ADDISON-WESLEY Mathematics 12

Selected Solutions — Chapter 3

3.1 Exercises, page 161

2. **a)** Answers may vary. The graph for the sunsets in Fairbanks has a higher maximum and a lower minimum value than the graph for the sunsets in Saskatoon. The graph for the sunsets in Mexico City has a much lower maximum and higher minimum than the graph for the sunsets in Saskatoon.

b) Answers may vary. The period of all three graphs is the same.

4. **c)** Answers may vary. The shadow would have a length of 0 m at noon. I would shift the graph vertically to have the graph touch the horizontal axis at noon.

5. **a)** Answers may vary. The graphs would have shorter periods; that is, the hills would be closer together.

b) Answers may vary. The graphs would have the same maximum and minimum values.

7. Explanations may vary. The deep breathing graph would have a higher maximum value, 5000 mL, compared to 2700 mL, for the regular breathing graph. The minimum value in the deep breathing graph would be 1000 mL compared to 2200 mL for the regular breathing graph. The period of the deep breathing function is 10 seconds compared to 5 seconds for the regular breathing function.

3.2 Exercises, page 168

12. Explanations may vary. For exercise 10 c, the angle is 1.8 radians. I substituted $\theta = 1.8$ and $r = 5$ cm into the formula that calculates the arc length, $a = r\theta$, to obtain $a = 9.0$ cm.

15. **a)** $\frac{\text{Area of a sector}}{\text{Area of a circle}} = \frac{\text{Sector angle}}{\text{Full rotation}}$

Let A represent the area of a sector with angle θ.
If the angle θ is measured in radians,

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$A = \frac{r^2\theta}{2}$$

b) If the angle θ is measured in degrees,

$$\frac{A}{\pi r^2} = \frac{\theta}{360^\circ}$$

$$A = \frac{\pi r^2\theta}{360^\circ}$$

c) When radian measure is used instead of degree measure for angles, the formula for area of a sector is much simpler.

16. **a)** The circumference of the tire is $\pi(64 \text{ cm})$ or 64π cm. Convert 100 km/h to cm/s.

$100 \text{ km/h} = 100 \times \frac{100\,000}{3600}$ cm/s

Divide this by the circumference of the tire to determine the number of rotations per second.

$$\frac{100\,000}{36 \times 64\pi}$$

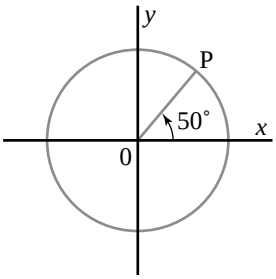
Selected Solutions — Chapter 3

Every rotation is 2π radians.
Hence the angular velocity is $\frac{100\ 000}{36 \times 64\pi} \times 2\pi \doteq 86.83$
The tire's angular velocity is approxmiately 86.83 radians/s.

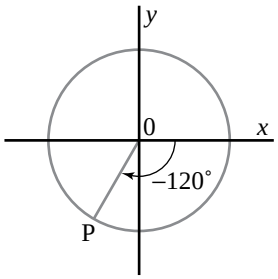
b) The circumference of the tire is πd cm. Convert x km/h to cm/s.
x km/h $= x \times \frac{100\ 000}{3600}$ cm/s
Divide this by the circumference of the tire to determine the number of rotations per second.
$\frac{1000x}{36 \times \pi\, d}$
Every rotation is 2π radians.
Hence the angular velocity is $\frac{1000x}{36\pi\, d} \times 2\pi = \frac{500x}{9d}$
The tire's angular velocity is $\frac{500x}{9d}$ radians/s.

3.3 Exercises, page 174

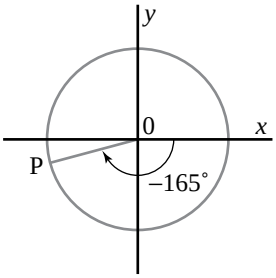
2. a)



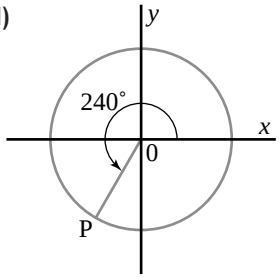
b)



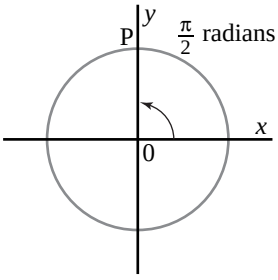
c)



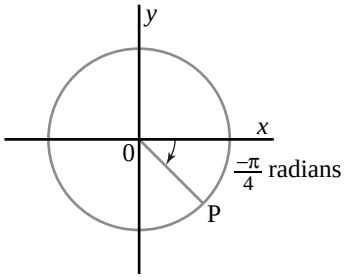
d)



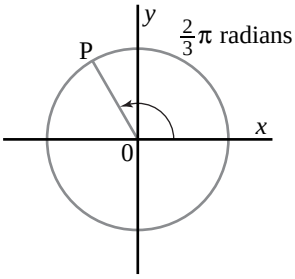
e)



f)



g)



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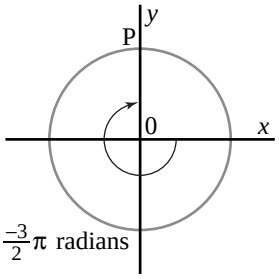
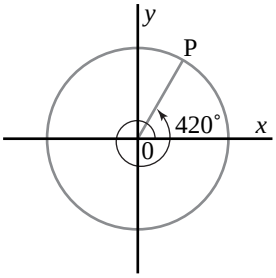
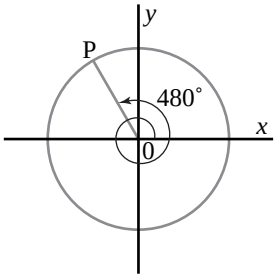


Selected Solutions — Chapter 3

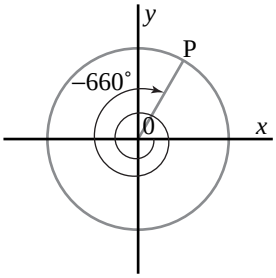
7. c)



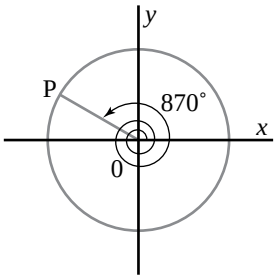
8. a) iii)



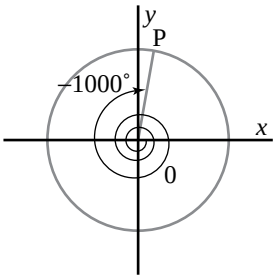
b) iii)



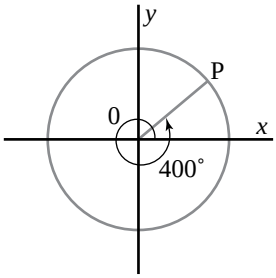
c) iii)



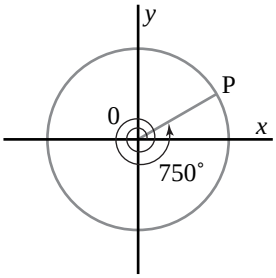
d) iii)



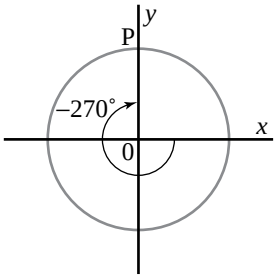
9. a)



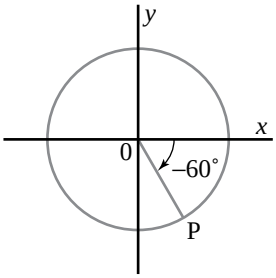
b)



c)

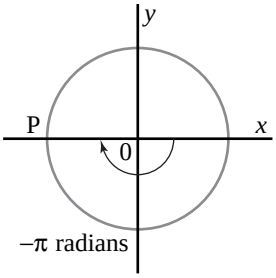


d)



Selected Solutions — Chapter 3

10. **a)**



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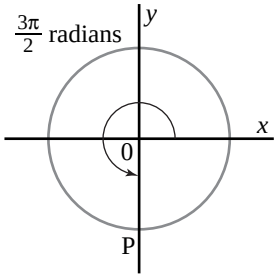
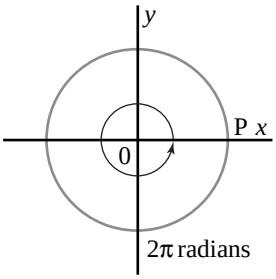


c)



