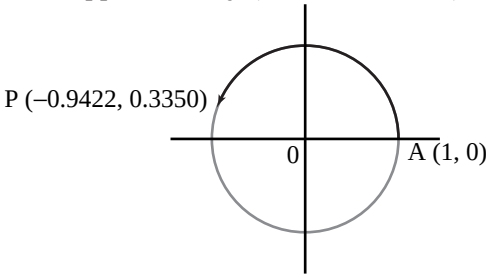
b) $\sin 1 \doteq 0.8415$ and $\cos 1 \doteq 0.5403$ mean that when the length of arc AP in a unit circle is 1 unit, the coordinates of P are approximately (0.5403, 0.8415).



c) $\sin 2 \doteq 0.9093$ and $\cos 2 \doteq -0.4161$ mean that when the length of arc AP in a unit circle is 2 units, the coordinates of P are approximately (–0.4161, 0.9093).

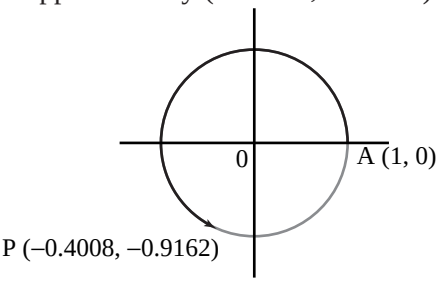


d) $\sin 2.8 \doteq 0.3350$ and $\cos 2.8 \doteq -0.9422$ mean that when the length of arc AP in a unit circle is 2.8 units, the coordinates of P are approximately (–0.9422, 0.3350).

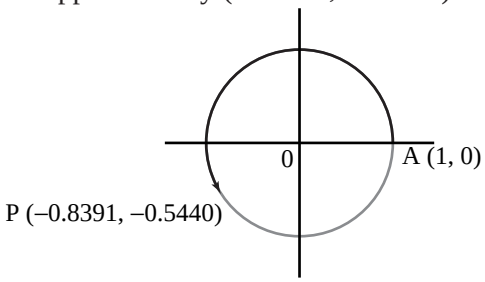


Selected Solutions — Chapter 4

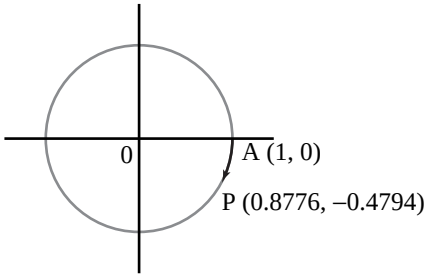
e) $\sin 4.3 \doteq -0.9162$ and $\cos 4.3 \doteq -0.4008$ mean that when the length of arc AP in a unit circle is 4.3 units, the coordinates of P are approximately (–0.4008, –0.9162).



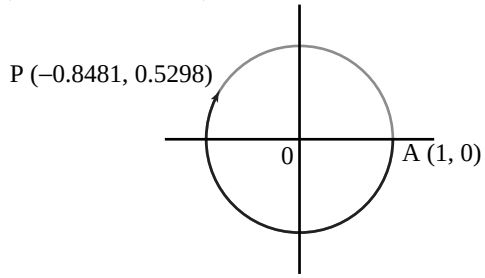
f) $\sin 10 \doteq -0.5440$ and $\cos 10 \doteq -0.8391$ mean that when the length of arc AP in a unit circle is 10 units, the coordinates of P are approximately (–0.8391, –0.5440).



g) $\sin(-0.5) \doteq -0.4794$ and $\cos(-0.5) \doteq 0.8776$ mean that when the length of arc AP in a unit circle is 0.5 units in the clockwise direction, the coordinates of P are approximately (0.8776, –0.4794).

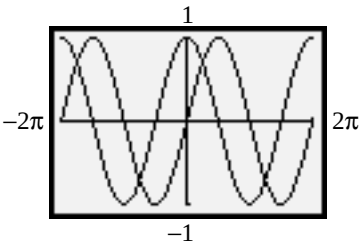


h) $\sin(-3.7) \doteq 0.5298$ and $\cos(-3.7) \doteq -0.8481$ mean that when the length of arc AP in a unit circle is 3.7 units in the clockwise direction, the coordinates of P are approximately (–0.8481, 0.5298).



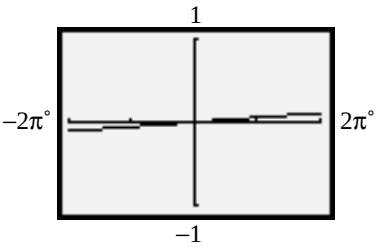
Selected Solutions — Chapter 4

6. a)



7. a) It would look like a straight line with a small slope passing through the origin. This is because the graph shows values only for $-6.28^\circ \le x \le 6.28^\circ$, which are very small.

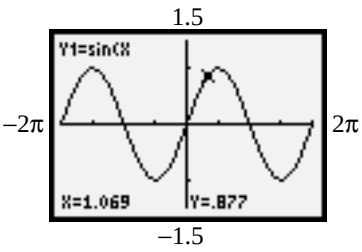
b)



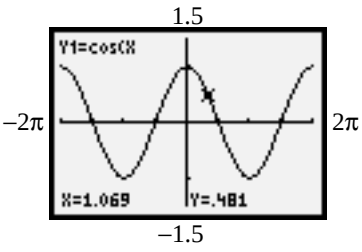
9. Explanations may vary. For image 1:

Since the distance travelled by the marked vertex is 1, the angle of the marked vertex is 1 radian. Since the angle of each vertex is $\frac{\pi}{2}$ greater than the preceding vertex, the angles of the other 3 vertices are $1 + \frac{\pi}{2}$ radians, $1 + \pi$ radians, and $1 + \frac{3\pi}{2}$ radians. To find the coordinates of the vertices, I took the cosine of the angle for the first coordinate, and the sine of the angle for the second coordinate. These were my results:
(–0.841, 0.54), (–0.54, –0.841), (0.841, –0.54).

13. a)



14. a)



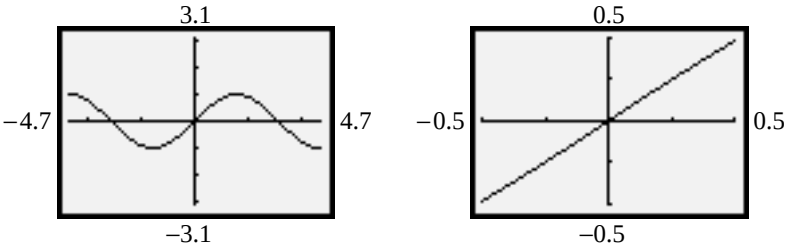
c) The cosine and sine graphs have positive values for $0 \le x \le \frac{\pi}{2}$ and $-2\pi \le x \le -\frac{3\pi}{2}$, or in the first and fourth quadrants. The common angles 1.069 and –5.214 are in these ranges, so they are the same for both exercise 13b and exercise 14b.

Selected Solutions — Chapter 4

15. a) The graph of $y = \sin x$ is above the graph of $y = \cos x$ at this point.

b) No. The graph of $y = \sin x$ is above $y = \cos x$ for $\frac{\pi}{4} + 2n\pi < x < \frac{5\pi}{4} + 2n\pi$, where n is any integer. It is below for $\frac{-3\pi}{4} + 2n\pi < x < \frac{\pi}{4} + 2n\pi$, where n is any integer.

16. a)



b) The x- and y-coordinates are equal. Refer to the large scale diagram of the unit circle. When x is small, the circle is almost a straight, vertical line, so the second coordinate of P, $\sin x$, is almost equal to x.

18. Prove that $\sin(x + 2\pi) = \sin x$.

$$\begin{aligned}\text{Left side} &= \sin(x + 2\pi)\\ &= \sin x \cos 2\pi + \cos x \sin 2\pi\\ &= \sin x\\ &= \text{Right side}\end{aligned}$$

Prove that $\cos(x + 2\pi) = \cos x$.

$$\begin{aligned}\text{Left side} &= \cos(x + 2\pi)\\ &= \cos x \cos 2\pi - \sin x \sin 2\pi\\ &= \cos x\\ &= \text{Right side}\end{aligned}$$

Exploring with a Graphing Calculator, page 236

1. f) No. Tracing will not land on the exact maximums, minimums, or intercepts.

2. f) Yes. The trace function will trace to the exact maximums, minimums, and intercepts.

3. The first window setting lets you trace to rational x-values. The second window setting lets you trace x-values which are multiples of π.

4. The functions have the same domain and range, they just use different variables. Since θ° equals x radians, the graphs are the same.

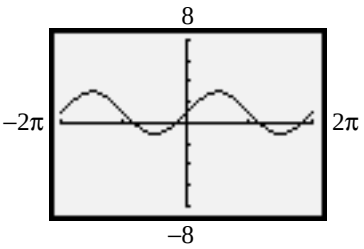
Selected Solutions — Chapter 4

Investigate, page 237

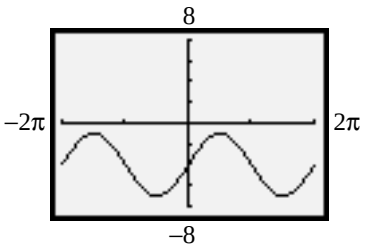
1. **c)** When 2 is replaced by $a \geq 1$, or $a \leq -1$, the graph is vertically expanded by a factor of a. When $-1 < a < 1$, the graph is vertically compressed by a factor of a. When $a < 0$, the graph is also reflected in the x-axis.
When 1 is replaced by d, the graph is vertically translated by d units.

3. The properties of vertical expansions, compressions, and translations are consistent regardless of the specific sinusoidal function to which they are applied. Therefore, the solution to exercise 1 part c holds here.

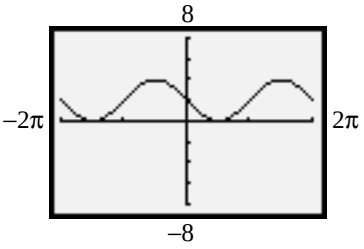
4. **a)**



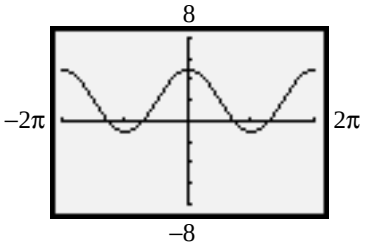
b)



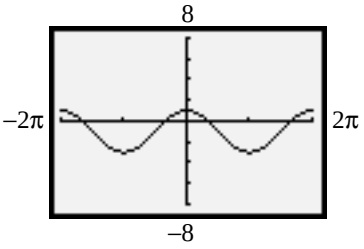
c)



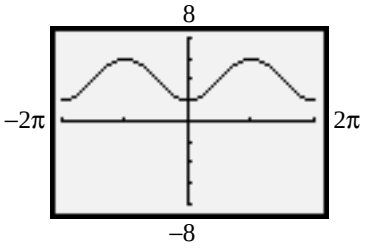
d)



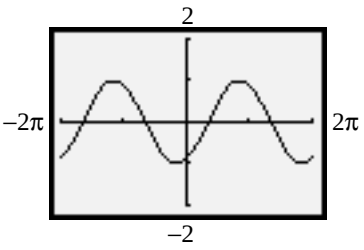
e)



f)



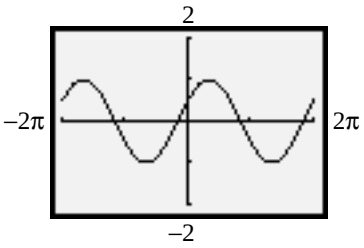
5. **a)**



b) When $\frac{\pi}{3}$ is replaced by c, the graph is horizontally translated c units right, when $c < 0$ and c units left, when $c > 0$.

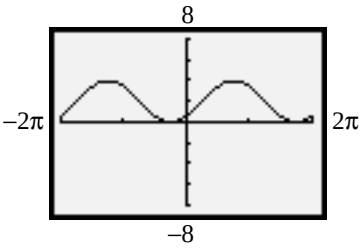
Selected Solutions — Chapter 4

7.

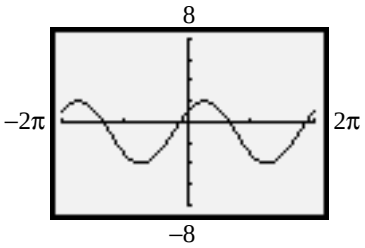


The properties of horizontal translations and application to specific sinusoidal functions is consistent regardless of the function. Therefore, the solution to exercise 5 part b holds here.

