Selected Solutions — Chapter 2

2.1 Exercises, page 69

1. **c)** **i)** The 1000 in the equation changes to the new value.

For an investment less than $1000, the graph is a vertical compression of the original graph.

For an investment greater than $1000, the graph is a vertical expansion of the original graph.

ii) The 1.08 in the equation changes to 1 plus the new value.

For an interest rate less than 8%, the graph increases more slowly than the original graph.

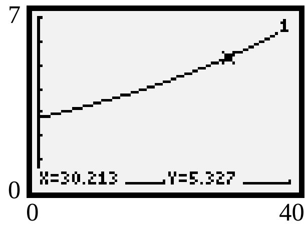
For an interest rate greater than 8%, the graph increases more rapidly than the original graph.

2. **b)** **i)** The 1.022 in the equation changes to the new value. The graph reaches high values more quickly.

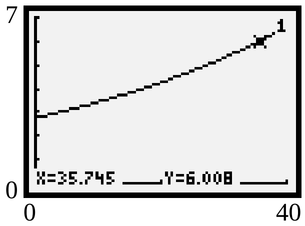
ii) The 1.022 in the equation changes to the new value. The graph reaches high values more slowly.

3. **a)** Refer to text page 66.

b)



c)



Modelling British Columbia's Population Growth, page 69

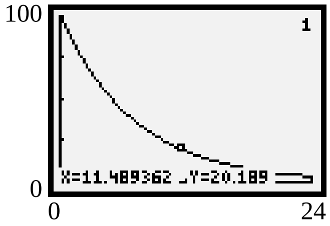
- The growth rate is constant.
- The actual growth rate is not constant. If the economy worsens or improves, the growth rate may decrease or increase, respectively. An epidemic of disease could cause a lower growth rate as well.
- The growth rate over several years was averaged.

4. **b)** The equation changes to $P = 100(0.9)^n$. The amount of light decreases more rapidly, so the graph decreases more rapidly.

5. The number in brackets in the equation will be greater since the substance will linger in the body for a longer period of time. The graph will take longer to reach lower values.

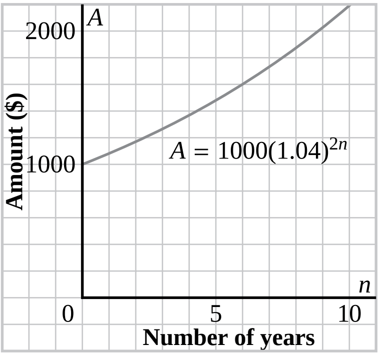
6. **a)** Refer to text page 66.

b)



Selected Solutions — Chapter 2

7. **a)**

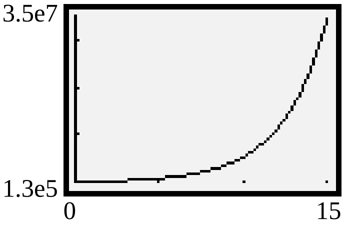


d) It is almost the same, but increases a tiny bit more quickly.

8. **a)** In 5 years, the first investment reaches \$1469.33, and the second investment reaches \$1480.24. The first investment reaches \$2500 after 11.9 years, and the second investment reaches \$2500 after 11.7 years. The answers are very close.

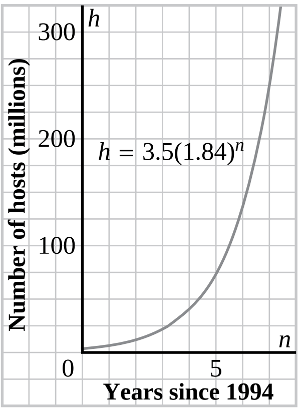
b) Both equations are of the form $A = 1000(1 + \frac{0.08}{x})^{xn}$, where x is the number of compounding periods per year and n is the number of years. In the first case, $x = 1$, in the second case, $x = 2$.

9. **a)**

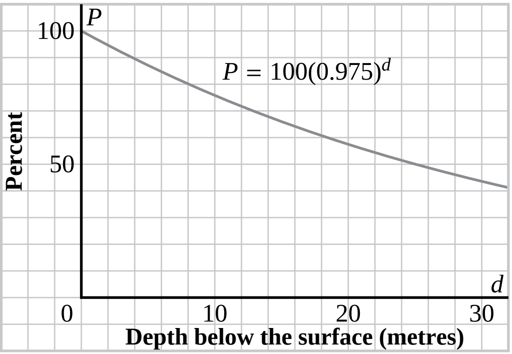


b) Assume the number of cellular phone subscribers continues to increase at the same rate.

10. **b)**



11. **b)**



c) The graph has a similar shape, but decreases more slowly.

12. **a)** The growth rate is 102%, so the equation of the graph is $A = 13.1(2.02)^n$. From 1987 to 2002 is 5 years, so

$$A = 13.1(2.02)^5$$
$$\doteq 440.6$$

This is very close to the amount given above the circle graph.

b) In 1997, Canada's market was worth 5.4% of \$13.1 billion, or \$707.4 million. In 2002, Canada's market will be worth 2.2% of \$435.1 billion, or \$9.5722 billion.

Thus,

$$9.5722 = 0.7074(1 + i)^5$$
$$i = \left(\frac{9.5722}{0.7074}\right)^{\frac{1}{5}} - 1$$
$$\doteq 0.684$$

Canada's growth rate is about 68.4%.

c) In 1997, Asia's market was worth 1.1% of \$13.1 billion, or \$144.1 million. In 2002, Asia's market will be worth 7.9% of \$435.1 billion, or \$34.3729 billion.

Thus,

$$34.3729 = 0.1441(1 + i)^5$$
$$i = \left(\frac{34.3729,}{0.1441}\right)^{\frac{1}{5}} - 1$$
$$\doteq 1.99$$

Asia's growth rate is about 199%.

Exploring With a Graphing Calculator, page 72

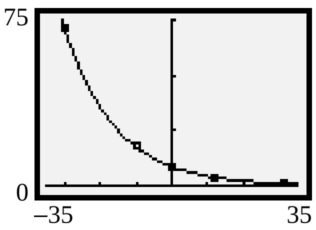
1. **b)** From the TI-83 ExpReg screen on p. 72 of the text, $y = 34.74(1.1266)^x$.

c) According to this model, the national debt increases by a factor of 1.1266 every year. This corresponds to a growth rate of 12.66% annually.

d) The model predicts that when $x = 1995 - 1970$, or $x = 25$, $y = 34.74(1.1266)^{25}$, or $y \doteq 684$ billion. Possible reasons why the national debt was less than this value in 1995 are increased taxes and reduced government spending.

Selected Solutions — Chapter 2

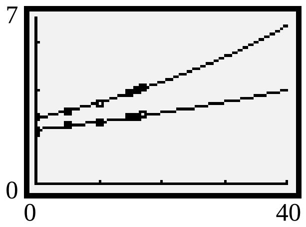
2. **a)**



b) According to this model, when $T = 20°C$, the time for the reaction is $t = 8.72(0.93)^{20}$, or approximately 2.0 s.

c) When $T = 30°C$, the time for the reaction is $t = 8.72(0.93)^{30}$, or approximately 1.0 s. Therefore, when the temperature increase by 10°C, the time is halved.

3. **a)**



b) The population of Alberta, according to the above model, is $y = 2.278(1.014)^{2011-1981}$, or approximately 3.457 million. The population of British Columbia, according to the equation above, is $y = 2.758(1.022)^{2011-1981}$, or approximately 5.298 million.

c) Alberta:

$$6 = 2.278(1.014)^x$$
$$1.014^x = \frac{6}{2.278}$$
$$x = \frac{\log 6 - \log 2.278}{\log 1.014}$$

Therefore the population of Alberta might reach 6 million in 1981 + 69.7, or in 2050.

British Columbia:

$$6 = 2.758(1.022)^x$$
$$1.022^x = \frac{6}{2.758}$$
$$x = \frac{\log 6 - \log 2.758}{\log 1.022}$$

Therefore the population of British Columbia might reach 6 million in 1981 + 35.7, or in 2016.

Modelling Population Growth, page 73

- Yes. These populations are based on real data.
- The economy, diseases.
- The graph of the province that is growing more quickly is steeper.
- Alberta: 1.38%, British Columbia: 2.23%

Investigate, page 74

1. **a)** 0.477, 1.813, 2.978, 3.679

b) 1, 2, 3, 4,... 0, –1, –2, –3,...

c) 1.301, 2.301, 3.301,... 1.477, 2.477, 3.477,...

d) 0.386, 1.386, 2.386, 3.386,... –0.614, –1.614, –2.614,...

e) 0.5, 1.0, 1.5,... 0, –0.5, –1.0,...

f) Do not exist.