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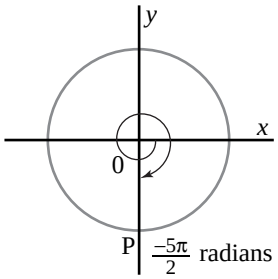
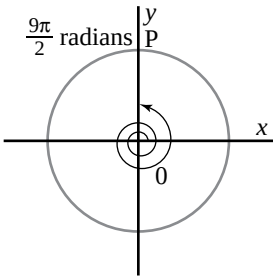
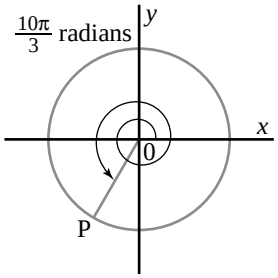


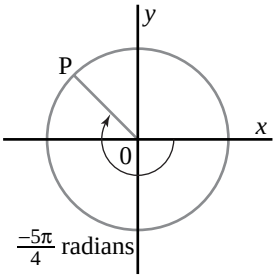
11. **a)**



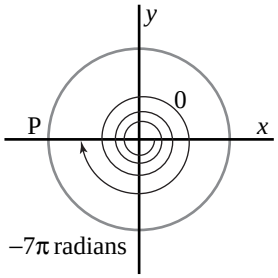
b)



c)

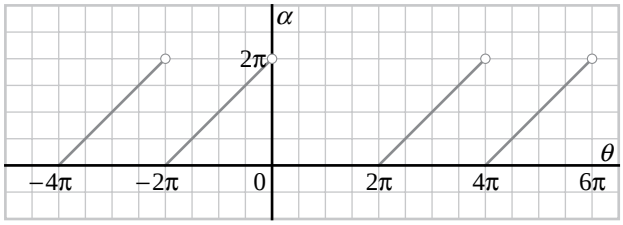


d)



12. Explanations may vary. For exercise 9a, the angle is 400°, which I wrote as 360° + 40°. I moved one complete rotation in the counterclockwise direction. I then measured 40° from the *x*-axis and drew the terminal arm from the origin to point P.

14. **a)**



b) The graph gives the domain $\theta < 0$ and $\theta \geq 2\pi$. The range is stated in the question as $0 \leq \alpha < 2\pi$. The period of the function for the defined domain is 2π.

Selected Solutions — Chapter 3

3.4 Exercises, page 180

4. The value of cos θ corresponds to the x-coordinate of P. Cos θ starts at 1 ($\theta = 0^\circ$), decreases to 0 ($\theta = 90^\circ$), decreases further to –1 ($\theta = 180^\circ$), increases to 0 ($\theta = 270^\circ$), and increases further to 1 ($\theta = 360^\circ$). The cycle repeats from $\theta = 360^\circ$ to $\theta = 720^\circ$. The value of sin θ corresponds to the y-coordinate of P. Sin θ starts at 0 ($\theta = 0^\circ$), increases to 1 ($\theta = 90^\circ$), decreases to 0 ($\theta = 180^\circ$), decreases further to –1 ($\theta = 270^\circ$), and increases to 0 ($\theta = 360^\circ$). The cycle repeats from $\theta = 360^\circ$ to $\theta = 720^\circ$.

9. **a)** Sine of an angle in the 2nd quadrant is positive.

b) Cosine of an angle in the 2nd quadrant is negative.

c) Cosine of an angle in the 3rd quadrant is negative.

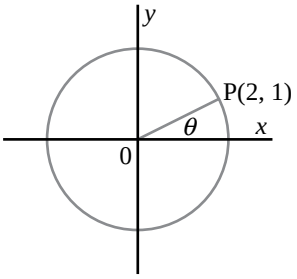
d) Sine of an angle in the 3rd quadrant is negative.

e) Cosine of an angle in the 4th quadrant is positive.

12. Explanations may vary. For part a, I read the coordinates of the vertex in the first quadrant, (0.866, 0.5) from the screen. Because circles and squares are symmetrical, the vertex in the second quadrant has the x-coordinate equal to the negative of the y-coordinate of the vertex in the first quadrant and the y-coordinate is equal to the x-coordinate of the vertex in the first quadrant. That is, the vertex in the second quadrant is (–0.5, 0.866). Using symmetry again, the x- and y-coordinates of the vertex in the third quadrant are the negatives of the x- and y-coordinates of the vertex in the first quadrant. That is, the vertex in the third quadrant is (–0.866, –0.5). Similarly, the x- and y-coordinates of the vertex in the fourth quadrant are the negatives of the x- and y-coordinates of the vertex in the second quadrant. That is, the vertex in the fourth quadrant is (0.5, –0.866).

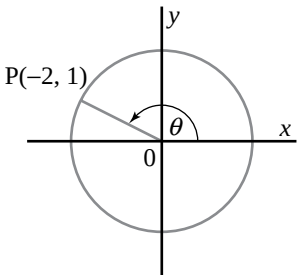
15. Explanations may vary. For exercise 13a: let P represent the point on the terminal arm and on the unit circle of the angle with measurement 30°. P is the vertex of the square in the first quadrant. A reflection in the y-axis of point P, would give me the point on the terminal arm and on the unit circle of the angle with measurement 150°. Thus, the reflected point has coordinates (–0.866, 0.5). Since the coordinates of any point on the unit circle may be expressed as (cos θ, sin θ), $\cos 150^\circ = -0.866$ and $\sin 150^\circ = 0.5$.

16. **a)**

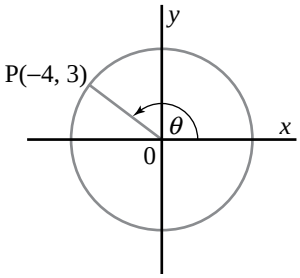


Selected Solutions — Chapter 3

17. a)



18. a)



19. a) For the TI-83, 999 999 999 999° or 999 999 999 999 radians are the largest measures of an angle accepted by the calculator in degree and radian mode.

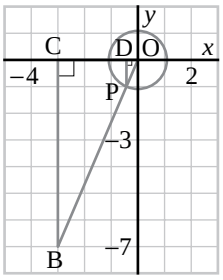
b) $999\,999\,999\,999^\circ = 999\,999\,999\,999 \times \frac{\pi}{180}$

$= 5\,555\,555\,556\pi$

$\doteq 17\,453\,292\,500$ radians

Therefore, the two angles in part a are not the same and furthermore, $999\,999\,999\,999^\circ < 999\,999\,999\,999$ radians.

20. a) Construct a triangle DPO, where P is a point on the unit circle, on the line segment OB. D is on the x-axis, where PD is perpendicular to the x-axis.



Since △CBO and △DPO have corresponding angles that are equal, the triangles are similar.

Therefore, $\frac{OD}{OP} = \frac{OC}{OB}$ ①

$\frac{OD}{1} = \frac{3}{\sqrt{58}}$

Also, $\frac{PD}{OP} = \frac{BC}{OB}$ ②

$\frac{PD}{1} = \frac{7}{\sqrt{58}}$

Therefore, from ① and ②, the coordinates of P are $\left(-\frac{3}{\sqrt{58}}, -\frac{7}{\sqrt{58}}\right)$.

Selected Solutions — Chapter 3

Since the coordinates of any point on the unit circle may be expressed as $(\cos\theta,\ \sin\theta)$, $\cos\theta = -\frac{3}{\sqrt{58}}$ and $\sin\theta = -\frac{7}{\sqrt{58}}$

b) Using $\triangle CBO$, $\sin\theta = -\frac{7}{\sqrt{58}}$ and $\cos\theta = -\frac{3}{\sqrt{58}}$.

The sine and cosine of the angle in part a are equal to the sine and cosine of the angle in part b.

Exploring with a Graphing Calculator, page 182

1. a) The left arrow key decreases the value of T by 10 until it reaches Tmin. The right arrow key increases the value of T by 10 until it reaches Tmax.

b) The trace feature starts at T = Tmin. The left arrow key decreases the value of T by the Tstep. The right arrow key increases the value of T by the Tstep. The trace feature stops at T = Tmax.

2. Yes. Since this is the graph of the unit circle, the x-coordinate is the cosine of the angle, and the y-coordinate is the sine.

3. a) The graph is 4 dots on the unit circle, at T = 30˚, 120˚, 210˚, and 300˚. These dots are vertices of a square.

b) The cursor jumps from T = 30˚ to T = 120˚ to T = 210˚ to T = 300˚ to T = 390˚.

c) The graph is a square.

4. a) Use Tmin = 30, Tmax = 390, and Tstep = 60.

b) Use Tmin = 18, Tmax = 378, and Tstep = 72.