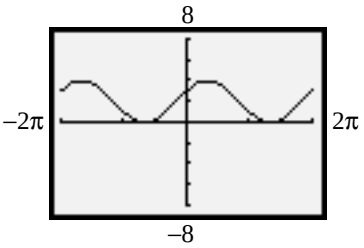
8. a)



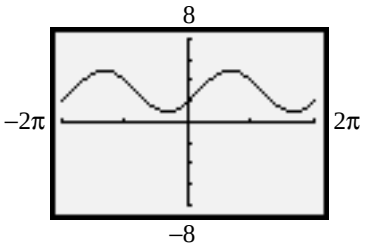
b)



c)



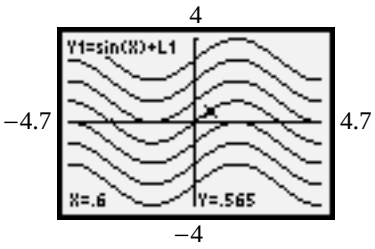
d)



4.2 Exercises, page 245

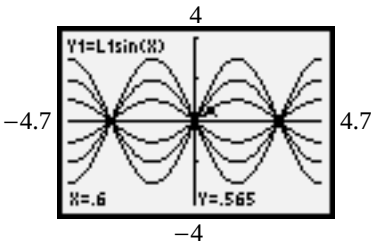
2. When a graph is translated vertically up, its shape does not change, but it moves up along the *y*-axis and its *y*-intercepts change. When a graph is expanded vertically, its shape changes, but the graph does not move up the *y*-axis and the *y*-intercepts do not change.

6. a)

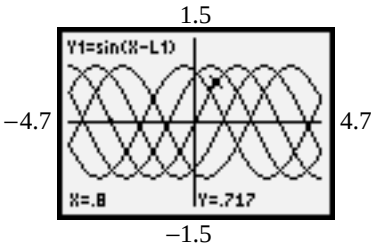


Selected Solutions — Chapter 4

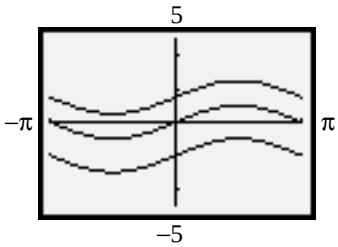
7. a)



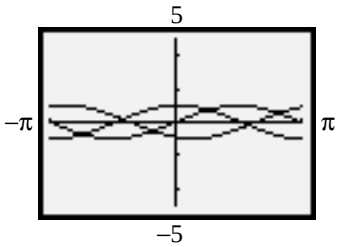
9. a)



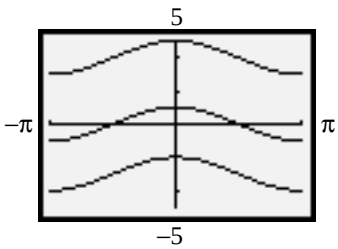
10. a)



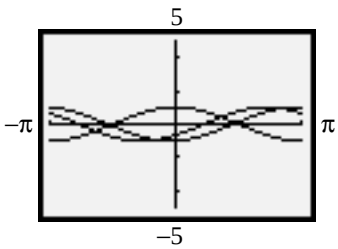
b)



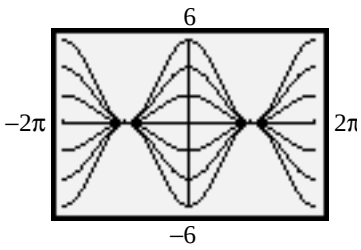
c)



d)



11. a)

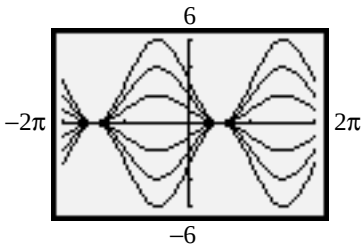


The graph of $y = 3\cos x + 3$ is a vertical expansion by a factor of 3, then a translation of 3 units up of the graph of $y = \cos x$.
The graph of $y = 2\cos x + 2$ is a vertical expansion by a factor of 2, then a translation of 2 units up of the graph of $y = \cos x$.
The graph of $y = \cos x + 1$ is a translation of 1 unit up of the graph of $y = \cos x$.
The graph of $y = -\cos x - 1$ is a reflection in the x-axis of the graph of $y = \cos x + 1$.

Selected Solutions — Chapter 4

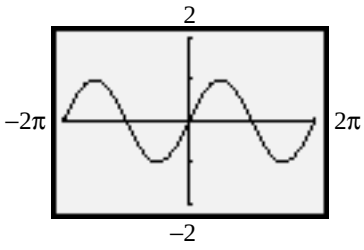
The graph of $y = -2\cos x - 2$ is a reflection in the x-axis of the graph of $y = 2\cos x + 2$.
The graph of $y = -3\cos x - 3$ is a reflection in the x-axis of the graph of $y = 3\cos x + 3$.

b)



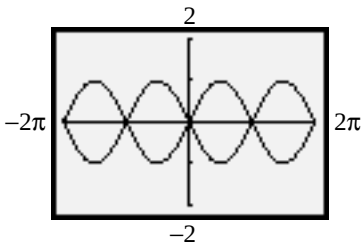
The graph of $y = 3\sin x - 3$ is a vertical expansion by a factor of 3, then a translation of 3 units down of the graph of $y = \sin x$.
The graph of $y = 2\sin x - 2$ is a vertical expansion by a factor of 2, then a translation of 2 units down of the graph of $y = \sin x$.
The graph of $y = \sin x - 1$ is a translation of 1 unit down of the graph of $y = \sin x$.
The graph of $y = -\sin x + 1$ is a reflection in the x-axis of the graph of $y = \sin x - 1$.
The graph of $y = -2\sin x + 2$ is a reflection in the x-axis of the graph of $y = 2\sin x - 2$.
The graph of $y = -3\sin x + 3$ is a reflection in the x-axis of the graph of $y = 3\sin x - 3$.

12. a)



The graphs all look like the graph of $y = \sin x$.

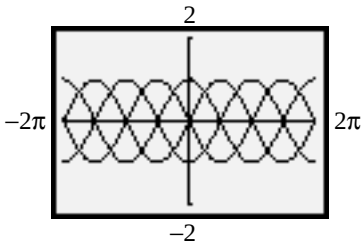
b)



The graph of $y = \sin(x - \pi)$ looks like the graph of $y = -\sin x$.
The graph of $y = \sin(x - 2\pi)$ looks like the graph of $y = \sin x$.
The graph of $y = \sin(x - 3\pi)$ looks like the graph of $y = -\sin x$.

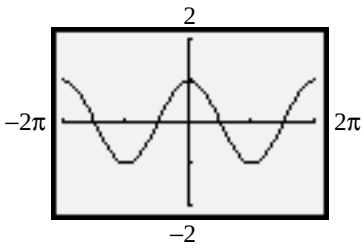
Selected Solutions — Chapter 4

c)

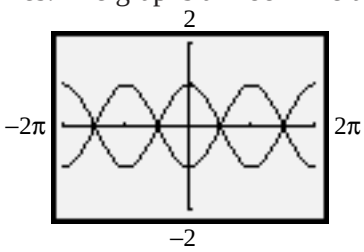


The graph of $y = \sin(x - \frac{\pi}{2})$ looks like the graph of $y = \sin x$ translated $\frac{\pi}{2}$ units right.
The graph of $y = \sin(x - \pi)$ looks like the graph of $y = -\sin x$.
The graph of $y = \sin(x - \frac{3\pi}{2})$ looks like the graph of $y = -\sin x$ translated $\frac{\pi}{2}$ units right.

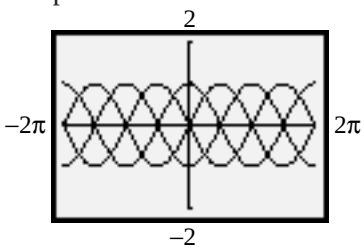
13. For part a:



Yes. The graphs all look like the graph of $y = \cos x$. For part b:



Yes. The graph of $y = \cos(x - \pi)$ looks like the graph of $y = -\cos x$.
The graph of $y = \cos(x - 2\pi)$ looks like the graph of $y = \cos x$.
The graph of $y = \cos(x - 3\pi)$ looks like the graph of $y = -\cos x$.
Notice that the graphs for parts a and b are the same as the graphs in exercise 12 parts a and b translated $\frac{\pi}{2}$ units right.
For part c:



The graph of $y = \cos(x - \frac{\pi}{2})$ looks like the graph of $y = \cos x$ translated $\frac{\pi}{2}$ units right.
The graph of $y = \cos(x - \pi)$ looks like the graph of $y = -\cos x$.

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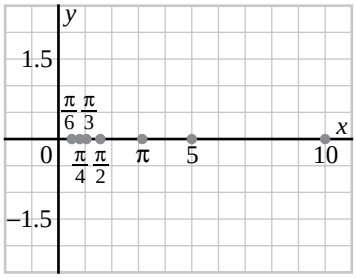
The graph of $y = \cos(x - \frac{3\pi}{2})$ looks like the graph of $y = -\cos x$ translated $\frac{\pi}{2}$ units right.

Notice that this graph is the same as the graph in exercise 12 part c.

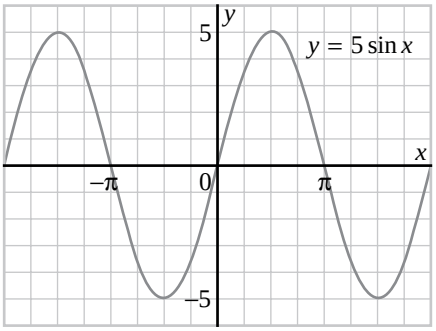
16. Each function of the form $y = \sin(x + b)$ or $y = \cos(x + b)$ can also be written as $y = \sin(x + b + 2n\pi)$ or $y = \cos(x + b + 2n\pi)$, where n is an integer, to give an infinite number of equations for the same graph. As well, any sine function can also be written as a cosine function, and vice versa:

$\sin(x + \frac{\pi}{2}) = \cos x$ and $\cos(x - \frac{\pi}{2}) = \sin x$.

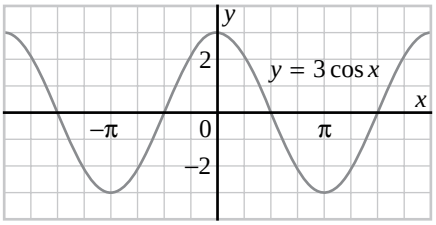
17.



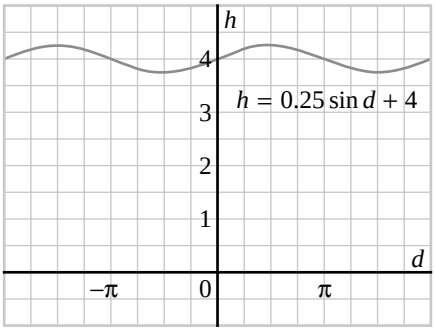
19. a)



b)



c)



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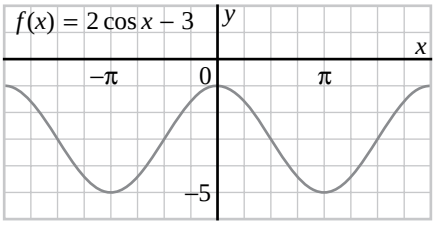
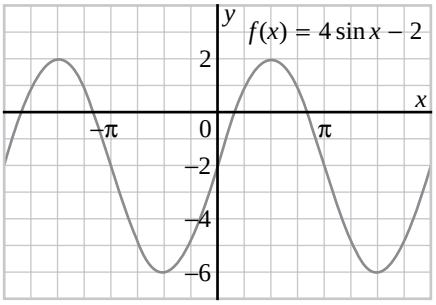
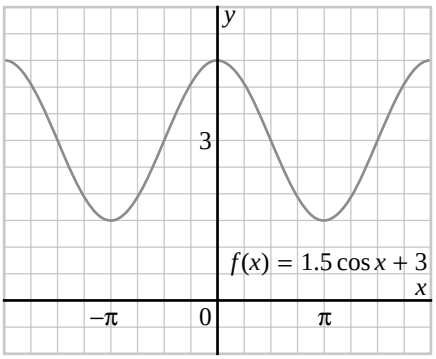
d)



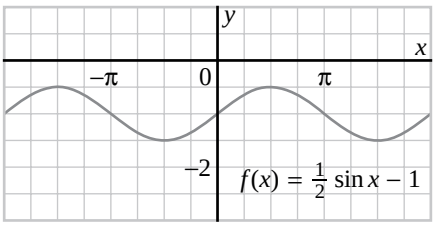
e)



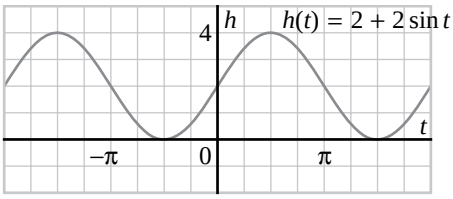
f)



g)



h)