2.2 Exercises, page 77

1. **a)** $10^{\log 23.6} = 10^{1.372\,91}$
$= 23.6$

b) $10^{\log 180} = 10^{2.255\,27}$
$= 180$

c) $10^{\log 5100} = 10^{3.707\,57}$
$= 5100$

d) $10^{\log 0.45} = 10^{-0.346\,79}$
$= 0.45$

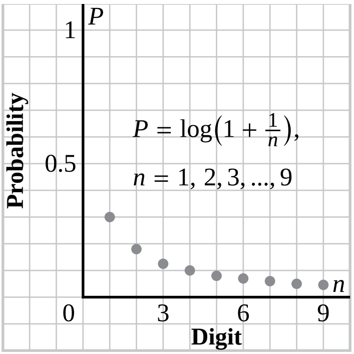
e) $10^{\log 0.072} = 10^{-1.142\,67}$
$= 0.072$

f) $10^{\log 0.0029} = 10^{-2.537\,60}$
$= 0.0029$

2. The logarithm of a number greater than 1 is positive, while the logarithm of a number greater than 0 and less than 1 is negative.

5. Explanations may vary.
For part c:
$\text{Log}_3 27$ is the exponent to which 3 must be raised to get 27.
I know that $3^3 = 27$, so $\log_3 27 = 3$.
For part g:
$\text{Log}_3 1$ is the exponent to which 3 must be raised to get 1.
I know that $3^0 = 1$, so $\log_3 1 = 0$.

9. **b)**



2.3 Exercises, page 83

1. **a)** $\log (5 \times 10^n) = \log 5 + \log 10^n$
$= \log 5 + n$

b) $\log 5^n = n \log 5$ by the laws of logarithms.

Selected Solutions — Chapter 2

2. **a)** In each case, $\log a + \log b = \log ab$, where $ab = 36$.

b) In each case, $\log a - \log b = \log \frac{a}{b}$, where $\frac{a}{b} = 2$.

3. Answers may vary
$\log 1 + \log 24$
$\log 2 + \log 12$
$\log 4 + \log 6$
$\log 8 + \log 3$

4. The results in parts a and b are the same, since

$$\begin{aligned} 2 \log 2 &= \log 2^2 \\ &= \log 4 \\ 3 \log 2 &= \log 2^3 \\ &= \log 8 \end{aligned}$$

and so on.

5. Answers may vary
L_1: log 3, log 9, log 27, log 81, log 243
L_2: log 3, 2 log 3, 3 log 3, 4 log 3, 5 log 3

14. **c)** Let $y = \log_3 20$.
Then $3y = 20$.
Take the logarithm to any base x of each side.

$$\begin{aligned} \log_x 3^y &= \log_x 20 \\ y \log_x 3 &= \log_x 20 \\ y &= \frac{\log_x 20}{\log_x 3} \end{aligned}$$

Thus, for any x, $\frac{\log_x 20}{\log_x 3} = \log_3 20$.

15. Let $s = \log_a x$ and $t = \log_a y$.

$$\begin{aligned} x &= a^s \\ y &= a^t \\ \frac{x}{y} &= \frac{a^s}{a^t} \\ &= a^s - t \end{aligned}$$

Now substitute to find $\log_a \frac{x}{y} = \log_a a^{s-t}$

$$\begin{aligned} &= s - t \\ &= \log_a x - \log_a y \end{aligned}$$

16. Let $s = \log_a x$.
$x = a^s$
Raise each side to the exponent $\frac{1}{n}$.

$$\begin{aligned} x^{\frac{1}{n}} &= (a^s)^{\frac{1}{n}} \\ x^{\frac{1}{n}} &= a^{\frac{s}{n}} \end{aligned}$$

Take the base-a logarithm of each side.

$$\begin{aligned} \log_a x^{\frac{1}{n}} &= \frac{s}{n} \\ \log_a x^{\frac{1}{n}} &= \frac{\log_a x}{n} \\ \log_a \sqrt[n]{x} &= \frac{1}{n} \log_a x \end{aligned}$$

20. **a)** $\log (33 \times 2^{259}) = \log 33 + 259 \log 2$
$\doteq 79.485\ 28$

b) 80; Since the base-10 logarithm is between 79 and 80, the number must be between 10^{79} and 10^{80}, giving it 80 digits.

Selected Solutions — Chapter 2

c) Divide the number by 10^{79} and take the logarithm

$$\begin{aligned}\log(33 \times 2^{259} \div 10^{79}) &= \log 33 + 259 \log 2 - 79\\ &\doteq 0.485\ 282\ 82\end{aligned}$$

Raising 10 to this exponent will give the first few digits.

$10^{0.485\ 282\ 82} \doteq 3.0569\ldots$

21. **a)** Assume such a bacterium exists and doubles every x seconds. There are 86 400 s in a day, so there are $\frac{86400}{x}$ doubling every day. Let d represent the number of doubling. The sum of the descendants is

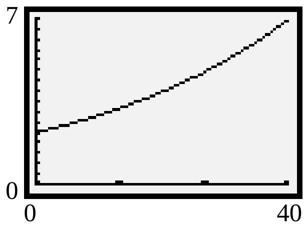
$$\begin{aligned}2 + 4 + 8 + 16 + \ldots + 2^d &= \frac{2(2^d - 1)}{2 - 1}\\ 76 \times 10^9 \times 10^{12} &= 2(2^d - 1)\\ 38 \times 10^{21} &= 2^d - 1\\ 2^d &= 38 \times 10^{21} + 1\\ d \log 2 &= \log(38 \times 10^{21} + 1)\\ d &= \frac{\log(38 \times 10^{21} + 1)}{\log 2}\\ &\doteq 75\end{aligned}$$

Thus, the bacterium doubles 75 times per day, or about every 19 min.

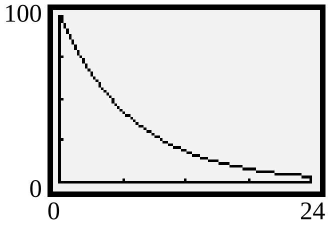
b) Answers may vary. In part a the assumptions are that there is a bacterium which doubles ever 19 minutes, that the conditions throughout the day remain constant, and that the bacterium does not die. It is not reasonable to assume that conditions remain constant and control against death.

Investigate, page 86

1. **a)**



4. **a)**



Modelling Canada's Population Growth, page 89

- The population growth is constant.
- The doubling time would decrease. The economy was strong.

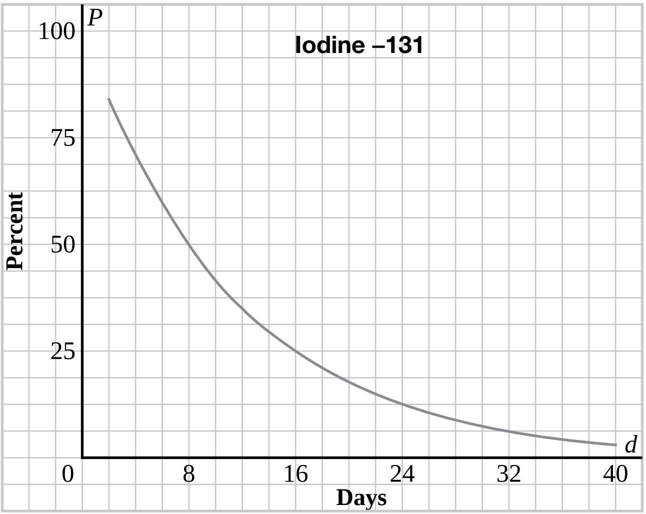
Selected Solutions — Chapter 2

2.4 Exercises, page 92

2. **b)** Answers may vary; improved medical care for infants, education and health care for pregnant women

7. In Exercise 5, the interest rate stays the same and the number of compounding periods increases, so the number of years for the investment to double decreases. In Exercise 6, the interest rate decreases, but the number of compounding periods increases to compensate, so the doubling time stays the same.

11. **a)** Graphs may vary. For iodine-131:



12. **a)** $1.0124 = 2^x$

$$\begin{aligned} x &= \frac{\log 1.0124}{\log 2} \\ &= 0.0178 \end{aligned}$$

$$\begin{aligned} P &\doteq 29.6 \times (2^{0.0178})^n \\ &= 29.6 \times 2^{0.0178n} \end{aligned}$$

$$\begin{aligned} P &= 29.6 \times 2^{\frac{n}{56}} \\ &\doteq 29.6 \times 2^{0.0179n} \end{aligned}$$

The two equations are almost the same, so the first equation can be converted to the second equation.

b) $2 = 1.0124^x$

$$\begin{aligned} x &= \frac{\log 2}{\log 1.0124} \\ &= 56.24 \end{aligned}$$

$$\begin{aligned} P &\doteq 29.6 \times (1.0124^{56.24})^{\frac{n}{56}} \\ &\doteq 29.6 \times 1.0124^n \end{aligned}$$

The two equations are almost the same, so the second equation can be converted to the first equation.

c) The two equations are mathematically equivalent, except for rounding.