5. a) $T = 180 - 18^\circ$, or 162˚ for the vertex in the second quadrant. So multiply the x-value of the known vertex by –1 to get the coordinate (–0.951, 0.309). Therefore, $\cos 162^\circ = -0.951$ and $\sin 162^\circ = 0.309$.

b) Reflect the graph in the x- and y-axis. The known vertex would be in the third quadrant and have $T = 180 + 18^\circ$, or 198˚. Its coordinate would be (–0.951, –0.309). Therefore, $\cos 198^\circ = -0.951$ and $\sin 198^\circ = -0.308$.

c) Reflect the graph in the x-axis. The known vertex would be in the fourth quadrant with $T = 360 - 18^\circ$, or 342˚. Its coordinate would be (0.951, –0.309). Therefore, $\cos 342^\circ = 0.951$ and $\sin 342^\circ = -0.309$.

d) Reflect the graph in the $y = x$ line and the new coordinate will be (y, x), or (0.309, 0.951). Its angle is $T = 90 - 18^\circ$, or 72˚. Therefore, $\cos 72^\circ = 0.309$ and $\sin 72^\circ = 0.951$.

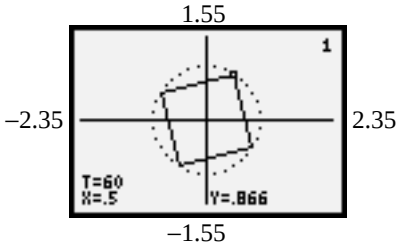
e) Reflect the image in part d in the y-axis and reverse the sign of the x-coordinate. The new angle will be $T = 180 - 72^\circ$, or 108˚. Therefore $\cos 108^\circ = -0.309$ and $\sin 108^\circ = 0.951$.

f) Reflect the image in part d in the x-axis and reverse the sign of the y-coordinate. The new angle will be $T = 360 - 72^\circ$, or 288˚. Therefore $\cos 288^\circ = 0.309$ and $\sin 288^\circ = -0.951$.

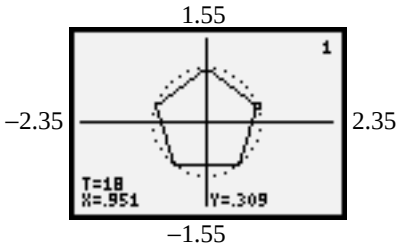
Selected Solutions — Chapter 3

6. a) Explanations may vary. Different window settings are required for each of the circle and polygon.

b) i) Answers may vary. Store a dotted unit circle first. To get the square follow exercise 4 part a. Recall the unit circle.



ii) Answers may vary. Store a dotted unit circle first. To get the square follow exercise 4 part b. Recall the unit circle.



7. When angle t is in standard position, $\cos t$ equals $\frac{x}{r}$ and $\sin t$ equals $\frac{y}{r}$, where (x, y) lies on a circle of radius r. To get $x = \cos t$ and $y = \sin t$, $r = 1$. That is, the points (x, y) lie on the unit circle.

8. a) Use Tmin = 18, Tmax = 738, and Tstep = 144.

b) Store a dotted unit circle first. To get the star, use Tmin = 10, Tmax = 1450, and Tstep = 160. Recall the unit circle.

9. a) $T = 180 - 10^\circ$, or 170° for the second vertex in the second quadrant. So multiply the x-value of the known vertex by –1 to get the coordinate (–0.985, 0.174). Therefore, $\cos 170^\circ = -0.985$ and $\sin 170^\circ = 0.174$.

b) Reflect the graph in the x-axis. The vertex in part a would be in the third quadrant and have $T = 180 + 10^\circ$, or 190°. Its coordinate would be (–0.985, –0.174). Therefore, $\cos 190^\circ = -0.985$ and $\sin 190^\circ = -0.174$.

c) Reflect the graph in the x-axis. The known vertex would be in the fourth quadrant with $T = 360 - 10^\circ$, or 350°. Its coordinate would be (0.985, –0.174). Therefore, $\cos 350^\circ = 0.985$ and $\sin 350^\circ = -0.174$.

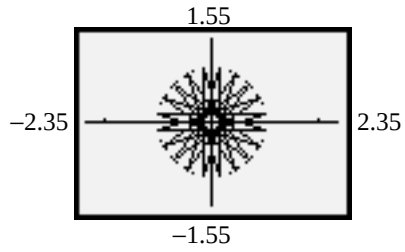
d) Reflect the graph in the $y = x$ line and the image of the known coordinate will be (y, x), or (0.174, 0.985). Its angle is $T = 90 - 10^\circ$, or 80°. Therefore, $\cos 80^\circ = 0.174$ and $\sin 80^\circ = 0.985$.

e) Reflect the image in part d in the y-axis and reverse the sign of the x-coordinate. The new angle will be $T = 90 + 10^\circ$, or 100°. Therefore $\cos 100^\circ = -0.174$ and $\sin 100^\circ = 0.985$.

Selected Solutions — Chapter 3

f) Reflect the image in part d in the x-axis and reverse the sign of the y-coordinate. The new angle will be $T = 270 + 10^\circ$, or 280°. Therefore $\cos 280^\circ = 0.174$ and $\sin 280^\circ = -0.985$.

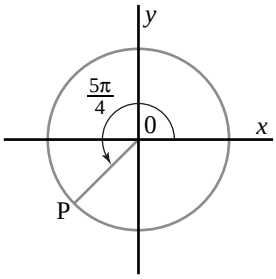
10. Answers may vary.



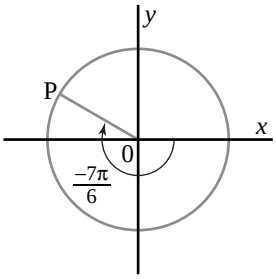
Window Settings: Tmin = 9, Tmax = 3249, Tstep = 162

3.5 Exercises, page 188

2. a) i)



ii)



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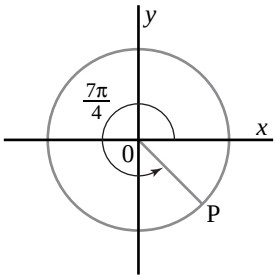

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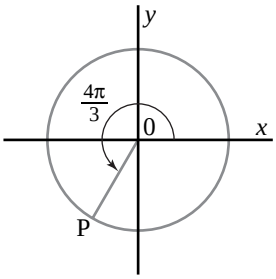

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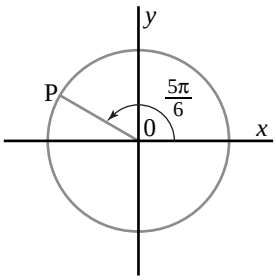

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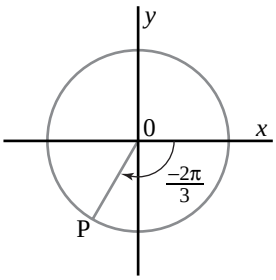

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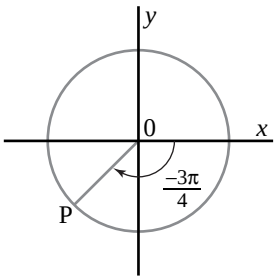
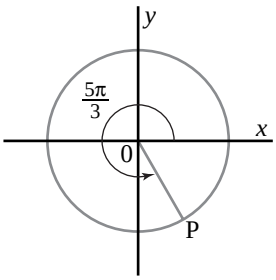


viii)



Selected Solutions — Chapter 3

5. **a)** $\sin 0^\circ = 0$, or $\frac{\sqrt{0}}{2}$; $\sin 30^\circ = \frac{1}{2}$, or $\frac{\sqrt{1}}{2}$; $\sin 45^\circ = \frac{\sqrt{2}}{2}$; $\sin 60^\circ = \frac{\sqrt{3}}{2}$; $\sin 90^\circ = 1$, or $\frac{\sqrt{4}}{2}$. Therefore, the pattern holds.

b) $\cos 0^\circ = 1$, or $\frac{\sqrt{4}}{2}$; $\cos 30^\circ = \frac{\sqrt{3}}{2}$; $\cos 45^\circ = \frac{\sqrt{2}}{2}$; $\cos 60^\circ = \frac{1}{2}$, or $\frac{\sqrt{1}}{2}$; $\cos 90^\circ = 0$, or $\frac{\sqrt{0}}{2}$. The cosine ratios of the angles have the same pattern as the sine ratios but in reverse order.