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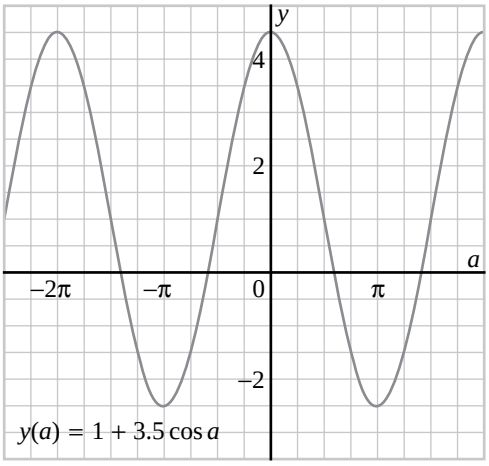
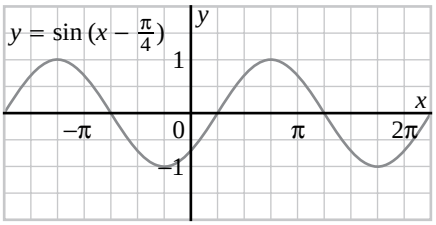
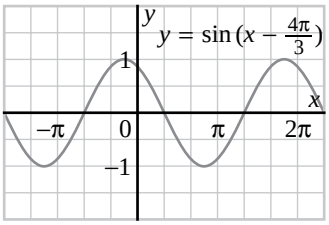
i)



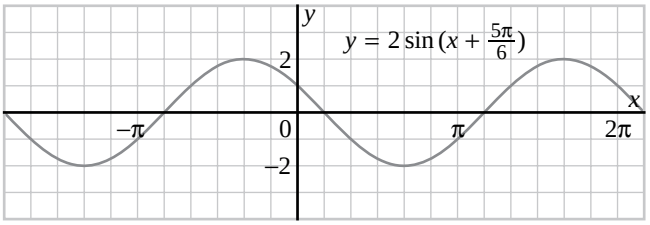
j)



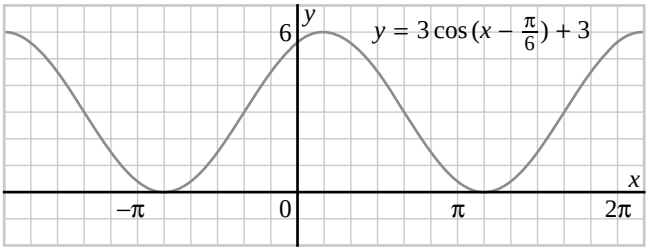
k)



l)



m)



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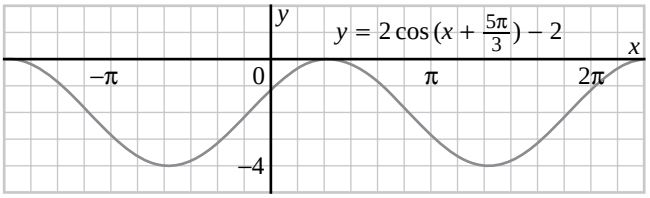
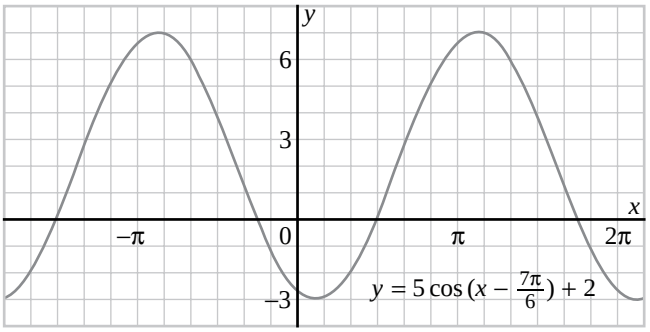
n)



o)



21. From the formulas on page 241 and in exercise 20, $a = \frac{M - m}{2}$ and $d = \frac{M + m}{2}$, where M represents the value of the maximum points and m represents the value of the minimum points. Since I know a and d, I can solve the equations for M and m.

$$a = \frac{M - m}{2}$$
$$2a = M - m \quad ①$$
$$d = \frac{M + m}{2}$$
$$2d = M + m \quad ②$$

I added 1 and 2 to obtain:

$$2a + 2d = 2M$$
$$M = a + d$$

I subtracted 1 from 2 to obtain:

$$2d - 2a = 2m$$
$$m = d - a$$

Therefore, $M = |a| + d$ and $m = d - |a|$. The absolute value signs are introduced for the $a < 0$ case. The position of maximum and minimum points remains constant. Where $a > 0$, the maximums occur at $x = \frac{3\pi}{2} + 2n\pi$ and the minimums occur at $x = \frac{\pi}{2} + 2n\pi$, for all integers n. Where $a < 0$, the coordinates of the maximum and minimum points are reversed, because the graph is reflected in the x-axis.

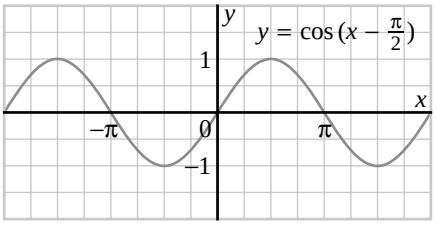
22. The maximum and minimum values are the same for $y = a\cos x + d$ and $y = a\sin x + d$:

$M = |a| + d$; $m = d - |a|$

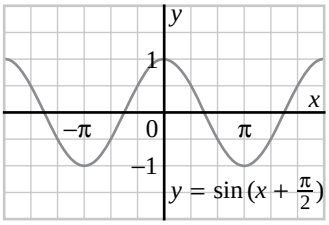
Where $a > 0$, the maximums occur at $x = 2n\pi$, and the minimums occur at $x = \pi + 2n\pi$, where n is any integer. Where $a < 0$, the coordinates of the maximum and minimum points are reversed.

Selected Solutions — Chapter 4

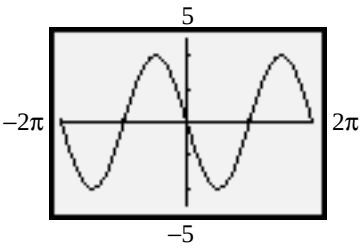
23. a)



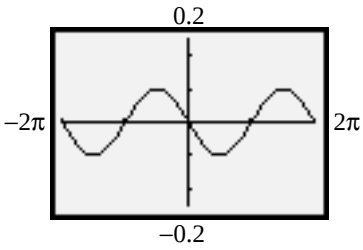
24. a)



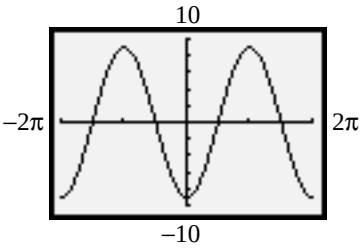
25. a)



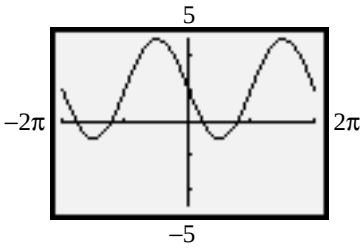
b)



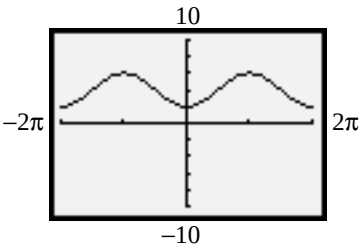
c)



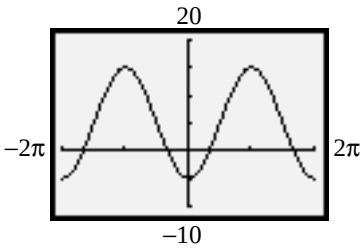
d)



e)



f)



27. There are two functions of the form $y = \sin x + p$ whose graphs just touch the x-axis. One has a maximum of 0, the other has a minimum of 0. The graph has an amplitude of 1, so

$M = a + d$

$0 = 1 + d$

$d = -1$

One function is $y = \sin x - 1$.

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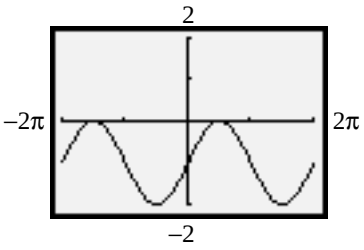
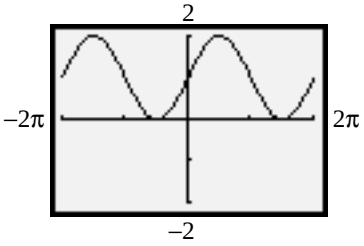


$m = d - a$
$0 = d - 1$
$d = 1$
The other function is $y = \sin x + 1$.



28. a) Since $a > 0$, $M = a + d$. The maximum occurs when $x - c = \frac{\pi}{2} + 2n\pi$, or $x = \frac{\pi}{2} + 2n\pi + c$.

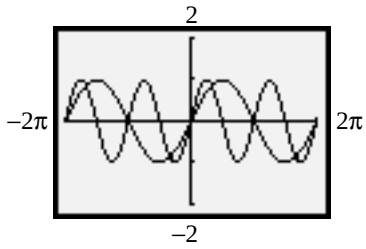
b) $m = d - a$. The minimum occurs when $x - c = \frac{3\pi}{2} + 2n\pi$, or $x = \frac{3\pi}{2} + 2n\pi + c$.

29. a) The amplitude is 1 and $M = 3$, so $d = 2$ by the formula $M = a + d$. The function $y = \sin(x - c) + 2$ has y-intercept $(0, 2)$ and maximum value 2 at $x = \frac{\pi}{2} + 2n\pi$, where n is any integer. So a translation of this graph by $\frac{\pi}{2}$ units left will result in the maximum value 3 when $x = 0$. Thus, the function $y = \sin(x + \frac{\pi}{2}) + 2$ satisfies the constraints.

b) The answer is not unique since $y = \sin(x + \frac{\pi}{2} + 2n\pi) + 2$, where n is any integer, will produce the same result.

Investigate, page 249

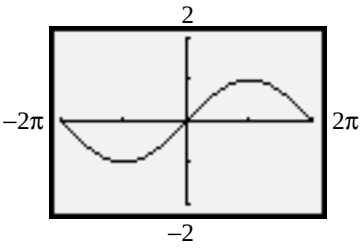
1. a)



b) The graph of any function $y = f(2x)$ is a horizontal compression by a factor of $\frac{1}{2}$ of the graph of $y = f(x)$.

Selected Solutions — Chapter 4

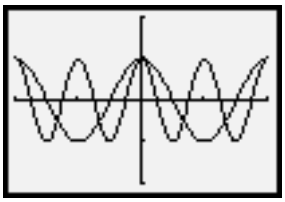
2. a)



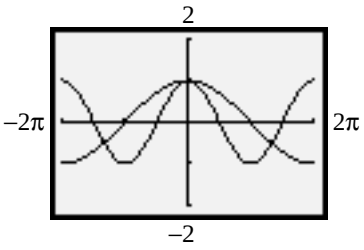
b) The graph of any function $y = f(\frac{1}{2}x)$ is a horizontal expansion by a factor of 2 of the graph of $y = f(x)$.

4. a) b is the amount by which the graph is horizontally compressed or expanded. It affects the period of the function. If $|b| > 1$, the graph is compressed and its period is less than 2π. If $0 < |b| < 1$, the graph is expanded and its period is greater than 2π. If $b < 0$, the graph is also reflected in the y-axis.

5.

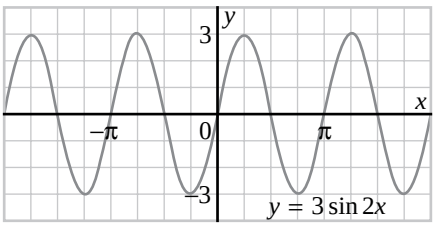


The graph of any function $y = f(2x)$ is a horizontal compression by a factor of $\frac{1}{2}$ of the graph of $y = f(x)$.

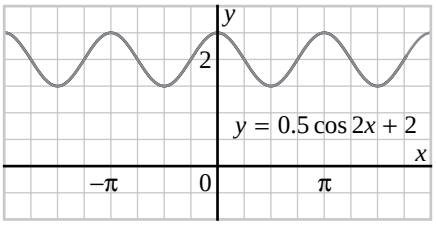


The graph of any function $y = f(\frac{1}{2}x)$is a horizontal expansion by a factor of 2 of the graph of $y = f(x)$. If $b > 1$, the graph is compressed. See exercise 4 part b for a discussion of b.