13. **a)** The number of people the arable land can support is $\frac{3.2 \times 10^9}{0.4}$, or 8×10^9. The growth equation is $P = 6(1.015)^n$, where P is in billions.

Solve the equation

$$8 = 6(1.015)^n$$
$$n = \frac{\log 4 - \log 3}{\log 1.015}$$
$$\doteq 19$$

The demand will exceed the supply in 2017.

b) i) $8 = 6(1.0075)^n$

$$n = \frac{\log 4 - \log 3}{\log 1.0075}$$
$$\doteq 38.5$$

The demand will exceed the supply in 2036.

ii) The number of people the arable land can support is $\frac{3.2 \times 10^9}{0.2}$, or 1.6×10^{10}.

$$16 = 6(1.015)^n$$
$$n = \frac{\log 8 - \log 3}{\log 1.015}$$
$$\doteq 66$$

The demand will exceed the supply in 2064.

iii) $16 = 6(1.0075)^n$

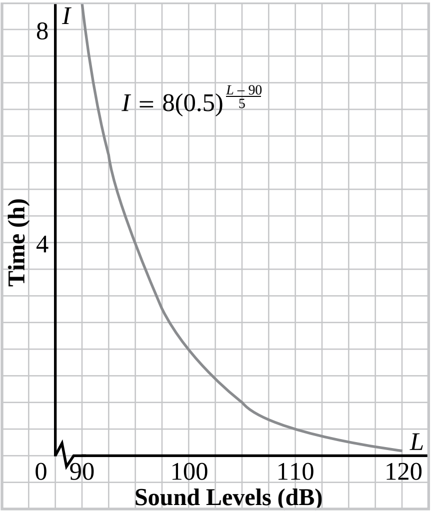
$$n = \frac{\log 8 - \log 3}{\log 1.0075}$$
$$\doteq 131$$

The demand will exceed the supply in 2129.

2.5 Exercises, page 99

8. Explanations may vary. For part i:
I used the equation $I = I_0 \times 10^R$.
$R = 7.3 - 4.6$, or 2.7.
Thus, $I = I_0 \times 10^{2.7}$, or $I \doteq I_0 \times 501$.
So, the 1946 earthquake was about 500 times more intense than the 1997 earthquake.

17. c)



19. b) The snow melt might release more pollution into the stream.

21. c) If the alkalinity of the swimming pool water is much more or less than that of the eye, it would damage the eye.

Selected Solutions — Chapter 2

Mathematical Modelling, page 102

1. a) From 1960 to 1970 is 10 years. Assuming there was 1 component on a chip in 1960, there were 2^{10} or 1024 components on a chip in 1970. So the information in the quotation has been rounded.

b) The equation which represents an annual doubling in the number of components is $C = 2^n$, where $0 < n < 10$ and represents the number of years since 1960.

2. a) From 1970 to 1992 is 22 years, or $\frac{22}{1.5}$ periods of 1.5 years. There were $1000 \times 2^{\frac{22}{1.5}}$ or approximately 26 007 978 components on a chip in 1992.

b) i) Following the example of part a, $C = 1000 \times 2^{\frac{n}{1.5}}$, where $n < 22$.

ii)

$$\begin{aligned} 1\,000\,000\,000 &= 1000 \times 2^{\frac{n}{1.5}} \\ \frac{1\,000\,000\,000}{1000} &= 2^{\frac{n}{1.5}} \\ n &= 1.5 \times \frac{\log 1\,000\,000}{\log 2} \\ n &\doteq 29.9 \end{aligned}$$

Therefore, the statement will be true if 1992 is changed to 1970 + 29.9, or late 1999.

3. $C = 1000(2^{\frac{n}{1.5}})$

1998 is 28 years after 1970.

Thus,

$$\begin{aligned} C &= 1000(2^{\frac{28}{1.5}}) \\ &\doteq 416\,127\,661 \end{aligned}$$

Thus, the model does not work for 1998.

4. a) Let n be the number of years since 1972. When $n = 0$, $I = 60\,000$. Every successive year, I doubles. Thus, the equation $I = 60\,000\left(2^{\frac{n}{1.5}}\right)$ models the situation.

b) For 1990, $n = 18$.

$$\begin{aligned} I &= 60\,000 \times 2^{\frac{18}{1.5}} \\ &\doteq 246 \text{ million} \end{aligned}$$

Round this down to 200 million and the statement is consistent.

5. Assume exponential growth. Let C represent the cheapness of computer power. Let n represent the number of years, and r the growth power.

$$\begin{aligned} C &= r^{\frac{n}{10}} \\ 8000 &= r^{\frac{30}{10}} \\ 8000 &= r^3 \\ r &= 20 \\ C &= 20^{\frac{n}{10}} \end{aligned}$$

a) i) $C_{10} = 20$

$C_{30} = 8000$

Computer power is 400 times cheaper now than 20 years ago.

ii) $C_{20} = 20^2$
$= 400$

Computer power is 20 times cheaper now than 10 years ago.

b) i) The cost in 10 years will be 20 times cheaper than now.

ii) The cost in 20 years will be 20 × 20 or 400 times cheaper than now.

6. If the car is 8000 times less expensive than it was in 1971, then it would cost $\frac{\$7000}{8000}$, or about $0.875 in 2001. This is less than $2.

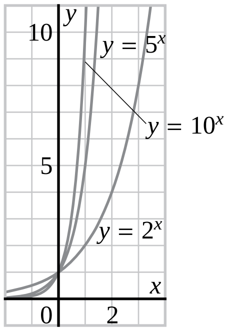
If the car doubles its performance power every 18 months and speed is a measure of performance, then the speed in 2001 would be $(150 \times 2^{\frac{30}{1.5}})$ km/h, or about 157 286 400 km/h. The speed of sound is 1192 km/h, so these two numbers do not compare.

If the car doubles its performance power every 18 months, and fuel consumption is a measure of performance, then the car would travel $(8 \times 2^{\frac{30}{1.5}})$ km on 1 L of gas, or about 8 388 608 km on 1 L. Suppose there is 1 mL of gas in a thimble. Then the car would travel $\frac{8\,388\,608}{1000}$ km, or 8389 km on one thimble of gas. This is much greater than 1000 km sited in the textbook.

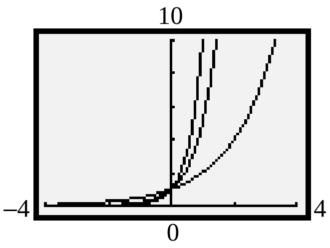
7. Answers may vary; there is no need to have the same progress; automobiles have existed for a longer time

2.6 Exercises, page 109

2. a), b)

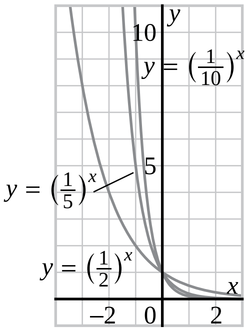


c)

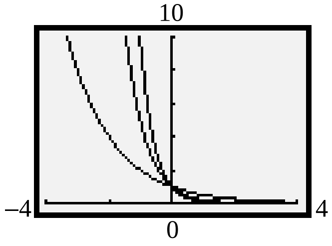


Selected Solutions — Chapter 2

3. a)



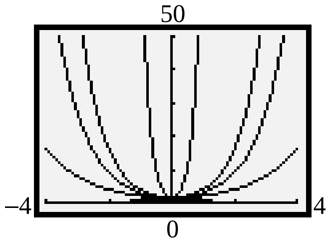
b)



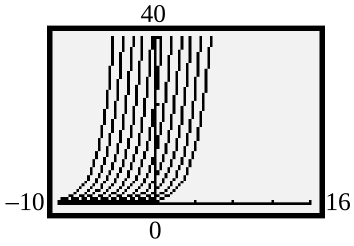
5. b) i) For $a > 1$, as a increases, the graph of $y = a^x$ gets steeper.

ii) The graph of $y = \left(\frac{1}{a}\right)^x$ is a reflection in the y-axis of the graph of $y = a^x$.

c)

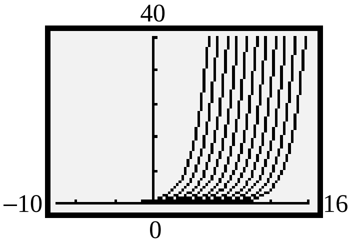


6. a) The graph of $y = 1 \times 2^x$ is the graph on page 104.
The graph of $y = 2 \times 2^x$ is a translation of 1 unit left of the graph of $y = 2^x$.
The graph of $y = 4 \times 2^x$ is a translation of 2 units left of the graph of $y = 2^x$.
The graph of $y = 8 \times 2^x$ is a translation of 3 units left of the graph of $y = 2^x$.
The graph of $y = 2^{10} \times 2^x$ is a translation of 10 units left of the graph of $y = 2^x$.