7. **a)** Left side $= \sin 2A + \sin 2B + \sin 2C$
$= \sin 90^\circ + \sin 90^\circ + \sin 180^\circ$
$= 1 + 1 + 0$
$= 2$

Right side $= 4 \sin A \sin B \sin C$
$= 4 \sin 45^\circ \sin 45^\circ \sin 90^\circ$
$= 4\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(1)$
$= 2$

Hence, Left side = Right side

b) **i)** Left side $= \sin 2A + \sin 2B + \sin 2C$
$= \sin 60^\circ + \sin 120^\circ + \sin 180^\circ$
$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + 0$
$= \sqrt{3}$

Right side $= 4 \sin A \sin B \sin C$
$= 4 \sin 30^\circ \sin 60^\circ \sin 90^\circ$
$= 4\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)(1)$
$= \sqrt{3}$

Hence, Left side = Right side

ii) Left side $= \sin 2A + \sin 2B + \sin 2C$
$= \sin 120^\circ + \sin 120^\circ + \sin 120^\circ$
$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$
$= \frac{3\sqrt{3}}{2}$

Right side $= 4 \sin A \sin B \sin C$
$= 4 \sin 60^\circ \sin 60^\circ \sin 60^\circ$
$= 4\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$
$= \frac{3\sqrt{3}}{2}$

Hence, Left side = Right side

9. Explanations may vary. I numbered the vertices from 1 to 8, with 1 being the marked vertex. Moving counterclockwise I read the first vertex from the screen, (0.924, 0.383). The second vertex, is a reflection in the line $y = x$ of the first vertex, or (0.383, 0.924). The third vertex is a reflection in the y-axis of the second vertex, or (–0.383, 0.924). The fourth vertex is a reflection in the line $y = -x$ of the third vertex, or (–0.924, 0.383). The fifth vertex is a reflection in the x-axis of the fourth vertex, or (–0.924, –0.383). The sixth

Selected Solutions — Chapter 3

vertex is a reflection in the line $y = x$ of the fifth vertex, or (–0.383, –0.924). The seventh vertex is a reflection in the y-axis of the sixth vertex, or (0.383, –0.924). The eighth vertex is a reflection in the line $y = -x$ of the seventh vertex, or (0.924, –0.383).

10. b) ii) The exact values cannot be calculated because the exact values for the reference angle, $\frac{\pi}{8}$, are not known.

12. a) Let Q be the point where the terminal arm of $\pi + \theta$ intersects the unit circle. Thus, the coordinates of Q are $(-x, -y)$.
$\sin(\pi + \theta) = -y$, $\cos(\pi + \theta) = -x$

b) Let R be the point where the terminal arm of $2\pi - \theta$ intersects the unit circle. Thus, the coordinates of R are $(x, -y)$.
$\sin(2\pi - \theta) = -y$, $\cos(2\pi - \theta) = x$

Exploring with a Graphing Calculator, page 190

2. The left screen has more points on the circle and more triangles, corresponding to these points. The right screen has a point every 10° or every 5°.

3. The cosine graph is the same as the sine graph translated to the left 90°.

4. P(x, y) is a point on the unit circle; the cosine and sine values are equal whenever $x = y$. The unit circle intersects the line $y = x$ at two points, which correspond to $\theta = 45°$ and $\theta = 225°$. The graph $y = \sin\theta$ and $y = \cos\theta$ intersect at two points in the interval $0° < \theta < 360°$. The points of intersection occur when $\theta = 45°$ and $\theta = 225°$.

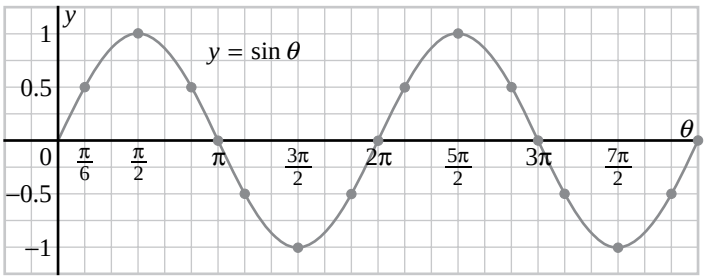
Exploring with a Graphing Calculator, page 191

1. Answers may vary. One advantage is that two complete periods are graphed. One disadvantage is that the trace function does not show integer values for the domain.

6. Answers may vary. One advantage is that two complete periods are graphed. One disadvantage is that the trace function does not show integer values for the domain.

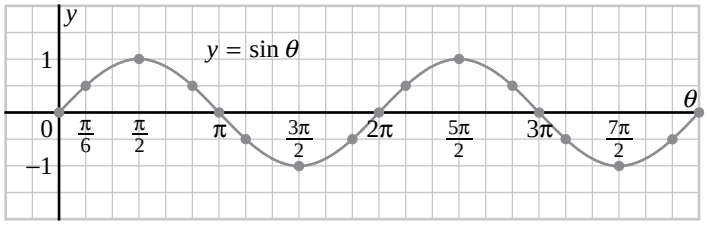
3.6 Exercises, page 196

1. a)

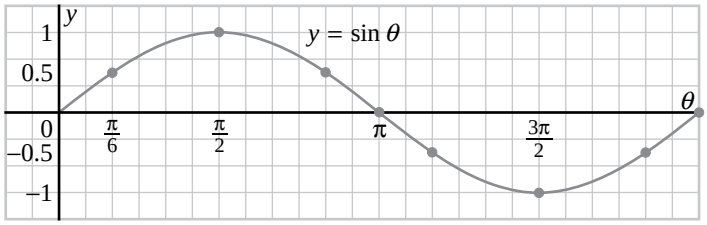


Selected Solutions — Chapter 3

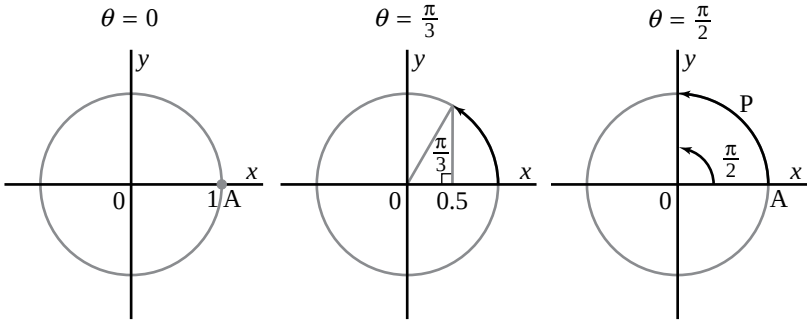
b)

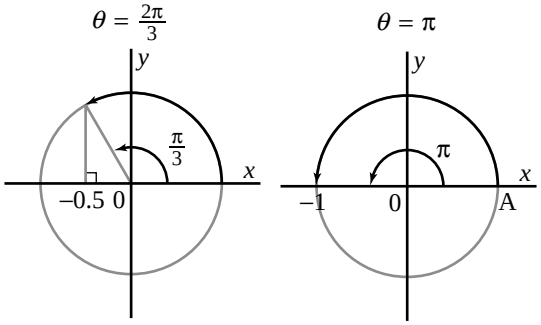


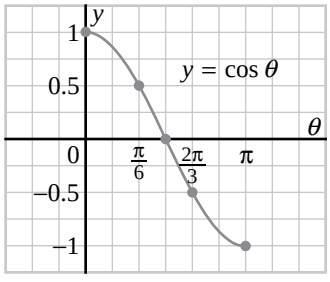
c)



2.

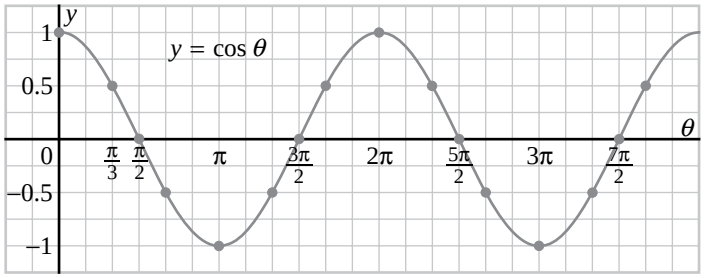




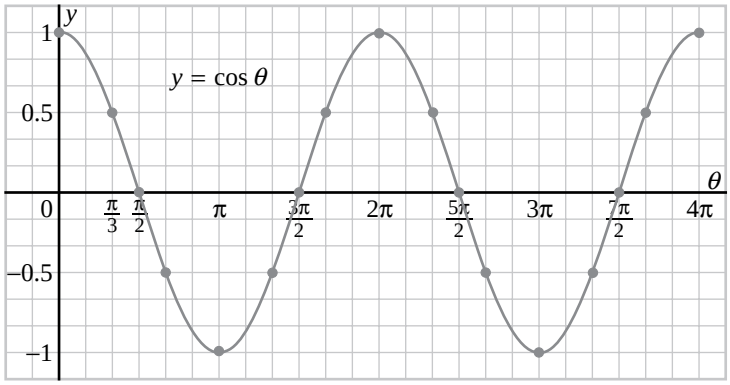


Selected Solutions — Chapter 3

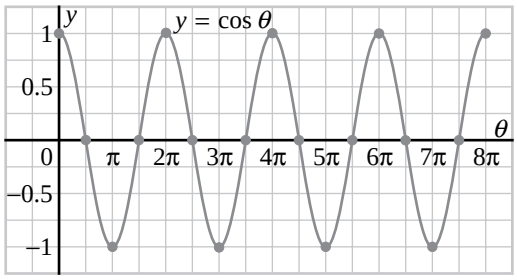
3. a)