6. a)



b)



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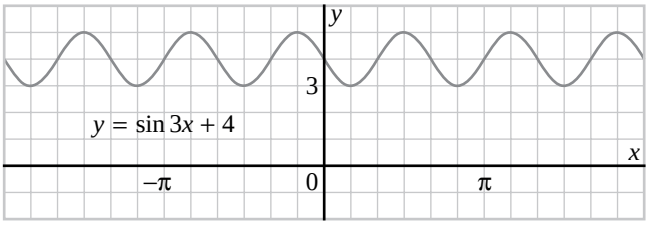
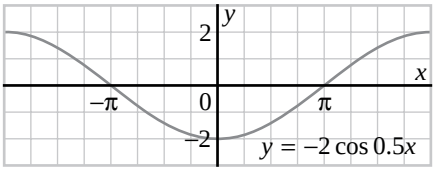
c)

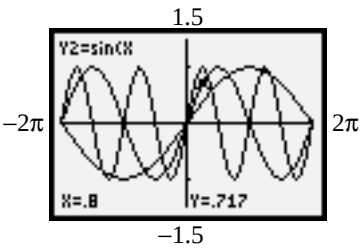


d)

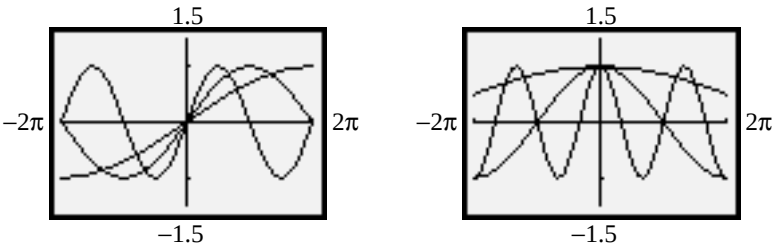


4.3 Exercises, page 254

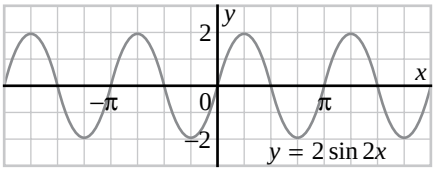
5. a)



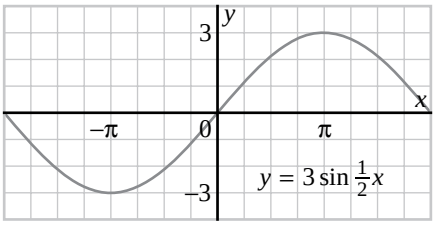
6. a)



9. a)

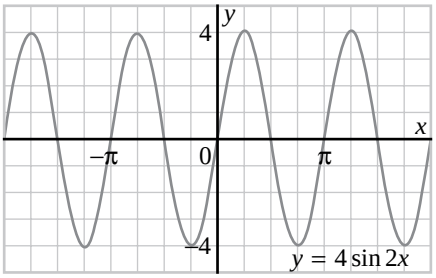


b)

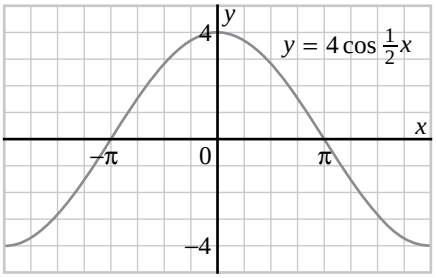


Selected Solutions — Chapter 4

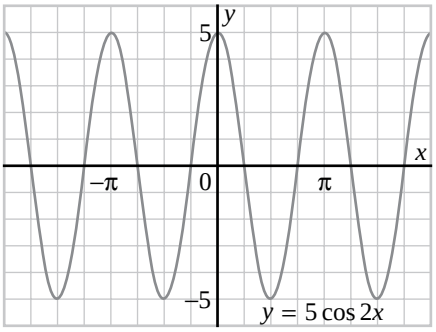
c)



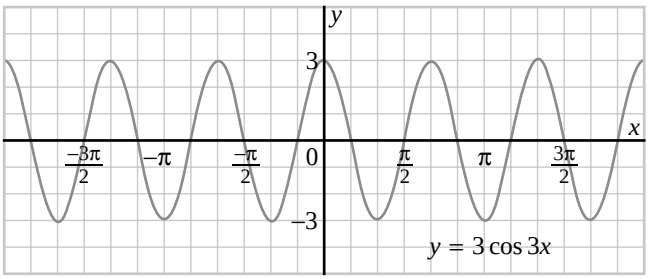
d)



e)

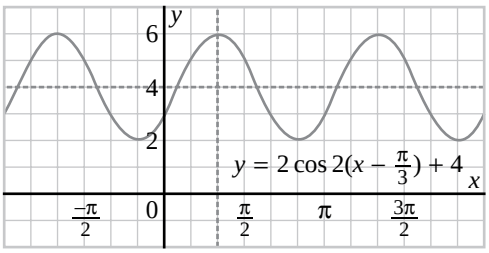


f)



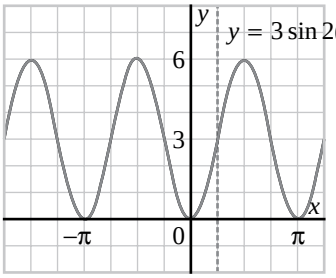
10. First sketch the graph of $y = \sin x$. For each point (x, y) on the graph of $y = \sin x$, graph the corresponding point $(\frac{x}{b}, ay)$ on the graph of $y = a \sin bx$. Connect the points with a smooth curve.

14. c)



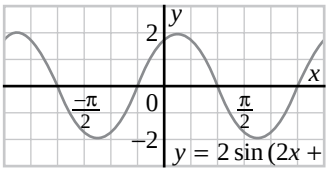
Selected Solutions — Chapter 4

15. c)



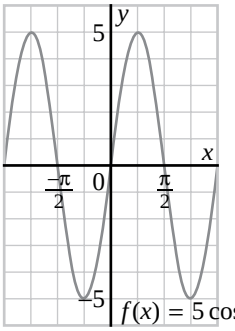
$y = 3\sin 2(x - \frac{\pi}{4}) + 3$

16. a)



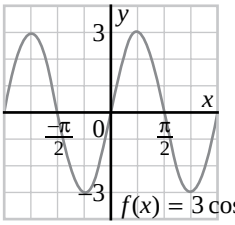
$y = 2\sin(2x + \frac{\pi}{3})$

b)



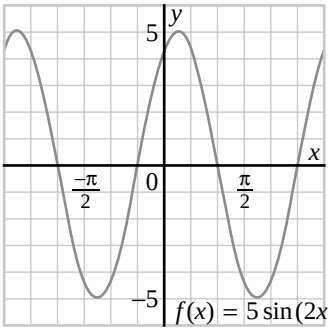
$f(x) = 5\cos(2x - \frac{\pi}{2})$

c)



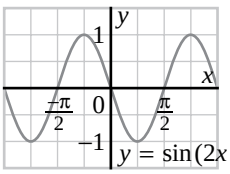
$f(x) = 3\cos(2x - \frac{\pi}{2})$

d)



$f(x) = 5\sin(2x + \frac{\pi}{3})$

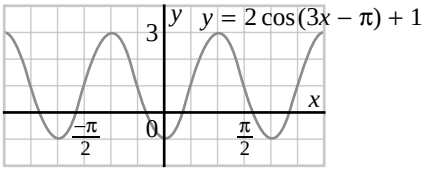
17. a)



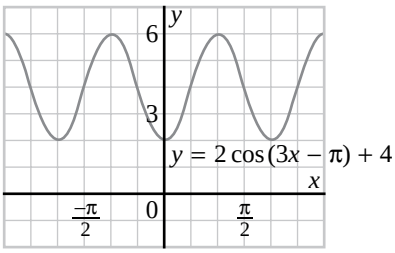
$y = \sin(2x - \pi)$

Selected Solutions — Chapter 4

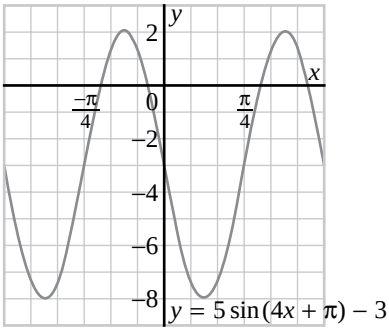
b)



c)

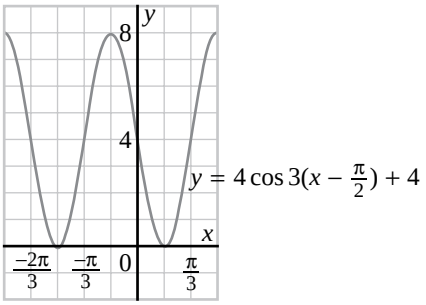


d)

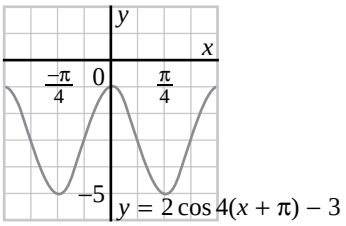


18. Sketch the graph of $y = \sin x$. For each point (x, y) on the graph of $y = \sin x$, graph the corresponding point $(\frac{1}{b}x + c, ay + d)$ on the graph of $y = a \sin b(x - c) + d$. That is, translate c units horizontally, horizontally compress or expand by a factor of $\frac{1}{b}$, translate d units vertically, and vertically compress or expand by a factor of a. Connect the points with a smooth curve.

19. a)

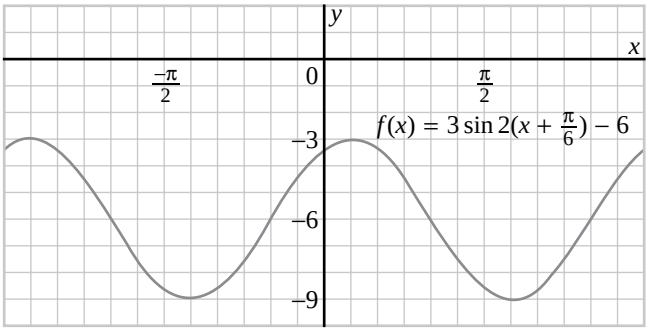


b)

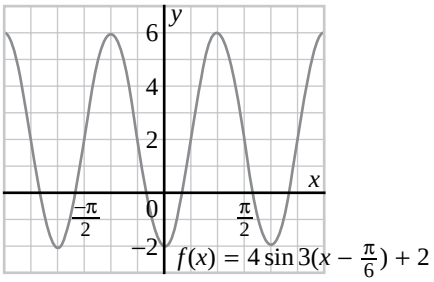


Selected Solutions — Chapter 4

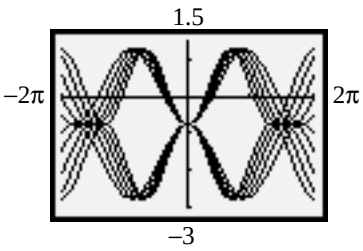
c)



d)



21.



22. The graph of $y = \sin(4x + 1)$ is a translation of 1 unit left and then a horizontal compression by a factor of $\frac{1}{4}$ of the graph of $y = \sin x$.
The graph of $y = \sin 4x + 1$ is a horizontal compression by a factor of $\frac{1}{4}$ and then a translation of 1 unit up of the graph of $y = \sin x$.

23. **a)** The maximum value is $|a| + d$. Where $a > 0$, it occurs when $b(x - c) = \frac{\pi}{2} + 2n\pi$, or $x = \frac{\frac{\pi}{2} + 2n\pi}{b} + c$. Where $a < 0$, the graph is reflected in the x-axis and therefore, it occurs when $b(x - c) = \frac{3\pi}{2} + 2n\pi$, or $x = \frac{\frac{3\pi}{2} + 2n\pi}{b} + c$.

b) The minimum value is $d - |a|$. Where $a > 0$, it occurs when $x = \frac{\frac{3\pi}{2} + 2n\pi}{b} + c$. Where $a < 0$, the graph is reflected in the x-axis and therefore, it occurs when $x = \frac{\frac{\pi}{2} + 2n\pi}{b} + c$.

24. **a)** The maximum value is $|a| + d$. Where $a > 0$, it occurs when $b(x - c) = 2n\pi$, or $x = \frac{2n\pi}{b} + c$. Where $a < 0$, the graph is reflected in the x-axis and therefore, it occurs when $b(x - c) = \pi + 2n\pi$, or $x = \frac{\pi + 2n\pi}{b} + c$.

Selected Solutions — Chapter 4

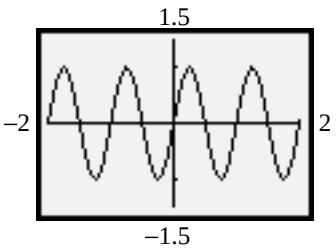
b) The minimum value is $d - |a|$. Where $a > 0$, it occurs when $x = \frac{\pi + 2n\pi}{b} + c$. Where $a < 0$, the graph is reflected in the x-axis and therefore, it occurs when $x = \frac{2n\pi}{b} + c$.

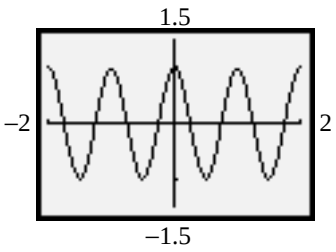
25. $y = 3\sin 2(x + \frac{\pi}{2})$
$= 3\sin(2x + \pi)$
$= 3(\sin 2x \cos \pi + \cos 2x \cos \pi)$
$= 3(-\sin 2x)$
$= -3\sin 2x$
$y = 3\cos 2(x + \frac{\pi}{4})$
$= 3\cos(2x + \frac{\pi}{2})$
$= 3(\cos 2x \cos \frac{\pi}{2} - \sin 2x \sin \frac{\pi}{2})$
$= 3(-\sin 2x)$
$= -3\sin 2x$
Thus, $y = 3\sin 2(x + \frac{\pi}{2})$ and $y = 3\cos 2(x + \frac{\pi}{4})$ represent the same function.