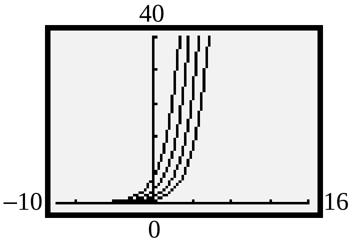


b) The graph of $y = 1 \times 2^x$ is the graph on page 104.
The graph of $y = \frac{1}{2} \times 2^x$ is a translation of 1 unit right of the graph of $y = 2^x$.

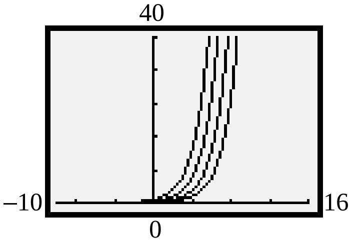
The graph of $y = \frac{1}{4} \times 2^x$ is a translation of 2 units right of the graph of $y = 2^x$.
The graph of $y = \frac{1}{8} \times 2^x$ is a translation of 3 units right of the graph of $y = 2^x$.
The graph of $y = \frac{1}{2^{10}} \times 2^x$ is a translation of 10 units right of the graph of $y = 2^x$.



7. **a)** The graph of $y = 2^x$ is the graph on page 104.
The graph of $y = 2^{x+1}$ is a translation of 1 unit left of the graph of $y = 2^x$.
The graph of $y = 2^{x+2}$ is a translation of 2 units left of the graph of $y = 2^x$.
The graph of $y = 2^{x+3}$ is a translation of 3 units left of the graph of $y = 2^x$.



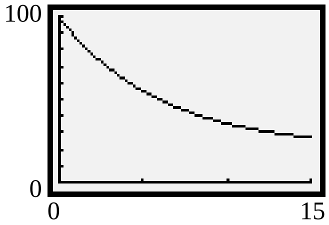
b) The graph of $y = 2^x$ is the graph on page 104.
The graph of $y = 2^{x-1}$ is a translation of 1 unit right of the graph of $y = 2^x$.
The graph of $y = 2^{x-2}$ is a translation of 2 units right of the graph of $y = 2^x$.
The graph of $y = 2^{x-3}$ is a translation of 3 units right of the graph of $y = 2^x$.



8. For part a:
$y = 1 \times 2^x$ in exercise 6 is the same as $y = 2^x$ in exercise 7.
$y = 2 \times 2^x$ in exercise 6 is the same as $y = 2^{x+1}$ in exercise 7.
$y = 4 \times 2^x$ in exercise 6 is the same as $y = 2^{x+2}$ in exercise 7.
$y = 8 \times 2^x$ in exercise 6 is the same as $y = 2^{x+3}$ in exercise 7.

For part b:
$y = 1 \times 2^x$ in exercise 6 is the same as $y = 2^x$ in exercise 7.
$y = \frac{1}{2} \times 2^x$ in exercise 6 is the same as $y = 2^{x-1}$ in exercise 7.
$y = \frac{1}{4} \times 2^x$ in exercise 6 is the same as $y = 2^{x-2}$ in exercise 7.
$y = \frac{1}{8} \times 2^x$ in exercise 6 is the same as $y = 2^{x-3}$ in exercise 7.
The observations for parts a and b confirm the algebraic property of exponents which states $y = 2^n \times 2^x$ is equivalent to $y = 2^{x+n}$.

9. a)



Modelling Cooling and Warming, page 111

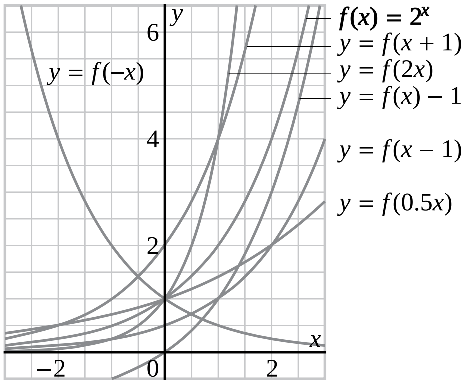
- As t increases, the temperature becomes closer and closer to 20˚C, or room temperature.
- 20˚C is room temperature. 79 is the difference between room temperature and the temperature of the hot chocolate. 0.85 is the rate of cooling per minute.
- The constant which represents the difference between room temperature and the temperature of the cold substance would have to be negative so that the graph would approach room temperature from below. For example, it might be $T = -79 \times 0.85^t + 20$.

10. b)

$$\begin{aligned} y &= 2^x \\ &\doteq (10^{0.301})^x \\ &= 10^{0.301x} \end{aligned}$$

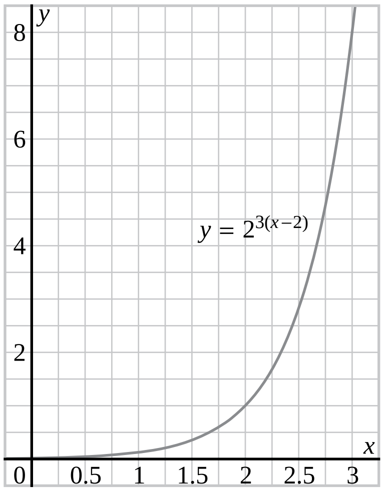
Thus, $k \doteq 0.301$.

14. a), b)

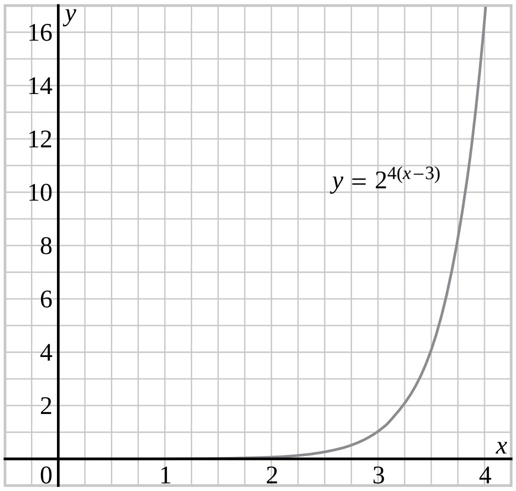


Selected Solutions — Chapter 2

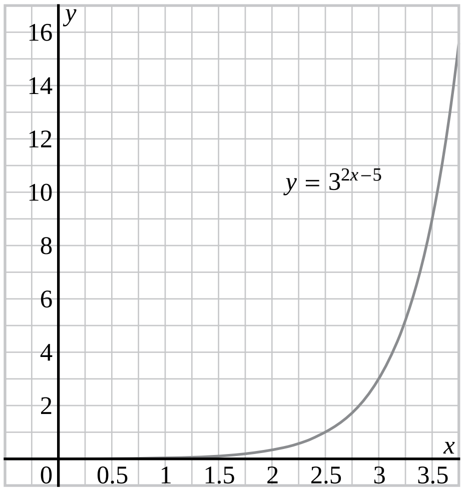
15. a)



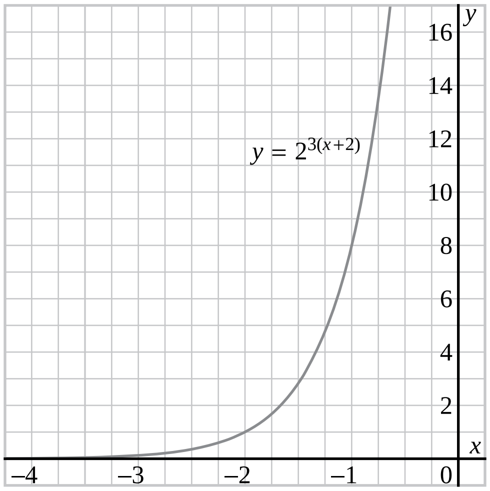
16. a)



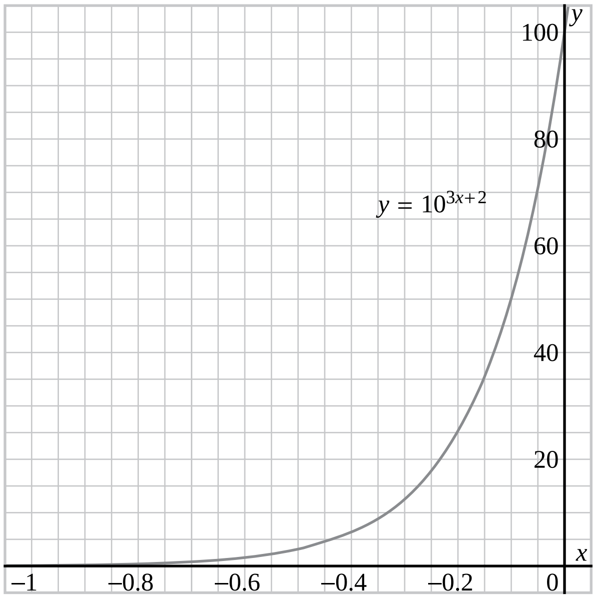
b)



c)



d)