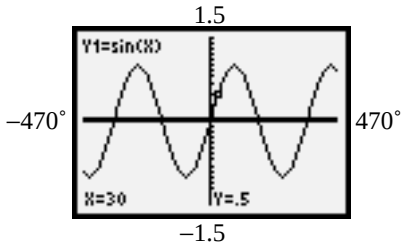
b)



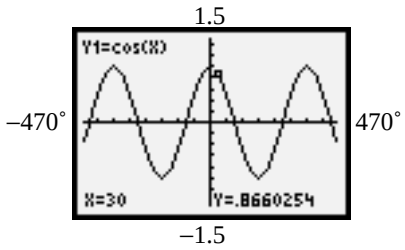
c)



7. a)



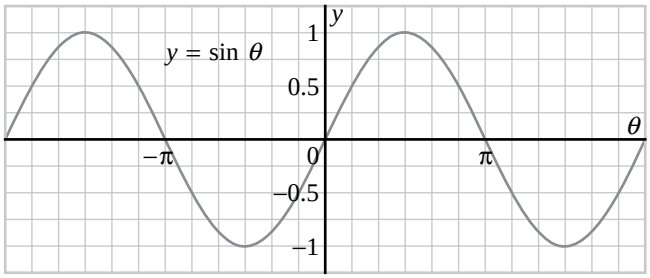
8. a)



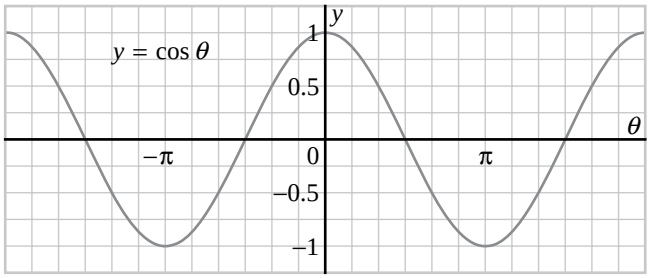
Selected Solutions — Chapter 3

c) Angles that are coterminal with 30° satisfy exercise 7b and 8b since the sine and cosine of an angle are positive in the first quadrant. The other angles are unique to sine and cosine since sine is positive in the second quadrant (cosine is negative) and cosine is positive in the fourth quadrant (sine is negative). That is, another angle that has the same sine as 30° is 150° and any angles coterminal with 150°. Another angle that has the same cosine as 30° is –30° and any angle coterminal with it.

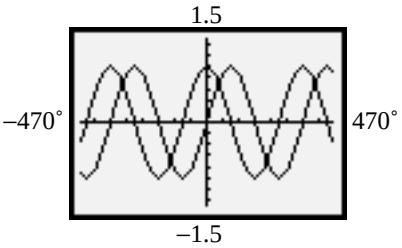
9.



11.



13.



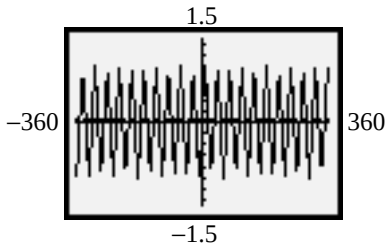
a) The cosine graph appears to be a translation of the sine graph 90° to the left.

b) Sin θ = cos θ when θ is an odd multiple of 45° and the terminal arm of θ is in the first or third quadrant.

c) By examining the graph, you can see which is greater.

14. **a)** The graph has many periods of $y = \sin\theta$. One period is approximately 6.28 radians. Between –360 radians and 360 radians there are approximately $\frac{720}{6.28}$ periods or 115 periods.

b)



Selected Solutions — Chapter 3

15. a) The graph is almost a straight line, with a very small positive slope, through the origin.

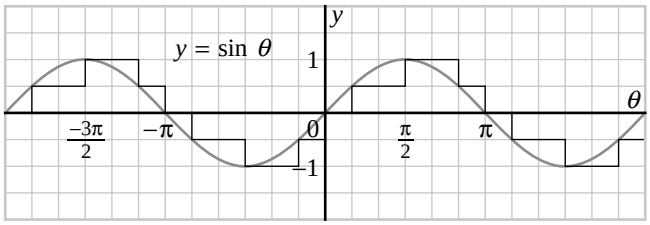
b) The graphs are the same provided all angles are in degrees or all angles are in radians.

16. a) The y-coordinate in exercise 5 is greater than the y-coordinate in exercise 4; that is, $\cos 40^\circ > \sin 40^\circ$.

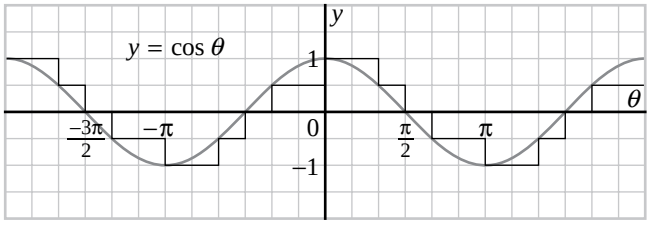
b) It is true for some additional values of x, but not all values of x. Returning to the unit circle to explain, for the first coordinate, $y = \cos\theta$, to be greater than the second, $y = \sin\theta$, the point P on the circle must be further from the x-axis than the y-axis. This only occurs half of the time.

Mathematics File, page 199

1.

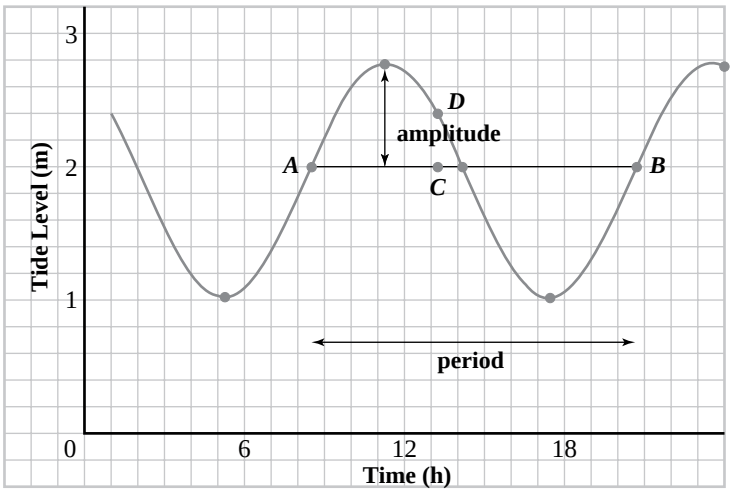


2.



Mathematical Modelling, page 201

1. a)



b) $A = \left(\frac{5.15 + 11.33}{2}, \frac{1.15 + 2.79}{2}\right) = (8.24, 1.97)$

c) $B = \left(\frac{17.51 + 23.69}{2}, \frac{1.15 + 2.79}{2}\right) = (20.6, 1.97)$

Selected Solutions — Chapter 3

f) Period: $17.51 - 5.15 = 12.36\,\text{h}$

g) Amplitude: $\frac{2.79 - 1.15}{2} = 0.82\,\text{m}$

2. a) Let 8:30 A.M. be the point C on the tide graph. Let D be the level of the tide at point C on the tide graph. Let Q and R be the corresponding points on the sine graph. Follow the method used in the text.

$C = \frac{8.5 - 8.24}{12.36} \times 2\pi \doteq 0.1322.$

$D = \sin 0.1322$, or approximately 0.1318.

The distance from C to D is 0.1318×0.82 m, or approximately 0.11 m.

Thus the level of D is 1.97 m + 0.11 m, or 2.08 m.

b) Let 9:30 A.M. be the point C on the tide graph. Let D be the level of the tide at point C on the tide graph. Let Q and R be the corresponding points on the sine graph. Follow the method used in the text.

$C = \frac{9.5 - 8.24}{12.36}\ 2\pi \doteq 0.6405.$

$D = \sin 0.6405$, or approximately 0.5976.

The distance from C to D is 0.5976 0.82 m, or approximately 0.49 m.

Thus the level of D is 1.97 m + 0.49 m, or 2.46 m.

c) Let 2:15 P.M. be the point C on the tide graph. Let D be the level of the tide at point C on the tide graph. Let Q and R be the corresponding points on the sine graph. Follow the method used in the text.