26. $f(x) = 3\sin(2x + \pi)$
$= -3\sin 2x$, from exercise 25
$f(x) = 3\cos(2x + \frac{\pi}{2})$
$= -3\sin 2x$, from exercise 25
Thus, $f(x) = 3\sin(2x + \pi)$ and $f(x) = 3\cos(2x + \frac{\pi}{2})$ represent the same function.

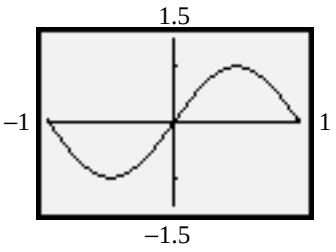
Investigate, page 260

1. d)

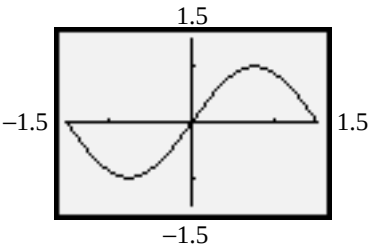




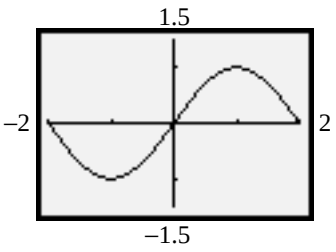
2. a)



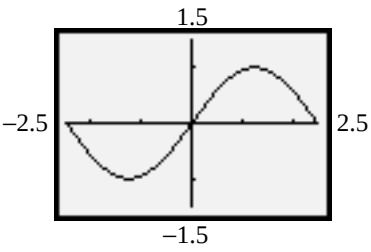
b)



c)



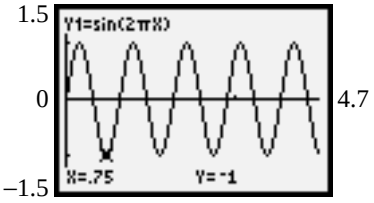
d)



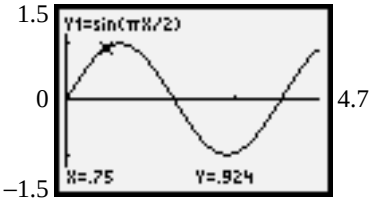
Selected Solutions — Chapter 4

4.4 Exercises, page 267

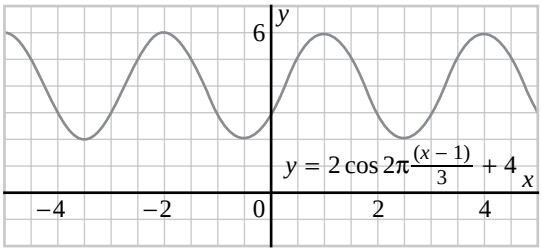
12. a)



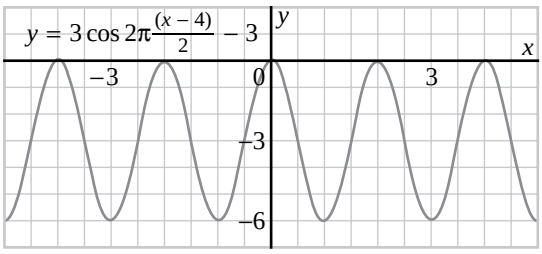
13. a)



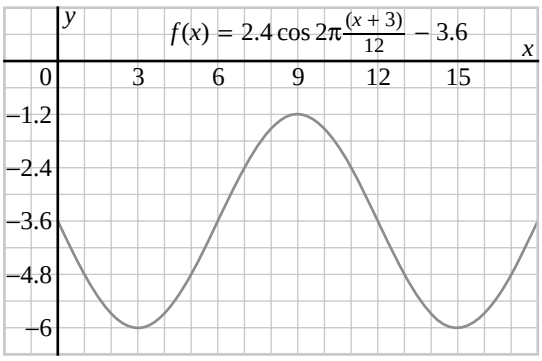
14. a)



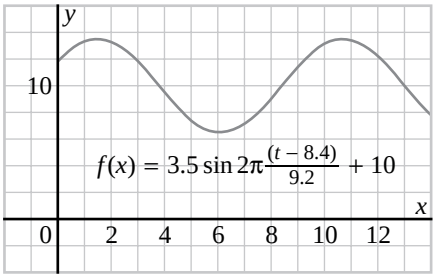
b)



c)

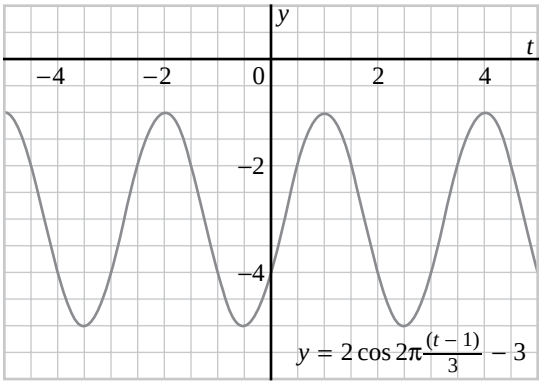


d)

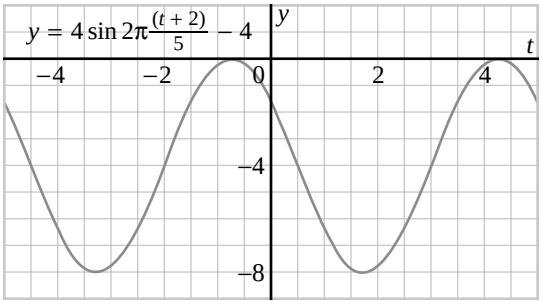


Selected Solutions — Chapter 4

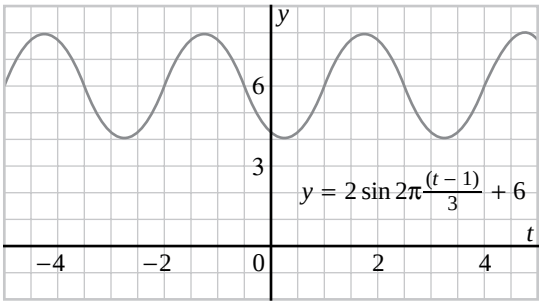
15. a)



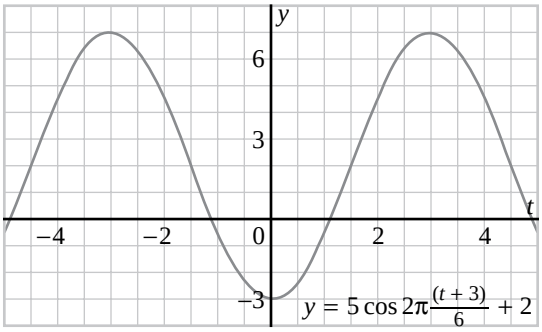
b)



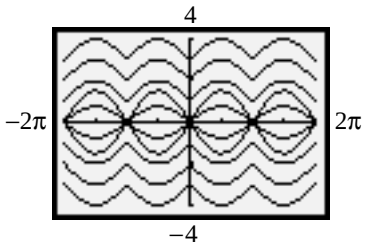
c)



d)



19.

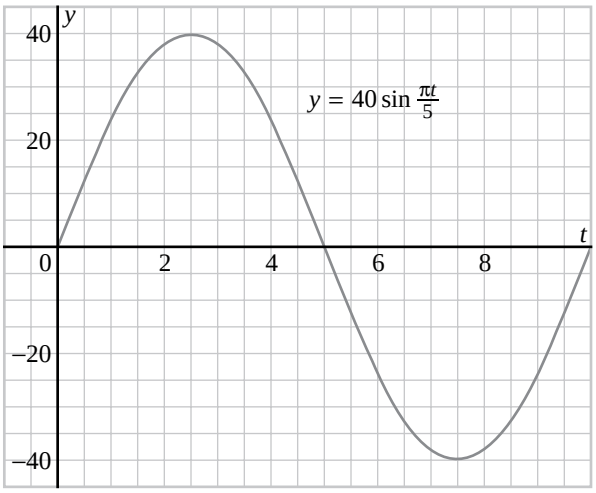


Selected Solutions — Chapter 4

Modelling the Volume of Air in the Lungs

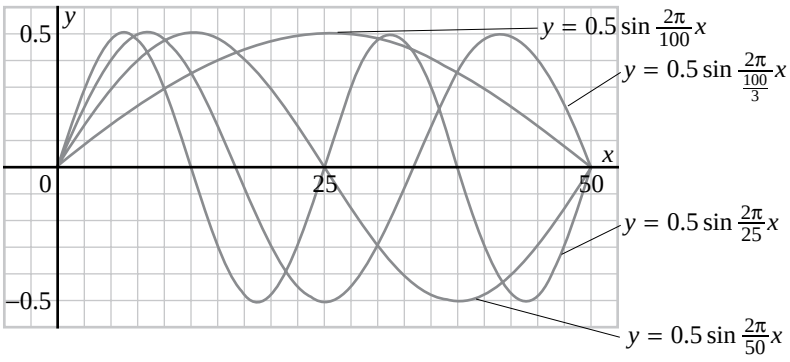
- Answers may vary; size of person, exertion, lung condition
- If a person had a smaller lung capacity, the graph would have a smaller scale, and if the person had a greater lung capacity, the graph would have a larger scale. Similarly, if a person over exerted themselves the scale could be larger. If the lungs were in poor condition due to smoke inhalation, emphysema, or other factors, the graph could have a smaller scale.

21. a)



24. The period of the function is immediately apparent when the equation is in the form $y = \sin \frac{2\pi}{p}x$.

25. b)



Modelling a Vibrating Guitar String

- The x represents the distance from one end of the string. The y represents the amount the string vibrates from its straight position.
- Strings of different tightness would attain greater or lesser amplitudes, since they vibrate differently.
- A tighter string might produce a graph with a smaller amplitude, and a looser string might produce a graph with a larger amplitude.

Selected Solutions — Chapter 4

26. $y = 3\cos 2\pi\frac{(t-5)}{8} + 2$

$= 3\cos\left(\frac{\pi t}{4} - \frac{5\pi}{4}\right) + 2$

$= 3\left(\cos\frac{\pi t}{4}\cos\frac{5\pi}{4} + \sin\frac{\pi t}{4}\sin\frac{5\pi}{4}\right) + 2$

$= 3\left(\cos\frac{\pi t}{4}\left(-\frac{1}{\sqrt{2}}\right) + \sin\frac{\pi t}{4}\left(-\frac{1}{\sqrt{2}}\right)\right) + 2$

$= -\frac{3}{\sqrt{2}}\left(\cos\frac{\pi t}{4} + \sin\frac{\pi t}{4}\right) + 2$

$y = 3\sin 2\pi\frac{(t-3)}{8} + 2$

$= 3\sin\left(\frac{\pi t}{4} - \frac{3\pi}{4}\right) + 2$

$= 3\left(\sin\frac{\pi t}{4}\cos\frac{3\pi}{4} - \cos\frac{\pi t}{4}\sin\frac{3\pi}{4}\right) + 2$

$= 3\left(\sin\frac{\pi t}{4}\left(-\frac{1}{\sqrt{2}}\right) - \cos\frac{\pi t}{4}\left(\frac{1}{\sqrt{2}}\right)\right) + 2$

$= -\frac{3}{\sqrt{2}}\left(\sin\frac{\pi t}{4} + \cos\frac{\pi t}{4}\right) + 2$

Thus, $y = 3\cos 2\pi\frac{(t-5)}{8} + 2$ and $y = 3\sin 2\pi\frac{(t-3)}{8} + 2$ represent the same function.

Mathematical Modelling, page 270

1. $a = \frac{M-m}{2}$

$= 4.56$

$d = \frac{M+m}{2}$

$= 12.24$

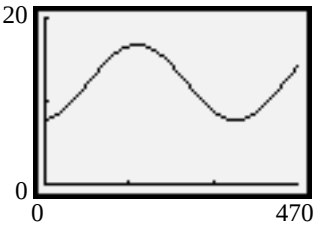
$p = 355 - (-10)$

$= 365$

$c = 172$

2. Substitute the values of a, d, p, and c from exercise 1 into the equation $h = a\cos 2\pi\frac{n-c}{p} + d$, to obtain $h = 4.56\cos 2\pi\frac{n-172}{365} + 12.24$.

3.