18. **a)** Right side $= 2^{x_1+x_2}$

$= (2^{x_1})(2^{x_2})$

Left side $= y_1y_2$

$= (2^{x_1})(2^{x_2})$

Therefore Left side = Right side

b) Right side $= 2^{x_1-x_2}$

$= \frac{2^{x_1}}{2^{x_2}}$

Left side $= \frac{y_1}{y_2}$

$= \frac{2^{x_1}}{2^{x_2}}$

Therefore Left side = Right side

c) Right side $= 2^{2x_1}$

$= (2^{x_1})^2$

Left side $= y_1^2$

$= (2^{x_1})^2$

Therefore Left side = Right side

d) Right side $= 2^{-x_1}$

$= \frac{1}{2^{x_1}}$

Left side $= \frac{1}{y_1}$

$= \frac{1}{2^{x_1}}$

Therefore Left side = Right side

19. Use the results of Modelling Cooling and Warming on page 111. The difference between body temperature and room temperature when the police first found the body is 27°C – 20°C or 7°C. Let r represent the rate of cooling per hour. Hence, an equation modelling the situation has the form $T = 7 \times r^t + 20$. To determine r, we use the fact that the temperature was 25°C after 1 h. Substitute 1 for t and 25 for T to obtain

$$\begin{aligned} 25 &= 7 \times r^1 + 20 \\ r &= \frac{5}{7} \\ r &\doteq 0.7143 \end{aligned}$$

Hence, an equation modelling the situation is $T = 7 \times 0.7143^t + 20$, where t is in hours. To determine the probable time of death substitute 37 for T to obtain $37 = 7 \times 0.7143^t + 20$. Solve for t:

$$\begin{aligned} 0.7143^t &\doteq 2.4286 \\ t &= \frac{\log 2.4286}{\log 0.7143} \\ t &\doteq -2.6373 \end{aligned}$$

The probable time of death was 2.64 h before 10 P.M., or about 7:22 P.M.

20. **a)** Use the results of Modelling Cooling and Warming on page 111. The difference between the original temperature and the temperature of the freezer is 95°C – (–10°C) or 105°C. Let r represent the rate of cooling per hour. Hence, an equation modelling the situation is $T = 105 \times r^t - 10$. To determine r, we use the fact that it took 90 min for the water to freeze. Substitute 90 for t and 0 for T to obtain

$$\begin{aligned} 0 &= 105 \times r^{90} - 10 \\ r^{90} &= \frac{10}{105} \\ r &\doteq 0.9742 \end{aligned}$$

Hence, an equation modelling the situation is $T = 105 \times 0.9742^t - 10$, where t is in minutes. To determine how long it would take water at 20°C to freeze, substitute 20 for T and solve for t:

$$\begin{aligned} 20 &= 105 \times 0.9742^t - 10 \\ 0.9742^t &= \frac{30}{105} \\ t &\doteq \frac{\log 0.2857}{\log 0.9742} \\ t &\doteq 47.9295 \end{aligned}$$

This means that it would take about 48 min for water at 95°C to drop to 20°C. Since we know that it takes 90 min for water at 95°C to freeze, it must take 42 min for water at 20°C to freeze.

b) No. The tray with the 20°C water froze about 3.5 times faster than the tray with the 90°C water.

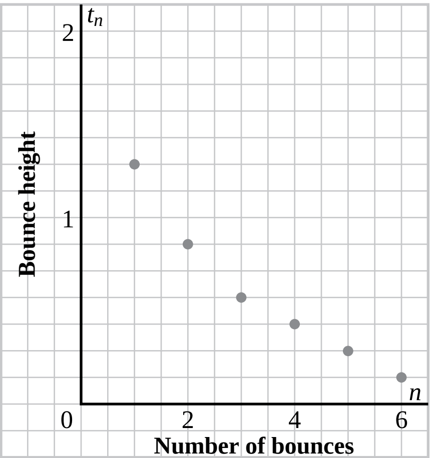
Selected Solutions — Chapter 2

21. Assume the rate of cooling of coffee is the same as that of hot chocolate. The temperature of coffee as it cools is $T = 79 \times 0.85^t + 20$, as explained in exercise 9, page 110. If the coffee is allowed to cool naturally, according to this formula, the coffee will be 35.6˚C after 10 min. Add the cream at this point and the temperature of the coffee will drop again. Consider the other scenario. If cream is added as the coffee is being poured, the difference coefficient will drop. This is true since the difference between the coffee and cream mixture and room temperature will be decreased. After 10 min the temperature of the coffee and cream will be less than 35.6˚C but greater than 35.6˚C less the temperature of the cream. Thus, coffee that has been cooled and then creamed will be cooler after 10 min.

2.7 Exercises, page 117

4. Explanations may vary. After several generations, there is repetition. For example, a few great-great grandparents may have the same great grandparents.

10. e)



Modelling Bounce Heights, page 117

- No. The height approaches 0, but never actually reaches it.
- Answers may vary. Friction is an important factor.

11. Note the relationship between every other term.

$$2 \times 1.4 = 2.8$$
$$2 \times 2 = 4$$
$$2 \times 2.8 = 5.6, \text{ and so on.}$$

The relationship is $2 \times t_n = t_n + 2$, or $\sqrt{2} \times t_n = t_{n+1}$.

Thus the common ratio is $\sqrt{2}$.

14. **a)** $t_1 + t_2 = 3$
$a + ar = 3$
$a(1 + r) = 3$ ①
$t_3 + t_4 = \frac{4}{3}$
$ar^2 + ar^3 = \frac{4}{3}$
$ar^2(1 + r) = \frac{4}{3}$ ②
Substitute ① into ②.
$3r^2 = \frac{4}{3}$
$r^2 = \frac{4}{9}$
$r = \pm\frac{2}{3}$
When $r = \frac{2}{3}$:
$a(1 + \frac{2}{3}) = 3$
$a = 3 \times \frac{3}{5}$
$= \frac{9}{5}$
The sequence is $\frac{9}{5}, \frac{6}{5}, \frac{4}{5}, \frac{8}{15}, \ldots.$
Check: $\frac{9}{5} + \frac{6}{5} = 3; \frac{4}{5} + \frac{8}{15} = \frac{4}{3}$
When $r = -\frac{2}{3}$:
$a(1 - \frac{2}{3}) = 3$
$a = 9$
The sequence is $9, -6, 4, -\frac{8}{3}, \ldots.$
Check: $9 - 6 = 3; 4 - \frac{8}{3} = \frac{4}{3}$

b) $t_3 + t_4 = 36$
$ar^2 + ar^3 = 36$
$ar^2(1 + r) = 36$ ①
$t_4 + t_5 = 108$
$ar^3 + ar^4 = 108$
$ar^3(1 + r) = 108$ ②
Substitute ① into ②.
$36r = 108$
$r = 3$
Substitute r into ①.
$9a(4) = 36$
$a = 1$
The sequence is 1, 3, 9, 27, 81, ….
Check: $9 + 27 = 36; 27 + 81 = 108$

15. For the arithmetic sequence, $t_1 = a$, $t_2 = a + d$, $t_3 = a + 2d$. For the geometric sequence, $t_1 = a$, $t_2 = ar$, $t_3 = ar^2$.
Corresponding terms are equal.
$a + d = ar$
$r = \frac{a + d}{a}$ ①
$a + 2d = ar^2$ ②
Substitute ① into ②.

$$\begin{aligned} a + 2d &= a\left(\frac{a+d}{a}\right)^2 \\ a + 2d &= \frac{a^2 + 2ad + d^2}{a} \\ a^2 + 2ad &= a^2 + 2ad + d^2 \\ d^2 &= 0 \\ d &= 0 \end{aligned}$$

Substitute d into ①.

$$\begin{aligned} r &= \frac{a+d}{a} \\ &= \frac{a+0}{a} \\ &= 1 \end{aligned}$$

For the numbers to form both an arithmetic and a geometric sequence, the common difference must be 0 and the common ratio must be 1. In other words, $a, a, a, \ldots$ is both an arithmetic and a geometric sequence.

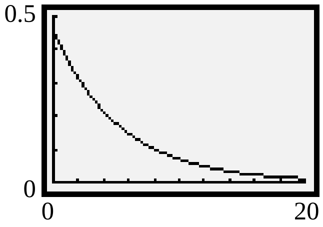
Exploring With a Graphing Calculator, page 120

1. **a)** The trace function gives $L \doteq 17.83$ when $n = 12$.

b) The table function lists $L \doteq 21.40$ when $n = 13$, so there must be at least 13 passes for $L = 20$.