$C = \frac{14.25 - 8.24}{12.36} \times 2\pi \doteq 3.0552.$

$D = \sin 3.0552$, or approximately 0.0863.

The distance from C to D is 0.0863×0.82 m, or approximately 0.07 m.

Thus the level of D is 1.97 m + 0.07 m, or 2.04 m.

d) Let 7:20 P.M. be the point C on the tide graph. Let D be the level of the tide at point C on the tide graph. Let Q and R be the corresponding points on the sine graph. Follow the method used in the text.

$C = \frac{19.33 - 8.24}{12.36} \times 2\pi \doteq 5.6393.$

$D = \sin 5.6393$, or approximately –0.6003.

The distance from C to D is -0.6003×0.82 m, or approximately –0.49 m.

Thus the level of D is 1.97 m – 0.49 m, or 1.48 m.

3. Let $x = 10.5$ and substitute into the equation.

$y = 0.82 \sin 2\pi \left(\frac{10.5 - 8.24}{12.36}\right) + 1.97$

$\doteq 2.72$ m

Thus the tide level is approximately 2.72 m.

Selected Solutions — Chapter 3

4. The amplitude is half of 21.5 − 16.9, or 2.3. The period is 365 days in a year. The starting value of x for a sine cycle is 80, since March 21 is the 80 day of the year. Finally the mean value of y is 16.9 + 2.3, or 19.2. Thus, the equation is

$y = 2.3 \sin 2\pi \left(\frac{x - 80}{365}\right) + 19.2$

May 24 is day 144 and so $x = 144$.

$y = 2.3 \sin 2\pi \left(\frac{144 - 80}{365}\right) + 19.2$

$\doteq 21.25$

Therefore, the time is approximately 21 − 12 hours, 0.25×60 minutes, or 9:15 P.M.

October 10 is day 283 and so $x = 283$.

$y = 2.3 \sin 2\pi \left(\frac{283 - 80}{365}\right) + 19.2$

$\doteq 18.41$

Therefore, the time is approximately 18 − 12 hours, 0.41×60 minutes, or 6:25 P.M.

5. To use the cosine function, A could have coordinates (11.33, 2.79) and B could have coordinates (23.69, 2.79). Since the cosine function and the sine function have the exact same shape, a similar equation can be developed; $y = 0.82 \cos 2\pi \left(\frac{x - 11.33}{12.36}\right) + 1.97$.

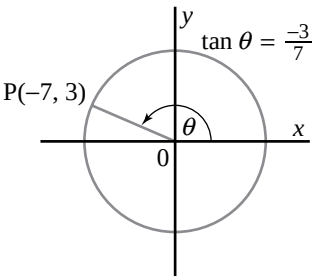
3.7 Exercises, page 209

6. $\text{Tan}\, 0^\circ = 0$; $\tan 30^\circ = \frac{\sqrt{3}}{3}$; $\tan 45^\circ = 1$; $\tan 60^\circ = \sqrt{3}$; $\tan 90^\circ$ = undefined

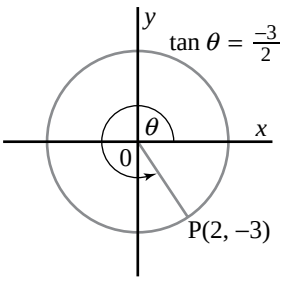
There does not appear to be a similar pattern.

7. **c)** If the coordinates of P are (x, y), then $\tan \theta = \frac{y}{x}$.

8. **a)**

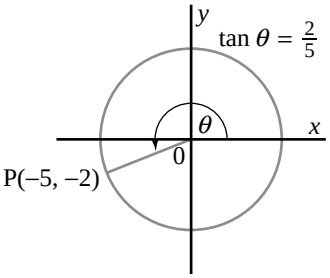


9. **a)**

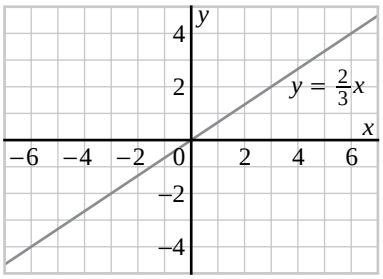


Selected Solutions — Chapter 3

10. a)



11. a)



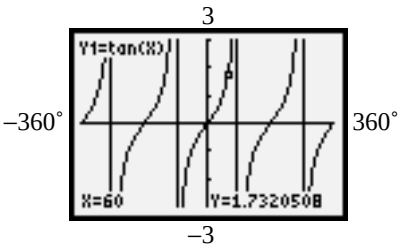
12. a) Since $\tan\theta = \frac{y}{x}$ and θ is in the first quadrant, I knew x and y were positive values. I expressed $\tan\theta$ as a decimal and let $y = \tan\theta$ and $x = 1$. Then I calculated $r = \sqrt{x^2 + y^2}$, $\sin\theta = \frac{y}{r}$, and $\cos\theta = \frac{x}{r}$.

b) Since $\sin\theta = \frac{y}{r}$ and θ is in the second quadrant, I knew y and r must be positive values. I expressed $\sin\theta$ as a decimal and let $y = \sin\theta$ and $r = 1$. Then I calculated $x = -\sqrt{r^2 - y^2}$, $\cos\theta = \frac{x}{r}$ and $\tan\theta = \frac{y}{x}$.

c) Since $\cos\theta = \frac{x}{r}$ and θ is in the third quadrant I knew x was a negative value and r a positive. I expressed $\cos\theta$ as a decimal and let $x = \cos\theta$ and $r = 1$. Then I calculated $y = -\sqrt{r^2 - x^2}$, $\sin\theta = \frac{y}{r}$, and $\tan\theta = \frac{y}{x}$.

Exploring with a Graphing Calculator, page 211

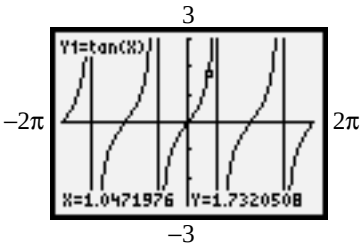
1. a)



Answers may vary. Yes, because $\tan 60^\circ$ lies in the first quadrant and is positive. The value of $\tan 60^\circ$ is $\sqrt{3}$ which is approximately 1.73.

Selected Solutions — Chapter 3

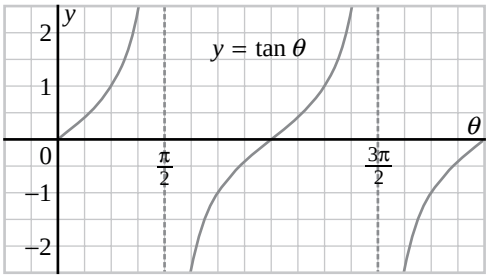
3. a)



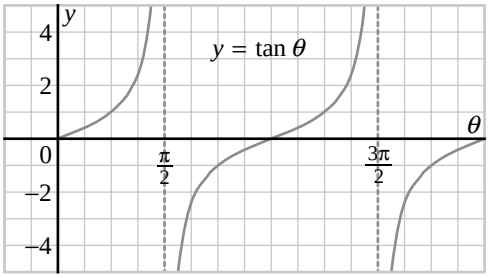
Answers may vary. Same as 1 above.

3.8 Exercises, page 213

1. a)



b)



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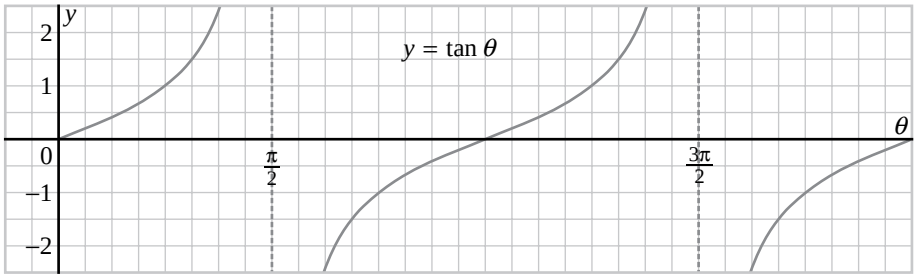
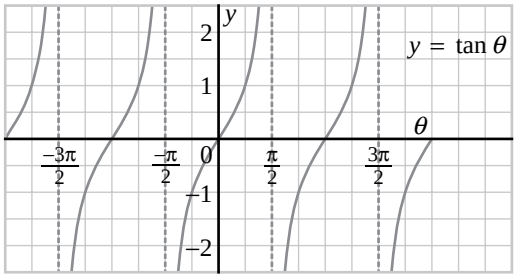


3. Explanations may vary. The period for the tangent function is 180˚. Therefore, I added 180˚ to 40˚ to get 220˚. I added 180˚ to 220˚ to get 400˚. I then subtracted 180˚ from 40˚ to get −140˚ and 180˚ from −140˚ to get −320˚.