4. **a)** March 1 is day 60, so let $n = 60$ and substitute to get $h = 4.56\cos 2\pi\frac{60-172}{365} + 12.24$, or approximately 10.646. Therefore, there are about 10.6 h of daylight on March 1.

b) July 15 is day 196, so let $n = 196$ and substitute to get $h = 4.56\cos 2\pi\frac{196-172}{365} + 12.24$, or approximately 16.416. Therefore, there are about 16.4 h of daylight on July 15.

c) November 11 is day 315, so let $n = 315$ and substitute to get $h = 4.56\cos 2\pi\frac{315-172}{365} + 12.24$, or approximately 8.694. Therefore, there are about 8.7 h of daylight on November 11.

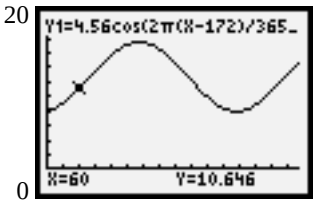
Selected Solutions — Chapter 4

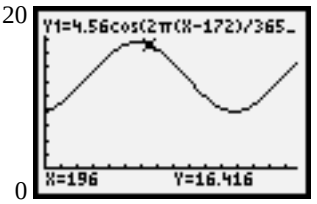
5. a) March 21 is day 80, so let $n = 80$ and substitute to get
$h = 4.56 \cos 2\pi\frac{80 - 172}{365} + 12.24$, or approximately 12.181.
June 21 is day 172, so let $n = 172$ and substitute to get
$h = 4.56 \cos 2\pi \times 0 + 12.24$, or 16.80.
September 21 is day 264, so let $n = 264$ and substitute to get
$h = 4.56 \cos 2\pi\frac{264 - 172}{365} + 12.24$, or approximately 12.181.

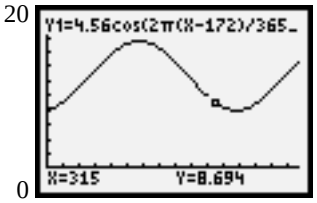
b) According to the model, there are approximately 12.18 h of daylight on March 21, 16.80 h on June 21, and 12.18 h on September 21. Discrepancies exist due to the fact that this is a model based on partial information, not an exact equation of the situation.

6. a) The amount of daylight on day 245, or September 2, is 13.649 h.

b)







7. a) Rewrite the table with cumulative days and hours from midnight in decimal form.

Date	–10	80	172	264	355
Sunset time	16.40	18.83	21.02	18.62	16.40
Sunrise time	8.72	6.60	4.22	6.32	8.72

i) $a = \frac{M - m}{2}$
$= 2.31$
$d = \frac{M + m}{2}$
$= 18.71$
$p = 355 - (-10)$
$= 365$
$c = 172$
Substitute to obtain
$s = 2.31 \cos 2\pi\frac{(n - 172)}{365} + 18.71$.

Selected Solutions — Chapter 4

ii) $a = \frac{M - m}{2}$
$= 2.25$
$d = \frac{M + m}{2}$
$= 6.47$
$p = 355 - (-10)$
$= 365$
$c = -10$
Substitute to obtain
$s = 2.25 \cos 2\pi \frac{(n + 10)}{365} + 6.47$.

b) The sunset time for March 1 is $s = 2.31 \cos 2\pi \frac{(60 - 172)}{365} + 18.71$, or approximately 17.902 h.
The sunrise time for March 1 is
$s = 2.25 \cos 2\pi \frac{(60 + 10)}{365} + 6.47$, or approximately 7.275 h.
The sunset time for July 15 is $s = 2.31 \cos 2\pi \frac{(196 - 172)}{365} + 18.71$, or approximately 20.826 h.
The sunrise time for July 15 is $s = 2.25 \cos 2\pi \frac{(196 + 10)}{365} + 18.71$, or approximately 4.402 h.
The sunset time for November 11 is
$s = 2.31 \cos 2\pi \frac{(315 - 172)}{365} + 18.71$, or approximately 16.914 h.
The sunrise time for November 11 is
$s = 2.25 \cos 2\pi \frac{(315 + 10)}{365} + 6.47$, or approximately 8.207 h.
These results conclude that there are 17.902 – 7.275, or approximately 10.627 h of sunlight on March 1. This is about 68 s less than the original calculation in exercise 4. There are 20.826 – 4.402, or 16.424 h of sunlight on July 15. This is approximately 29 s more sunlight than the estimation in exercise 4. There are 16.914 – 8.207, or 8.707 h on November 11. This is approximately 47 s more sunlight than the calculation in exercise 4.

2. The graph of hours of sunlight would not be affected by daylight savings time. The graphs for times of sunset and sunrise would have a vertical displacement of 1 during daylight savings time.

4.5 Exercises, page 276

2. a) i) 2.5 is the amplitude. Double it to find how much the depth of the water varies.

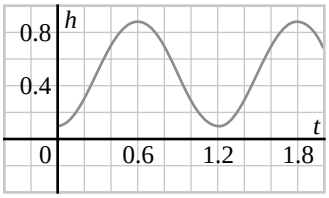
ii) 12.4 is the period. It is the time between consecutive maximums or minimums.

iii) 1.5 is the phase shift. It is the time at which the first mean depth occurs.

iv) 4.3 is the vertical displacement. It is the mean depth of the water.

Selected Solutions — Chapter 4

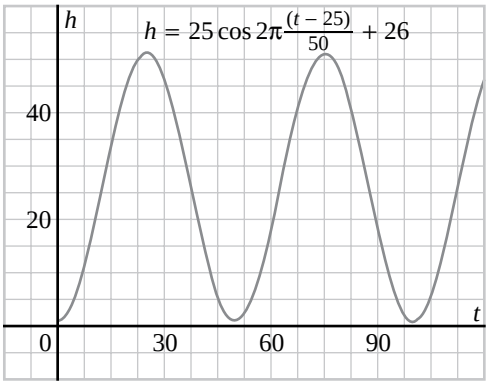
6. a)



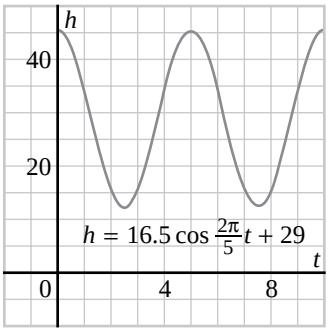
Modelling the Motion of a Spring

- The amplitude of the spring would decrease during subsequent cycles, till the spring would eventually stop.
- The graph would look something like this:
- Multiply the function by $\frac{1}{1+t}$. This creates a hyperbola shape with asymptotes at $t = -1$ and $h = 0$. The curve is sinusoidal.

7. a)



10. a)



12. Yes. Just substitute 360 for 2π in the equation. This converts the angle to degrees instead of radians.

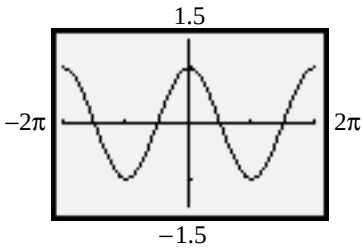
13. Yes. The function would be $y = A \sin[\frac{\pi}{2} + 2\pi\frac{(t-S)}{P}] + M$, where A, S, P, and M are defined in Example 1. This is true since $y = \sin(\frac{\pi}{2} + x)$ and $y = \cos x$ represent the same function.

Problem Solving, page 280

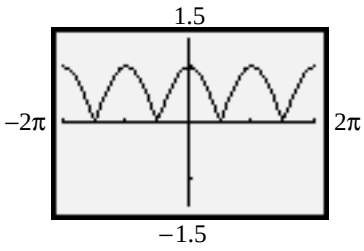
1. The first graph is $y = |\sin x|$, because all the y-values are greater than or equal to 0. The second graph is $y = \sin|x|$, because the y-value for $-x$ is equal to the y-value for x for all values of x.

Selected Solutions — Chapter 4

2. a) The graph of $y = \cos|x|$ would look like the graph of $y = \cos x$ for $x \geq 0$, and $y = \cos(-x)$, which is the same as $y = \cos x$, for $x < 0$.

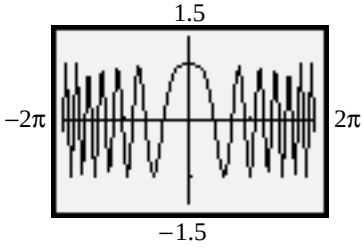


b) The graph of $y = |\cos x|$ would look like the graph of $y = \cos x$ with all the parts below the x-axis reflected in the x-axis.

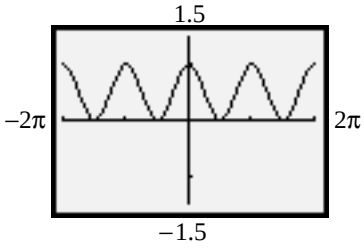


3. The second graph is $y = (\sin x)^2$, because all the y-values are greater than or equal to 0. The first graph is $y = \sin x^2$, because the y-value for $-x$ is equal to the y-value for x for all values of x.

4. a) The graph of $y = \cos x^2$ would look like the graph of $y = \cos x$ for values of x near 0, and then similar to the graph of $y = \sin x^2$ in exercise 3 for values of x away from 0.

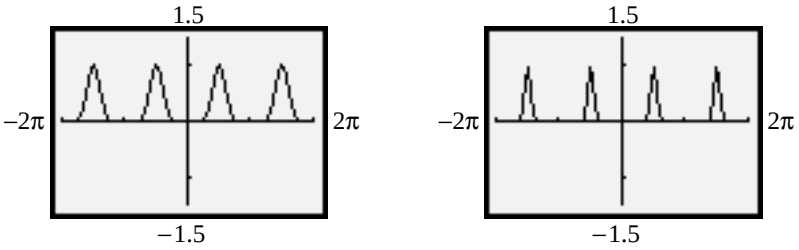


b) The graph of $y = (\cos x)^2$ would look like the graph of $y = (\sin x)^2$ in exercise 3, with a phase shift of $-\frac{\pi}{2}$.



Selected Solutions — Chapter 4

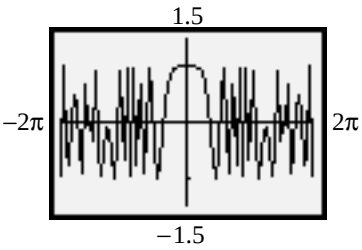
5. The graphs of $y = (\sin x)^n$ for $n = 4, 6, 8, \ldots$ get flatter near the minimums and pointier near the maximums as n increases, but otherwise look similar to the graph of $y = (\sin x)^2$. The graphs of $y = (\sin x)^{10}$ and $y = (\sin x)^{50}$ which follow, demonstrate this trend:



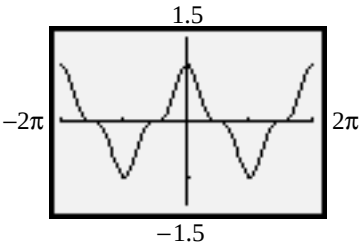
The same is true for $y = (\cos x)^n$ in comparison with $y = (\cos x)^2$.

6. Comparing to exercise 3, it appears that the first graph is the graph of $y = \sin x^3$, and the second graph is the graph of $y = (\sin x)^3$.

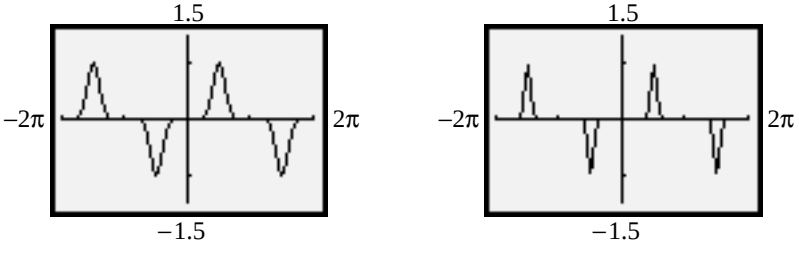
7. **a)** The graph of $y = \cos x^3$ would look like the graph of $y = \cos x$ for values near 0, and then similar to the graph of $y = \sin x^3$ in exercise 6 for values of x away from 0.



b) The graph of $y = (\cos x)^3$ would look like the graph of $y = (\sin x)^3$ in exercise 6, with a phase shift of $-\frac{\pi}{2}$.



8. The graphs of $y = (\sin x)^n$ for $n = 5, 7, 9, \ldots$ get flatter near the points where $y = 0$ and pointier near the maximums and minimums as n increases, but otherwise look similar to the graph of $y = (\sin x)^3$. The graphs of $y = (\sin x)^{11}$ and $y = (\sin x)^{51}$ which follow, demonstrate this trend:

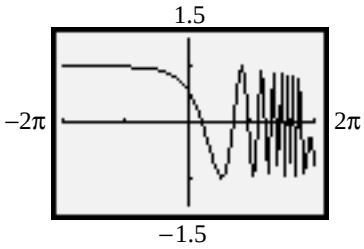


The same is true for $y = (\cos x)^n$ in comparison with $y = (\cos x)^3$.