2. **a)** A decrease in thickness of 17% corresponds to 100% – 17%, or 0.83. Therefore, $t = 0.45(0.83^n)$.

b)



c) The trace function gives $t \doteq 0.05$ when $n = 12$.

d) Let $t = 0.001$ and substitute it into the equation from part a.

$$\begin{aligned} 0.001 &= 0.45(0.83^n) \\ \frac{0.001}{0.45} &= 0.83^n \\ n &= \frac{\log 0.001 - \log 0.45}{\log 0.83} \\ n &\doteq 32.8 \end{aligned}$$

Therefore, the slab must pass through approximately 33 times.

3. **a)** The volume of the slab remains constant. Let l represent the original length, w the original width, and t the original thickness. Let r be the rate of decrease of the thickness, in decimal form. Thus,

$$\begin{aligned} lwt &= 1.2l \times w \times (1 - r)t \\ 1 &= 1.2(1 - r) \\ 1 - r &\doteq 0.83 \\ r &\doteq 0.17 \end{aligned}$$

The thickness decreases by 17% at every successive pass.

b) Let $n = 33$, as found in exercise 2 part d, and substitute it into the equation for L, to get $L = 2(1.2^{33})$, or 820.4 m long.

Selected Solutions — Chapter 2

2.8 Exercises, page 124

2. Explanations may vary. For part a:
I set $S = 1 + 2 + 4 + \ldots + 32$ and multiplied S by 2 to get
$2S = 2 + 4 + 8 + \ldots + 64$. I then subtracted S from $2S$ to get $S = 63$.

8. b) Player 1 and player 2 play, and one drops out.
The winner plays player 3, and one drops out.
The winner plays player 4, and one drops out.
…
The winner plays player 64, and one drops out.
This is 63 matches.

9. b) I would take Prize 2 and capitalize on the power of compound interest and other investment opportunities.

11. The algorithm for downloading to all calculators will follow the geometric sequence 1, 2, 4, 8, 16, ..., 2^n, where n represents the number of times a group of calculators download to another group. The first student will download to a second student by following the instructions below. Following the same procedure, the two of them will download to two more, who will download to four more, and so on until the class set of calculators has the program stored in memory. The total time for the whole class to complete the algorithm depends on the length of downloading the initial program. For example, a program with approximately 75 instructions takes 3 seconds to download.

Instructions for Receiver

1. Pres LINK and select 1:Receive.
2. Once the sendor has transmitted, confirm that the program is received.
3. Press PRGM to see the program in memory.

Instructions for Sendor

1. Press LINK and select 3:Prgm.
2. Move down to the program which will be transferred and press ENTER.
3. Select 1:Transmit from the TRANSMIT menu.
4. Wait for the confirmation that the program has be downloaded successfully.

12. e)

$$\begin{aligned}(r-1)(a + ar + ar^2 + \cdots + ar^{n-1}) &= a(r-1)(1 + r + r^2 + \cdots + r^{n-1})\\ &= a(r^n - 1)\\ a + ar + ar^2 + \cdots + ar^{n-1} &= \frac{a(r^n-1)}{r-1}\end{aligned}$$

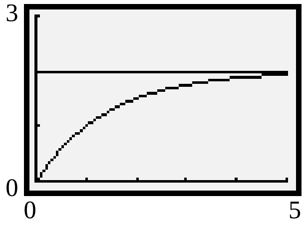
13. a)

$$\begin{aligned}Sn &= \frac{a(r^n-1)}{r-1}\\ &= \frac{1\left(\left(\frac{1}{2}\right)^2 - 1\right)}{\frac{1}{2} - 1}\\ &= 2\left(1 - \left(\tfrac{1}{2}\right)^n\right)\end{aligned}$$

Since $\left(\frac{1}{2}\right)^n > 0$ for all values of n, $1 - \left(\frac{1}{2}\right)^n < 1$ for all values of n, and $2\left(1 - \left(\frac{1}{2}\right)^n\right) < 2$ for all values of n.

b) As n becomes larger and larger, $\left(\frac{1}{2}\right)^n$ gets closer and closer to 0, $1-\left(\frac{1}{2}\right)^n$ gets closer and closer to 1, and $2\left(1-\left(\frac{1}{2}\right)^n\right)$ gets closer and closer to 2.

c)

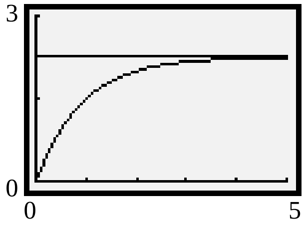


14. **a)**
$$\begin{aligned} Sn &= \frac{a(r^n-1)}{r-1} \\ &= \frac{1\left(\left(\frac{1}{3}\right)^n-1\right)}{\frac{1}{3}-1} \\ &= 1.5\left(1-\left(\tfrac{1}{3}\right)^n\right) \end{aligned}$$

Since $\left(\frac{1}{3}\right)^n > 0$ for all values of n, $1-\left(\frac{1}{3}\right)^n < 1$ for all values of n, and $1.5\left(1-\left(\frac{1}{3}\right)^n\right) < 1.5$ for all values of n.

b) As n becomes larger and larger, $\left(\frac{1}{3}\right)^n$ gets closer and closer to 0, $1-\left(\frac{1}{3}\right)^n$ gets closer and closer to 1, and $1.5\left(1-\left(\frac{1}{3}\right)^n\right)$ gets closer and closer to 1.5.

c)



2.9 Exercises, page 130

5. **b)** I wrote the series as $25\,000 + 0.95(25\,000) + 0.95^2(25\,000) + \cdots$. The first term is 25 000 and the ratio is 0.95. Thus, the sum of the series is $\frac{25\,000}{1-0.95}$ or 500 000. I assumed that the well will stay in production an infinitely long time, and that the ratio will stay the same.

c) The estimate is probably higher than the actual production, because the well will not stay in production an infinitely long time.

Modelling the Depletion of an Oil Well, page 130

- 14 months; No. This is not a very long time for an oil well to operate.
- It would be producing $0.95^{(25\times12)}(25\,000)$ or about 0.005 barrels by that time, so it would have been shut down.
- 0.5%; 5 000 000 barrels will be produced. After 25 years, the well would still produce 5000 barrels.

Selected Solutions — Chapter 2

6. c) I found the sum of the infinite series $2 + 0.63(2)(2) + 0.63^2(2)(2) + \cdots$ using the formula $S = \frac{a}{1-r}$.
The sum is $2 + \frac{0.63(2)(2)}{1-0.63}$ or about 8.81 m. I assumed that the ball bounces an infinite number of times.

d) The estimate is greater than the actual amount, because the ball will not bounce an infinite number of times.

7. One quarter of the area is removed at each step. The infinite series is
$\frac{1}{4}A + \frac{1}{4} \times \frac{3}{4}A + \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4}A + \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}A + \cdots$, or
$\frac{1}{4}A + \frac{3}{4} \times \frac{1}{4}A + (\frac{3}{4})^2 \times \frac{1}{4}A + (\frac{3}{4})^3 \times \frac{1}{4}A + \cdots$

$$\text{The sum is } \frac{\frac{1}{4}A}{1-\frac{3}{4}} = \frac{\frac{1}{4}A}{\frac{1}{4}} = A$$

The amount removed is A, or the original area of the triangle.

8. a) i) The perimeter of the triangle is 3 × 24 mm or 72 mm.
The length of each side of figure 2 is $\frac{1}{3}$ the length of the original side. There are 4 smaller lengths in the place of the original 24 mm side. Thus the perimeter of figure 2 is $3 \times 24 \text{ mm} \times \frac{4}{3}$, or 96 mm.

The length of each side of figure 3 is $\frac{1}{3}$ the length of the side in figure 2. There are 4 smaller lengths in the place of one length in figure 2. Thus the perimeter of figure 3 is $3 \times 24 \text{ mm} \times (\frac{4}{3})^2$, or 128 mm.