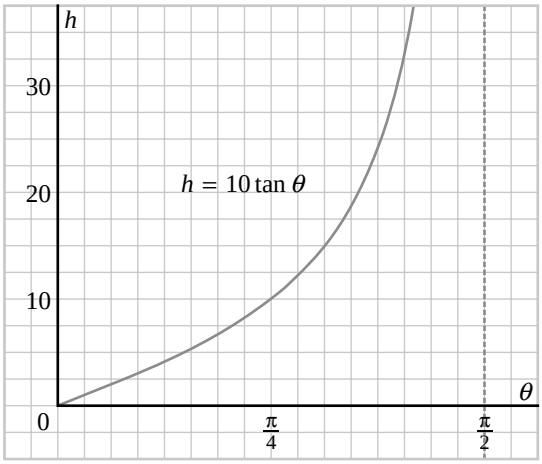
4. The relationship $\tan\theta = \frac{\sin\theta}{\cos\theta}$ is true for all θ except when $\cos\theta = 0$.

Selected Solutions — Chapter 3

5.



7. b)



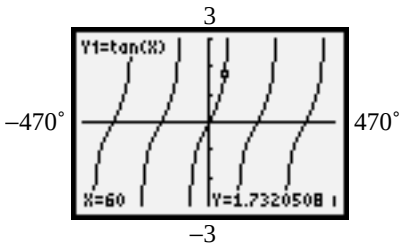
c) The value for $\tan\theta$ is undefined for $\frac{\pi}{2}$. For heights above 150 m, θ is approaching $\frac{\pi}{2}$. Thus $\tan\theta$ is approaching an infinite value, which is unrealistic for a rocket.

d) The graph is undefined for $\theta = \frac{\pi}{2}$. If $\theta = \frac{\pi}{2}$, the rocket would be directly above the observer, which is impossible.

e) To determine the maximum height of any model rocket in flight, I would use the formula $h = d\tan\theta$. I would substitute the distance from the launchpad in for d and the angle of elevation for the rocket's trajectory in for θ and solve for h.

f) When there is wind, the situation cannot be modelled with a right triangle.

8.



a) Both graphs are periodic and have the same θ-intercepts.

b) When $0 < \theta < 90^\circ$, $\tan\theta$ is greater than $\sin\theta$.

Examine the graph; $\tan\theta$ is above $\sin\theta$, in this interval.

Use the definition, $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cos\theta$ is always less than or equal to 1 and $\sin\theta$ is positive in this interval. Therefore, $\tan\theta$ is greater than $\sin\theta$ in this interval.

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Selected Solutions — Chapter 3

Problem Solving, page 215

1. For the square with vertical and horizontal sides, the x- and y-coordinates for point P are defined piecewise. By drawing right triangles, with point P (x, y) as a vertex, the formulas may be obtained.

For $0^\circ \le \theta \le 45^\circ$, $x = 1$, $y = \tan\theta$

$45^\circ \le \theta \le 135^\circ$, $x = \cot\theta$, $y = 1$

$135^\circ \le \theta \le 225^\circ$, $x = -1$, $y = -\tan\theta$

$225^\circ \le \theta \le 315^\circ$, $x = -\cot\theta$, $y = -1$

$315^\circ \le \theta \le 360^\circ$, $x = 1$, $y = \tan\theta$

For the square with vertical and horizontal diagonals, the formulas are developed.

In the 1st quadrant,

$x + y = 1$ ①

$y = x\tan\theta$ ②

Substitute ② into ①

$x + x\tan\theta = 1$

$x = \dfrac{1}{1 + \tan\theta}$

$x = \dfrac{\cos\theta}{\cos\theta + \sin\theta}$

Substitute the value of x into ②

$y = \dfrac{\sin\theta}{\cos\theta + \sin\theta}$

In the 2nd quadrant,

$-x + y = 1$ ①

$y = x\tan\theta$ ②

Substitute ② into ①

$-x + x\tan\theta = 1$

$x = \dfrac{1}{\tan\theta - 1}$

$x = \dfrac{\cos\theta}{\sin\theta - \cos\theta}$

Substitute the value of x into ②

$y = \dfrac{\sin\theta}{\sin\theta - \cos\theta}$

In the 3rd quadrant,

$x + y = -1$ ①

$y = x\tan\theta$ ②

Substitute ② into ①

$x + x\tan\theta = -1$

$x = \dfrac{-1}{1 + \tan\theta}$

$x = \dfrac{-\cos\theta}{\cos\theta + \sin\theta}$ or $x = \dfrac{\cos\theta}{-\cos\theta - \sin\theta}$

Substitute the value of x into ②

$y = \dfrac{-\sin\theta}{\cos\theta + \sin\theta}$ or $y = \dfrac{\sin\theta}{-\cos\theta - \sin\theta}$

Selected Solutions — Chapter 3

In the 4th quadrant,

$x - y = 1$ ①

$y = x \tan \theta$ ②

Substitute ② into ①

$x - x \tan \theta = 1$

$x = \frac{1}{1 - \tan \theta}$

$x = \frac{\cos \theta}{\cos \theta - \sin \theta}$

Substitute the value of x into ②

$y = \frac{\sin \theta}{\cos \theta - \sin \theta}$

All 4 functions for x and all 4 functions for y may be modified to get 1 function for x and 1 function for y.

In the 2nd quadrant, $\cos \theta < 0$; therefore, $-\cos \theta = |\cos \theta|$

In the 3rd quadrant, $\cos \theta < 0$ and $\sin \theta < 0$; therefore, $-\cos \theta = |\cos \theta|$ and $-\sin \theta = |\sin \theta|$

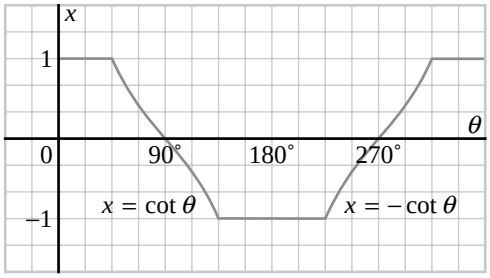
In the 4th quadrant, $\sin \theta < 0$; therefore, $-\sin \theta = |\sin \theta|$

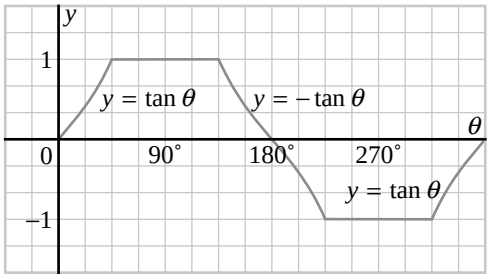
The point P may be defined

$x = \frac{\cos \theta}{|\cos \theta| + |\sin \theta|}$

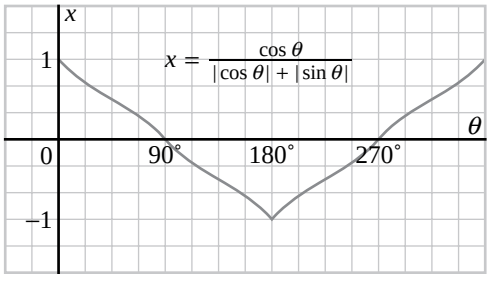
$y = \frac{\sin \theta}{|\cos \theta| + |\sin \theta|}$

2. For the square with vertical and horizontal sides:

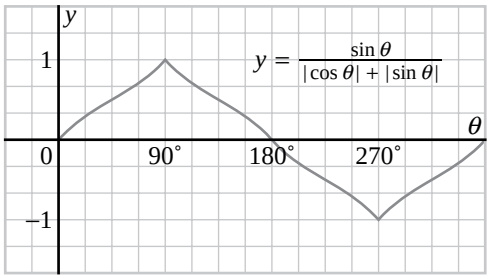




For the square with vertical and horizontal diagonals:



Selected Solutions — Chapter 3



3. For the square with vertical and horizontal sides:

The graph for the x-coordinate is periodic with period 360°. It has the same θ-intercepts and the same maximum and minimum values as the cosine graph. Thus it has the appearance of a "flattened" cosine graph.

The graph for the y-coordinate is periodic with period 360°. It has the same θ-intercepts and the same maximum and minimum values as the sine graph. Thus, it has the appearance of a "flattened" sine graph.