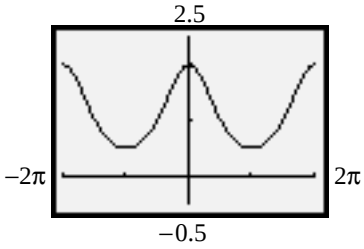
Selected Solutions — Chapter 4

9. The first graph is of $y = 2^{\sin x}$, since the y-values range from 2^{-1} to 2^1. The second graph is of $y = \sin 2^x$. As x gets large negatively, 2^x approaches 0, so $\sin 2^x$ approaches 0. As x gets large positively, 2^x gets very large and positive, and $\sin 2^x$ fluctuates between -1 and 1 faster and faster.

10. a) The graph of $y = \cos 2^x$ would approach 1 as x gets large negatively, and fluctuate between -1 and 1 faster and faster as x gets large positively.



b) The graph of $y = 2^{\cos x}$ would look like the graph of $y = 2^{\sin x}$ in exercise 9 with a phase shift of $-\frac{\pi}{2}$.



11. a) $\sin x \doteq x$ for small values of x, so $(\sin x)^2 \doteq x^2$ for small values of x. Let $q = x^2$ and follow the theory above. Thus, $\sin q \doteq q$ for small values of q, so $\sin x^2 \doteq x^2$ for small values of x^2, which is equivalent to small values of x.

b) Similar properties exist for all the pairs of sine functions, excluding the powers of 2. This is true, by application of the theory in part a, since $(\sin x)^n \doteq x^n$ and $\sin x^n \doteq x^n$, for small values of x. Therefore, the functions $y = \sin x^3$ and $y = (\sin x)^3$, in exercise 6 follow the pattern.
This property does not apply to any of the cosine functions since $\cos x \doteq 1$ for small values of x. Thus, $(\cos x)^n \neq \cos x^n \neq x^n$, for small values of x.

Exploring with a Graphing Calculator, page 283

1. b) This is just a model, so these values may not exactly match the results on page 270.

3. b) $y = \sin 1.571x$ represents $y = \sin \frac{\pi}{2}x$, or $y = \sin \frac{2\pi}{4}x$, an exact sine function with period 4.

Selected Solutions — Chapter 4

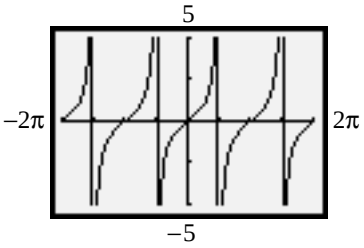
Investigate, page 284

2.

x	y
0	0
0.5	0.5
1	1.6
1.5	14.1
2	−2.2
2.5	−0.7
3	−0.1
3.5	0.4
4	1.2
4.5	4.6
5	−3.4
5.5	−1.0
6	−0.3

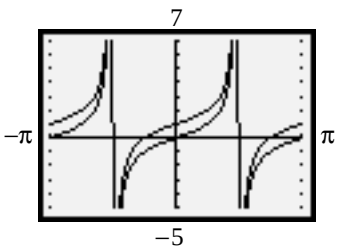
3. When $0 \le x < \frac{\pi}{2}$ and $\pi \le x < \frac{3\pi}{2}$, the graph of $\tan x$ increases from 0 to infinity and fails to reach the vertical asymptotes $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$, respectively. When $\frac{\pi}{2} \le x < \pi$ and $\frac{3\pi}{2} \le x < 2\pi$, the graph of $\tan x$ increases from negative infinity to 0. It never touches the vertical asymptote $x = \frac{\pi}{2}$.

4.

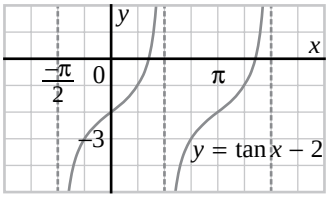


4.6 Exercises, page 287

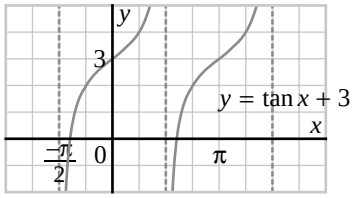
1. b)



2. a)

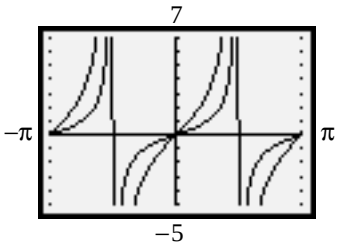


b)

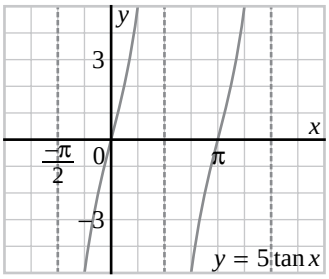


Selected Solutions — Chapter 4

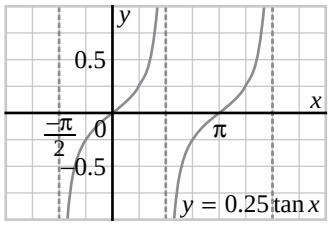
3. b)



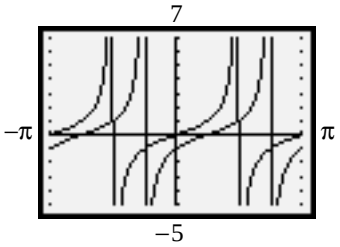
4. a)



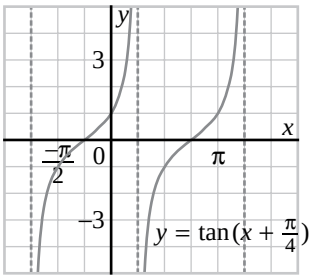
b)



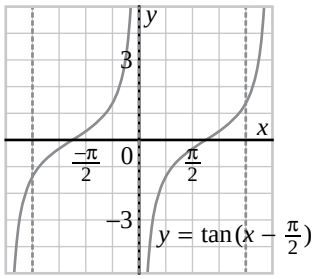
5. b)



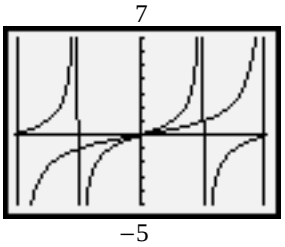
6. a)



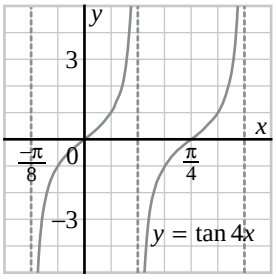
b)



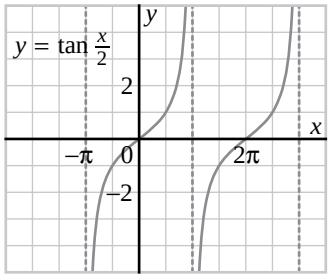
7. b)



8. a)

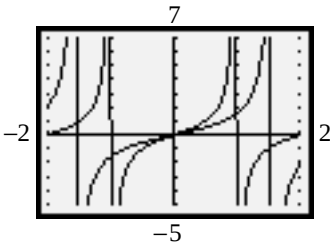


b)

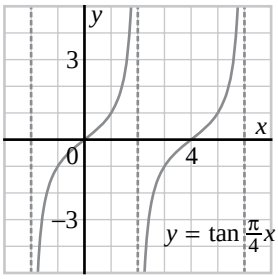


Selected Solutions — Chapter 4

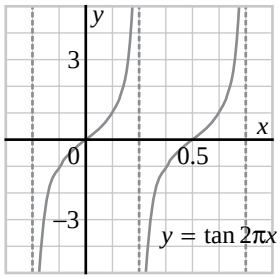
9. b)



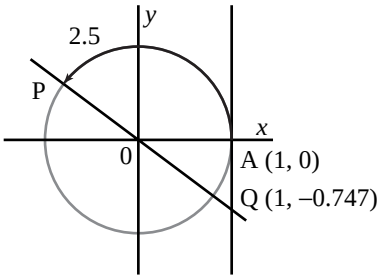
10. a)



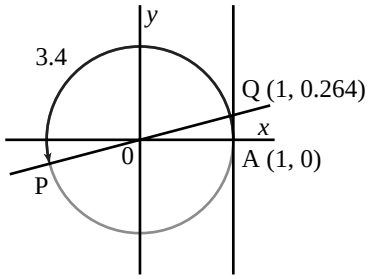
b)



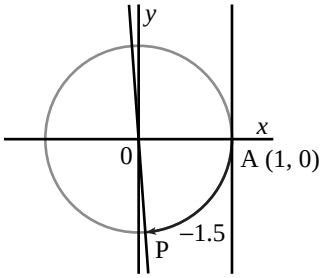
11. a)



b)



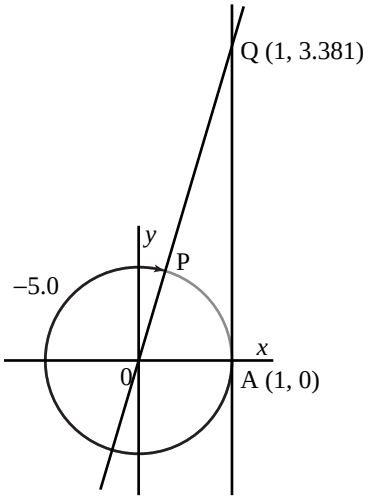
c)



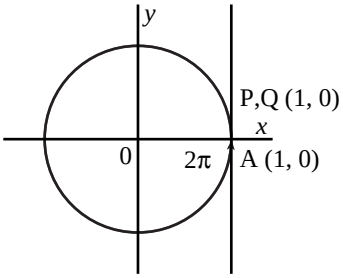
The line passing through the origin and P and the tangent line at A(1, 0) will eventually intersect each other at (1, −14.101).

Selected Solutions — Chapter 4

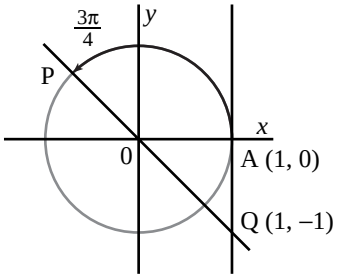
d)



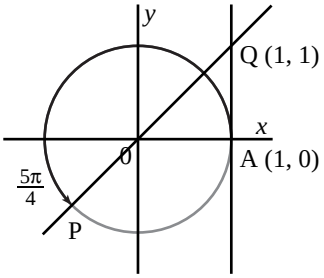
e)



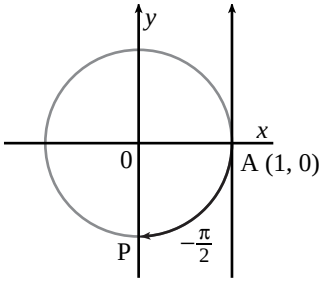
f)



g)

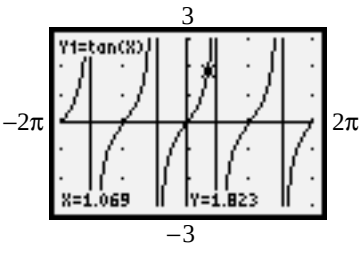


h)

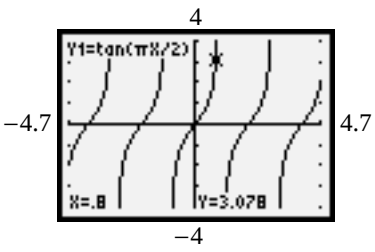


The line passing through O and P (or the *y*-axis) and the tangent line at A(1, 0) will never intersect, since they are parallel.

15. a)

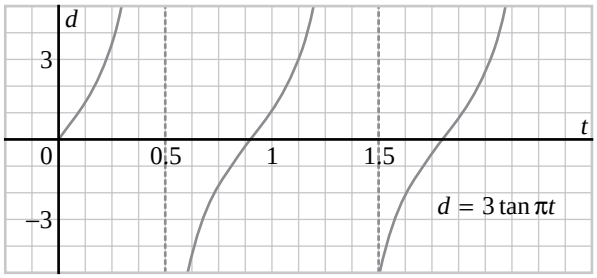


16. a)



Selected Solutions — Chapter 4

17. b)

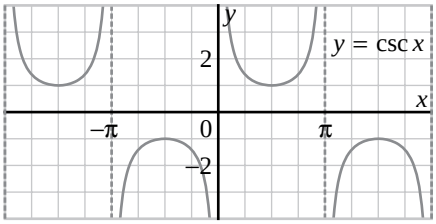


18. b) The coordinates are equal; that is, $x = \tan x$. By definition, $\tan x = \frac{\sin x}{\cos x}$, and $\sin x \doteq x$ and $\cos x \doteq 1$ when x is very small. Therefore, $\tan x \doteq \frac{x}{1}$, or $\tan x \doteq x$, when x is very small.