The length of each side of figure 4 is $\frac{1}{3}$ the length of the side in figure 3. There are 4 smaller lengths in the place of one length in figure 3. Thus the perimeter of figure 4 is $3 \times 24 \text{ mm} \times (\frac{4}{3})^3$, or $\frac{512}{3}$ mm.

ii) The perimeters form a geometric sequence with ratio $\frac{4}{3}$. The perimeter of the snowflake curve that results from continuing the steps forever is infinite, since $r > 1$.

b) i) Triangle: $144\sqrt{3}$ mm^2;
Figure 2: $1 + \frac{3}{9}$ of triangle, that is, $\frac{12}{9} \times 144\sqrt{3}$, or $192\sqrt{3}$ mm^2;
Figure 3: $1 + \frac{3}{9} + \frac{12}{81}$ of triangle, that is, $\frac{120}{81} \times 144\sqrt{3}$, or $\frac{640}{3}\sqrt{3}$ mm^2;
Figure 4: $1 + \frac{3}{9} + \frac{12}{81} + \frac{48}{729}$ of triangle, that is, $\frac{1128}{729} \times 144\sqrt{3}$, or $\frac{6016}{27}\sqrt{3}$ mm^2

ii) Starting with figure 2, the area is given by the sum of 1 and an infinite geometric series (with $a = \frac{3}{9}$ and $r = \frac{4}{9}$) times the area of the triangle. Since $r < 1$, this series has a sum to infinity. Substitute a and r into the formula for the sum of a geometric series, add this to 1, and multiply by the area of the triangle.

$$
\begin{aligned}
(S+1)\times 144\sqrt{3} &= \left(\frac{\frac{3}{9}}{1-\frac{4}{9}}+1\right)\times 144\sqrt{3}\\
&= 1.6\times 144\sqrt{3}\\
&\doteq 399.065
\end{aligned}
$$

Thus, the area of the snowflake curve approaches 399.1 mm^2.

c) The snowflake has infinite perimeter and finite area.

9. a) i) The segments AB, BC, CD, DE, ... form the sequence 1, x, x^2, x^3, ... The first term is 1, and the ratio is x. From the triangle, $x < 1$. Thus, the sum of the infinite series is $\frac{1}{1-x}$.

ii) The segments AB, CD, EF, ... form the sequence 1, x^2, x^4, ... The first term is 1, and the ratio is x^2. Since $x < 1$, $x^2 < 1$. Thus, the sum of the infinite series is $\frac{1}{1-x^2}$.

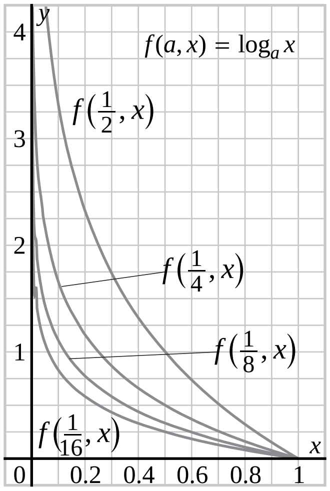
iii) The segments BC, DE, FG, ... form the sequence x, x^3, x^5, ... The first term is x, and the ratio is x^2. Since $x < 1$, $x^2 < 1$. Thus, the sum of the infinite series is $\frac{x}{1-x^2}$.

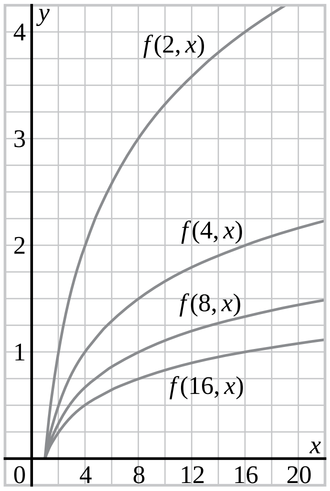
b) Answers may vary.

$$
\begin{aligned}
AC + CE + EG + \cdots &= y + yx^2 + y(x^2)^2 + \cdots\\
&= \frac{y}{1-x^2}\\
&= \frac{\sqrt{1-x^2}}{1-x^2}, \text{ by Pythagorus}
\end{aligned}
$$

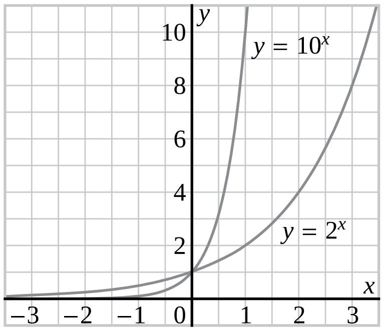
2.10 Exercises, page 138

3. a)

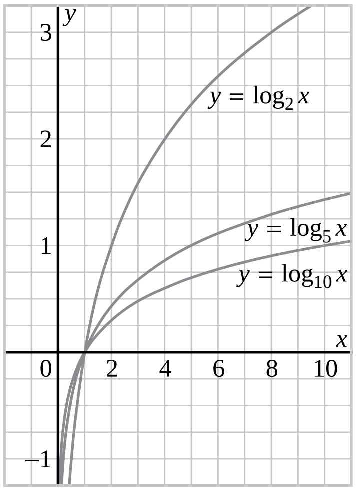




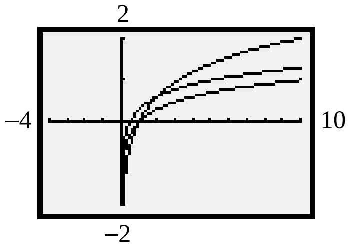
4. a)



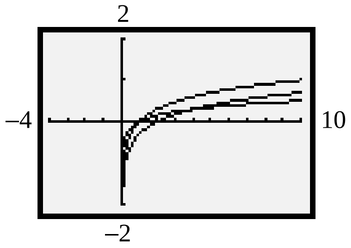
b), c)



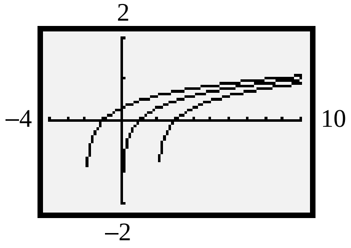
5. **a)** The graph of $y = 2 \log x$ is a vertical expansion by a factor of 2 of the graph of $y = \log x$. The graph of $y = \log 2x$ is a horizontal compression by a factor of $\frac{1}{2}$ of the graph of $y = \log x$.



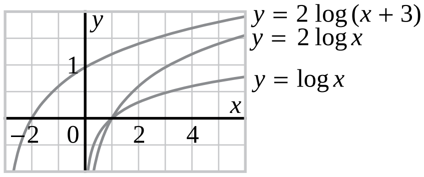
b) The graph of $y = \frac{1}{2} \log x$ is a vertical compression by a factor of $\frac{1}{2}$ of the graph of $y = \log x$. The graph of $y = \log \frac{1}{2}x$ is a horizontal expansion by a factor of 2 of the graph of $y = \log x$.



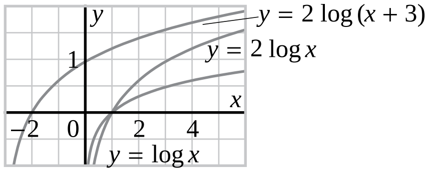
c) The graph of $y = \log (x - 2)$ is a translation 2 units right of the graph of $y = \log x$. The graph of $y = \log (x + 2)$ is a translation 2 units left of the graph of $y = \log x$.



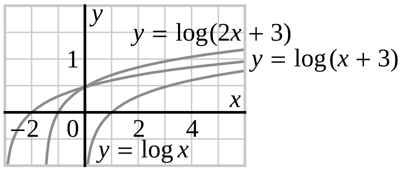
6. **a)**



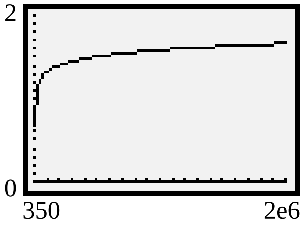
b)



c)



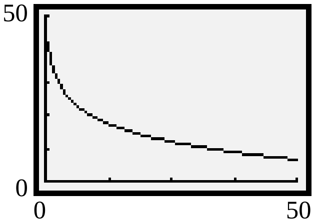
7. **a)**



Modelling the Pace of Life, page 139

- Answers may vary.
- Answers may vary; number of appointments or classes in a day, number of people with whom a person interacts, number of hours spent sleeping, number of hours spent in leisure activity

8. **a)**

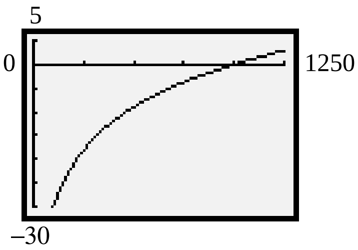


9. Explanations may vary. For exercise 8:
The graph of $S = -18 \log t + 84$ is a vertical expansion by a factor of 18, a reflection in the x-axis, and a translation of 84 units up of the graph of $y = \log x$.

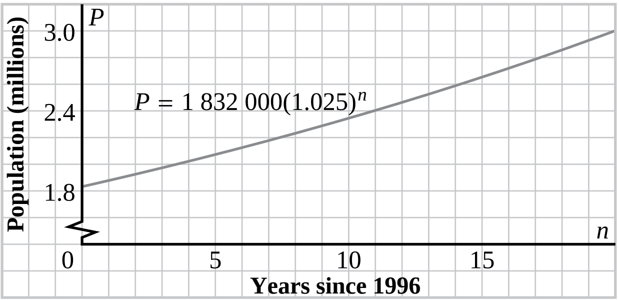
10. **b)** **i)** The money will be worth $1250 in 2.9 years.