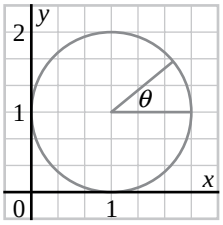
For the square with vertical and horizontal diagonals:

The graph for the x-coordinate is periodic with period 360°. It has the same θ-intercepts and the same maximum and minimum values as the cosine graph. It has the appearance of the cosine graph with different curvature.

The graph for the y-coordinate is periodic with period 360° and has the same θ-intercepts and the same maximum and minimum values as the sine graph. It has the appearance of a sine graph with different curvature.

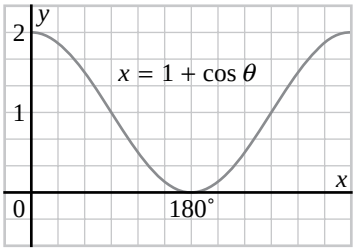
4. Solutions will vary.

This solution uses a unit circle with centre (1, 1).

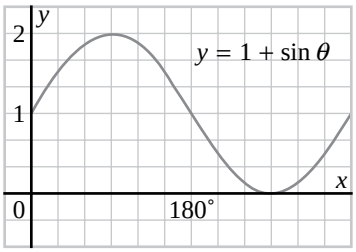


The most straightforward solution is obtained by changing the position of the angle. The angle θ has initial arm on $y = 1$ and the terminal arm is rotated about the centre of the circle (1, 1).

The point P has coordinates $x = 1 + \cos\theta$ and $y = 1 + \sin\theta$.



Selected Solutions — Chapter 3



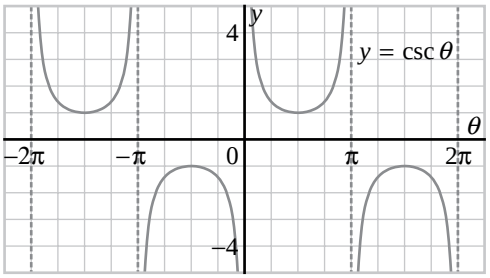
The graph of the x-coordinate is a vertical translation of 1 of the cosine graph. The graph of the y-coordinate is a vertical translation of 1 of the sine graph.

3.9 Exercises, page 218

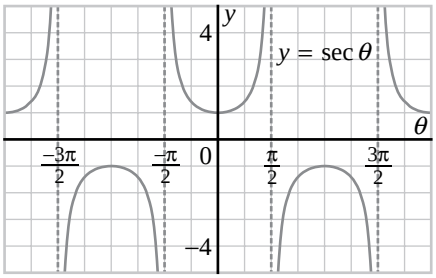
1. Answers may vary. Both $y = \csc\theta$ and $y = \sin\theta$ have periods of 2π. The intervals that $y = \csc\theta$ is positive or negative are the same as the intervals that $y = \sin\theta$ is positive or negative.

The graph of $y = \csc\theta$ has asymptotes while $y = \sin\theta$ does not have asymptotes. The graph of $y = \csc\theta$ does not have a maximum or minimum value while $y = \sin\theta$ has a maximum value of 1 and a minimum value of -1.

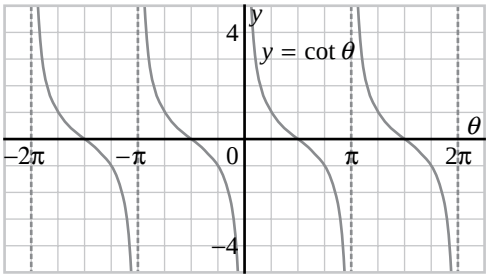
7. a)



b)



c)



Selected Solutions — Chapter 3

3 Review

2. a) The students had their calculators in radian mode instead of degree mode when they calculated the sine of the angles. Thus, the numbers are incorrect.

b)

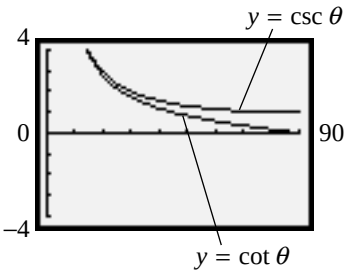
Angle of incidence, i (degrees)	Angle of refraction, r (degrees)	$\sin i$	$\sin r$	$\frac{\sin i}{\sin r}$
10	7	0.174	0.122	1.425
20	13	0.342	0.225	1.520
30	19	0.500	0.326	1.536
40	25	0.643	0.423	1.521
50	30	0.766	0.500	1.532
60	35	0.866	0.574	1.510
70	38	0.940	0.616	1.526
80	40	0.985	0.643	1.532

4. a) Yes. The circle the front wheel travels must always be larger than the circle the rear wheel travels. Otherwise, the wheels would hit each other.

b) The front wheel can turn 90° to the right. Beyond this point many bike wheels are unable to rotate.

d) Label the left end of the front wheel A.
$\angle BFA = \theta$ by opposite angles.
$\angle OFA = 90^\circ$ because the wheel is a tangent to the circle.
Thus, $\angle BFO = 90^\circ - \theta$.
$\angle OBF = 90^\circ$ because the rear wheel is tangent to the inner circle.
Thus, $\angle BOF = 90^\circ - (90^\circ - \theta)$, or θ.

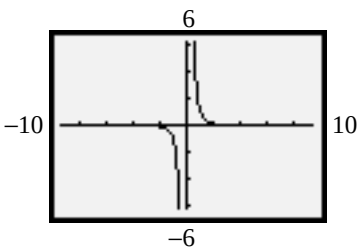
f)



6. b) The graphs are the same since the tangent of an angle in radians is the same as the tangent of the corresponding angle in degrees.

3 Cumulative Review

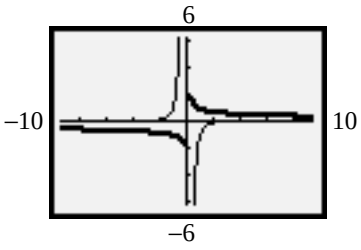
1. a)



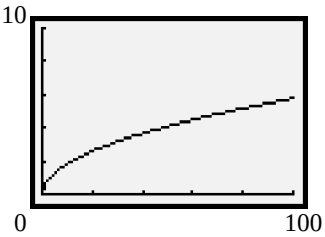
Selected Solutions — Chapter 3

b) The function is not defined for $x = 0$, so the y-axis is an asymptote. For large positive and negative values of x, the function approaches 0, so the x-axis is an asymptote. The function is negative for negative x, and positive for positive x.

c)



3. a)



5. b) $(\log_b a)(\log_a b) = 1$

$$\text{Left side} = (\log_b a)(\log_a b)$$
$$= \frac{\log a}{\log b} \times \frac{\log b}{\log a}$$
$$= 1$$
$$= \text{Right side}$$

6. $\log_a x$ is the exponent to which a must be raised to get x.
$a \log_a x$ is a raised to the exponent to which a must be raised to get x.
Thus, $a^{\log_a x} = x$.

7. a) $\log(10x) - \log x = \log\left(\frac{10x}{x}\right)$

$\log(10x) - \log x = \log 10$

$\log(10x) - \log x = 1$

b) $\log 6 + \log 10^x = \log 6 + x \log 10$

$\log 6 + \log 10^x = \log 6 + x$

9. Explanations may vary. For part c:

I divided each side by 100 to get $(0.95)^n = 0.5$.
Then, I took the logarithm of each side to get $n \log 0.95 = \log 0.5$.
Finally, I divided each side by $\log 0.95$ to get $n = \frac{\log 0.5}{\log 0.95}$.

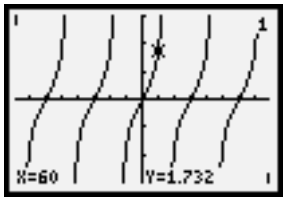
Selected Solutions — Chapter 3

12. c) All terms in the arithmetic sequence can be obtained by adding some multiple of 9 to the first term, 3. Every multiple of 9 gives a term of this sequence. In the geometric sequence, notice that the difference between the second term and the first term is 9, and the difference between the third term and the second term is 36. Both are multiples of 9. In fact, the difference between any 2 consecutive terms is a multiple of 9. We can see this as follows:

$$\begin{aligned} t_{n+1} - t_n &= 3 \times 4^n - 3 \times 4^{n-1} \\ &= (12 - 3)4^{n-1} \\ &= 9 \times 4^{n-1} \end{aligned}$$

Since the difference between any two consecutive terms is a multiple of 9, the difference between any term and the first term is a multiple of 9. Therefore, by the definition of the arithmetic sequence above, every term of the geometric sequence is a term in the arithmetic sequence.

14. a)



15. The tangent and cotangent graphs are reflections of one another following a 90° horizontal translation. Thus, the curves are very similar. Both curves approach vertical asymptotes. The graphs of $y = \tan x$ and $y = \cot x$ are different because the equations of the asymptotes are different.