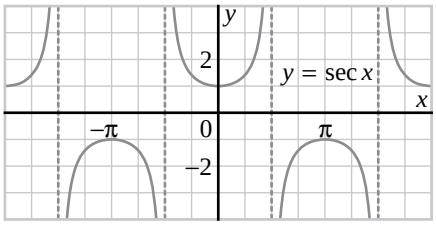
c) Recall that $\tan x = \frac{\sin x}{\cos x}$. For small values of x, $\cos x < 1$ and so $\frac{1}{\cos x} > 1$. Multiply both sides by $\sin x$ and simplify to arrive at the result, $\tan x > \sin x$, when x is very small.

4.7 Exercises, page 291

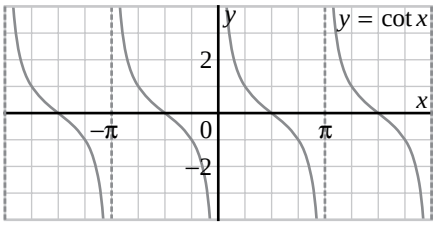
1.



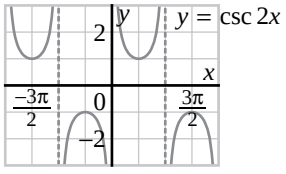
3.



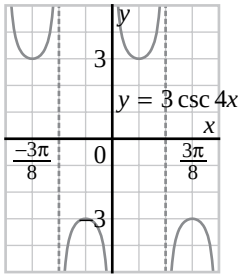
5.



7. a)

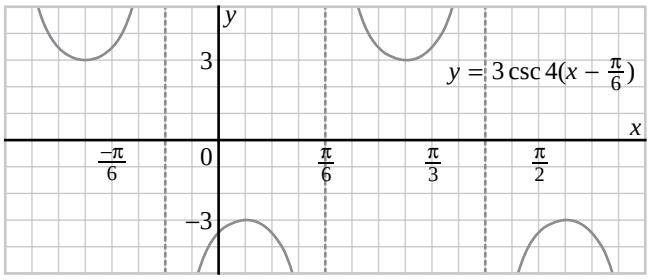


b)

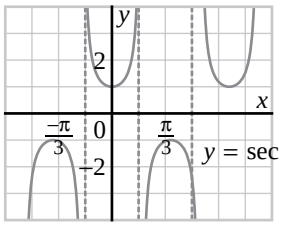


Selected Solutions — Chapter 4

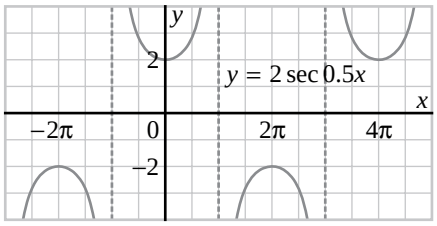
c)



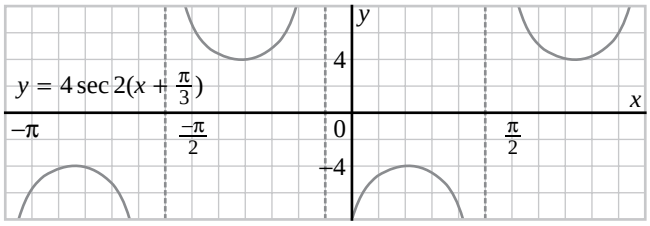
d)



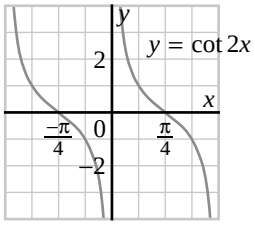
e)



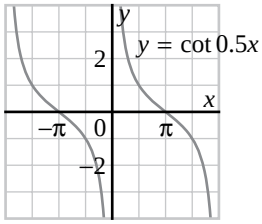
f)



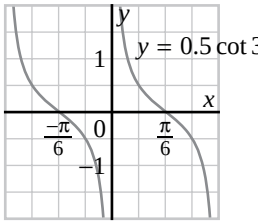
g)



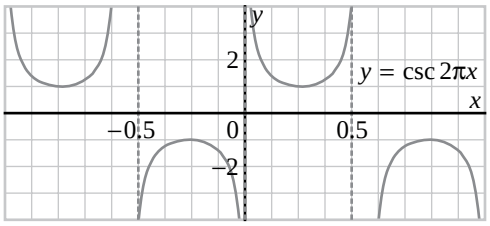
h)



i)

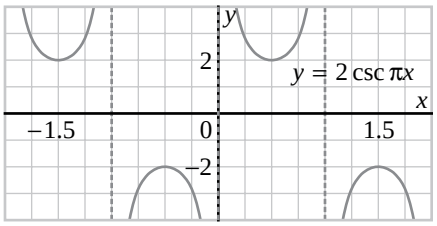


8. a)

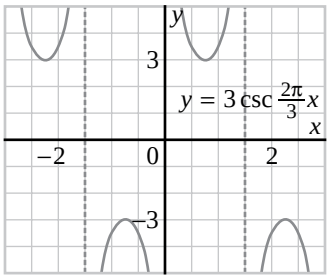


Selected Solutions — Chapter 4

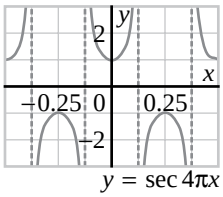
b)



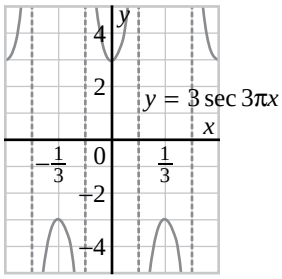
c)



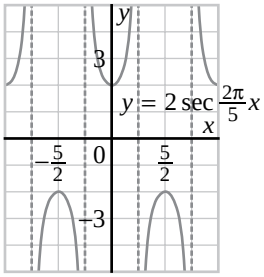
d)



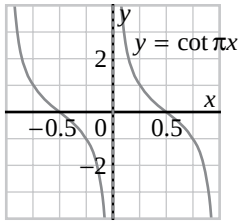
e)



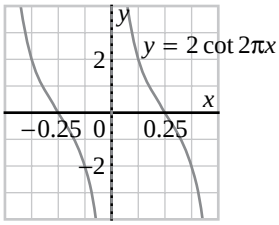
f)



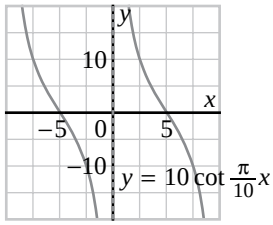
g)



h)



i)

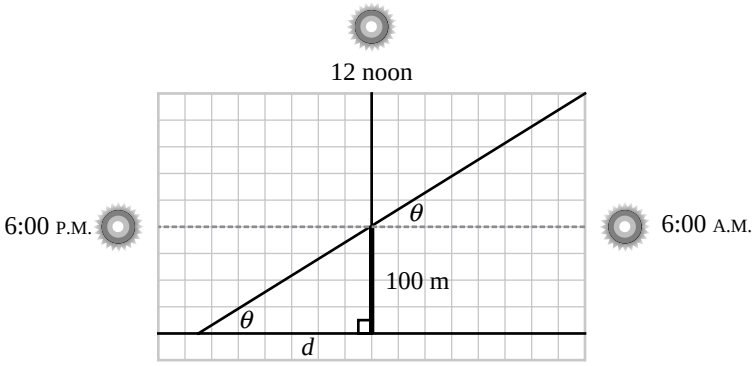


9. Explanations may vary. For exercise 8g:
First I graphed $y = \tan \pi x$ and then I reflected it in the y-axis.
I translated the graph 0.5 units left and arrived at the graph of $y = \cot \pi x$.

11. Explanations may vary. For part a:
I chose to represent the graph with a cosecant function since both are undefined at $x = 0$. The graph shows half a period, so the period is 6. There is no phase shift, but there is a vertical translation down 1 unit. Therefore, a possible equation of the graph is $y = \csc \frac{2\pi}{3}x - 1$, or $y = \csc \frac{\pi}{3}x - 1$.

Selected Solutions — Chapter 4

12. a) Draw a diagram to understand the situation.



$\frac{100}{d} = \tan\theta$

$d = 100\cot\theta$

The sun travels π radians in 12 h. Thus,

$\frac{\theta}{\pi} = \frac{t-6}{12}$

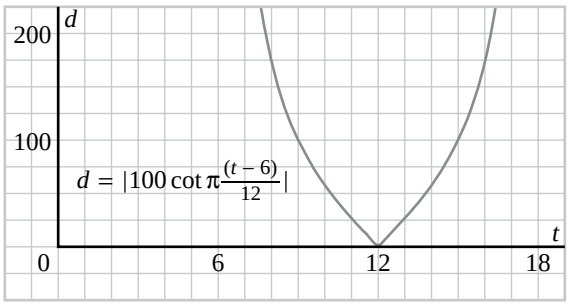
$\theta = \pi\frac{(t-6)}{12}$

Then substitute to get

$d = 100\cot\pi\frac{(t-6)}{12}$, $6 < t < 18$

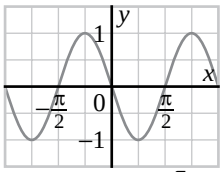
Notice that this equation gives negative values for $t > 12$, so the equation should be $d = |100\cot\pi\frac{(t-6)}{12}|$.

b)



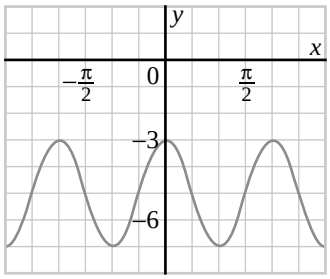
4 Review, page 293

5. a)



$y = \sin 2(x - \frac{\pi}{2})$

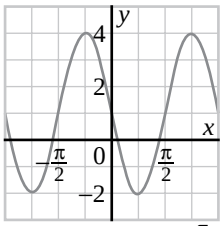
b)



$y = 2\sin 3(x - \frac{\pi}{2}) - 5$

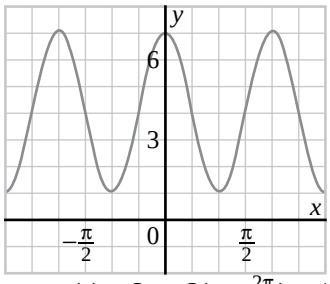
Selected Solutions — Chapter 4

c)



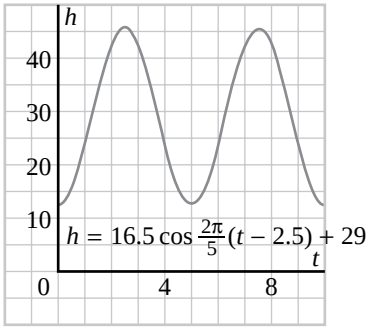
$g(x) = 3\cos 2(x + \frac{\pi}{4}) + 1$

d)



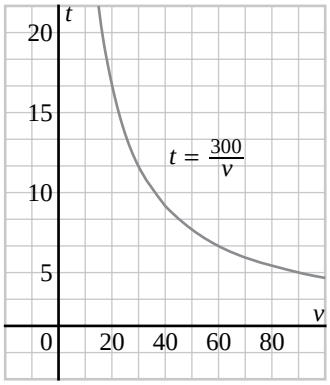
$g(x) = 3\cos 3(x - \frac{2\pi}{3}) + 4$

6. a)

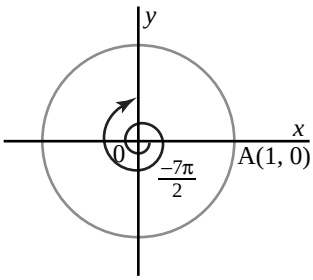


4 Cumulative Review, page 294

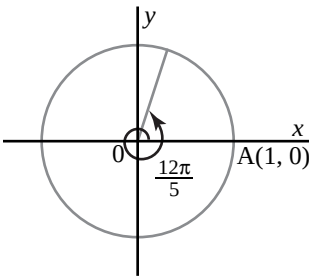
1. a)



8. a)

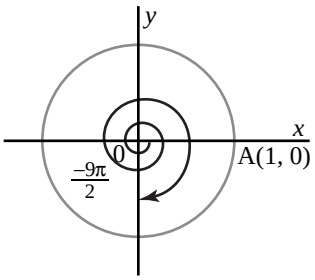


b)

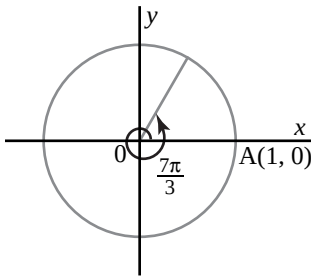


Selected Solutions — Chapter 4

c)



d)



9. **a)** The sine function is negative in the third quadrant.

b) The cosine function is negative in the third quadrant.

c) The tangent function is positive in the first quadrant.

d) The sine function is negative in the fourth quadrant.

e) The cosine function is negative in the second quadrant.

f) The sine function is positive for $0 < \theta < 180^\circ$.

g) The cosine function is positive in the first quadrant.

h) The tangent function is positive in the third quadrant.

10. **b)**