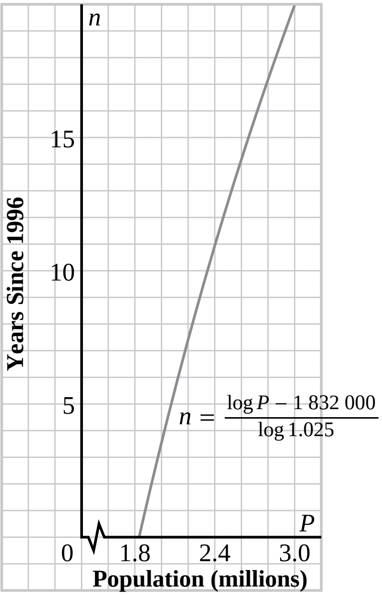
ii) The money was worth $350 about 13.6 years ago.

c)

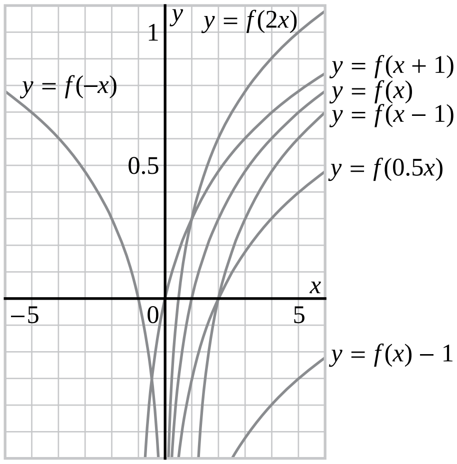

5
0
1250
–30

11. d)


P
3.0
2.4
1.8
Population (millions)
$P = 1\,832\,000(1.025)^n$
n
0
5
10
15
Years since 1996

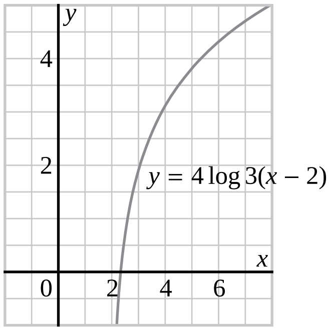

n
15
10
5
0
Years Since 1996
$n = \frac{\log P - 1\,832\,000}{\log 1.025}$
P
1.8
2.4
3.0
Population (millions)

12. a), b)

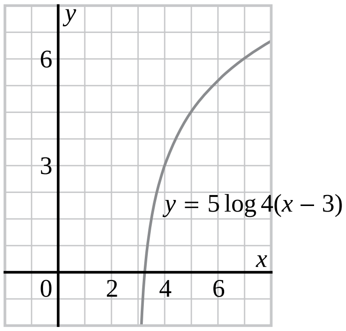

y
1
0.5
x
–5
0
5
$y = f(2x)$
$y = f(-x)$
$y = f(x + 1)$
$y = f(x)$
$y = f(x - 1)$
$y = f(0.5x)$
$y = f(x) - 1$

Selected Solutions — Chapter 2

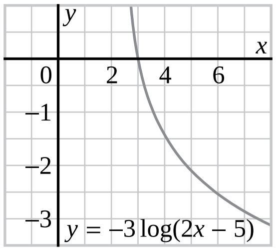
13.a)



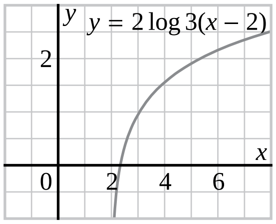
14. a)



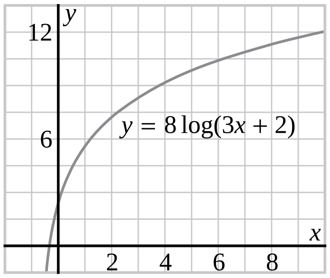
b)



c)

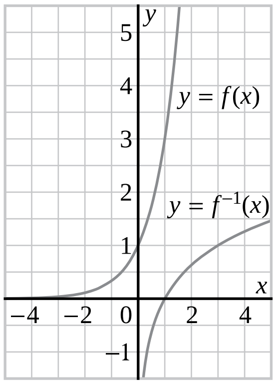


d)

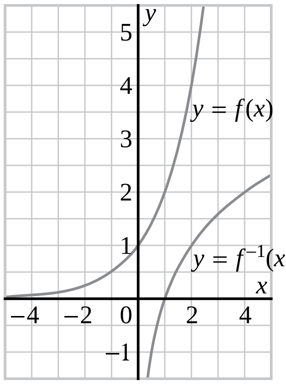


Selected Solutions — Chapter 2

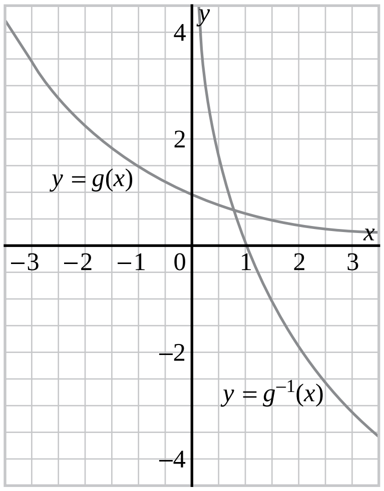
16. **a)**



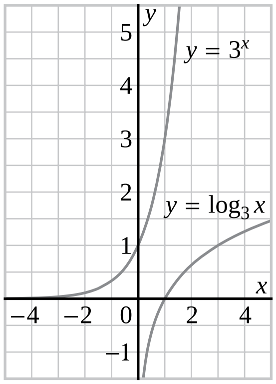
17. **a)**



b)



c)



18. **a)** Since $y = \log 2^x$ and $y = 2^x$ are inverse functions simply exchange the x- and y-coordinates of the points in exercise 18 page 112 to get the points which lie on the graph of $y = \log 2^x$.

b) For the first point:
Since $\log_2 (x_1x_2) = \log_2 x_1 + \log_2 x_2 = y_1 + y_2$, the point $(x_1x_2, y_1 + y_2)$ is on the graph.

19. From the graph in *Example 1*, page 135, the curves appear to intersect at $x = 0.5$. Use trial and error.
Write $y = \log_{\frac{1}{3}} x$ in exponential form.

$$\left(\frac{1}{3}\right)^y = x$$

Take the logarithm of each side.

$$y \log \frac{1}{3} = \log x$$

$$y = \frac{\log x}{\log \frac{1}{3}}$$

Use this equation to find values on a calculator.

x	0.5	0.55	0.548
$\left(\frac{1}{3}\right)^x$	0.5774	0.5465	0.5477
$\log_{\frac{1}{3}} x$	0.6309	0.5442	0.5475

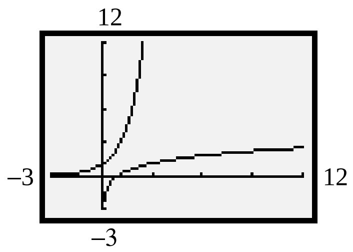
The graphs intersect at approximately (0.548, 0.548).

Mathematics File, page 141

2. **a)** This means $e^0 = 1$.
b) This means $e^{0.6931} \doteq 2$.
c) This means $e^1 = e$.
d) This means $e^{2.3026} \doteq 10$.
e) This means $e^{-0.6931} \doteq 0.5$.

3. **a)** This means $e^0 = 1$.
b) This means $e^{0.6931} \doteq 2$.
c) This means $e^1 = e$.
d) This means $e^{2.3026} \doteq 10$.
e) This means $e^{-0.6931} \doteq 0.5$.

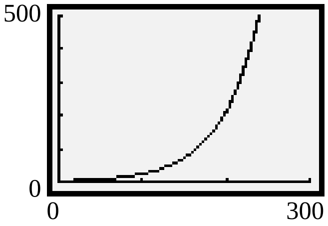
4. **a)**



Selected Solutions — Chapter 2

b) The graph of ln x is a reflection in the line $y = x$ of the graph of $y = e^x$.
$y = e^x$
Solve for x.
$\ln y = x$
Interchange x and y.
$y = \ln x$

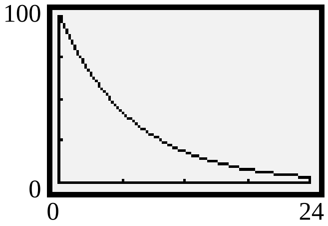
5. c)



6. The graphs are the same, but their equations are different.

7. c) Answers may vary. The country may not be able to sustain itself at this rate of growth.

8. b)



The graphs are the same.

c) The graph of $P = 100e^{-0.139n}$ is a vertical expansion by a factor of 100, a horizontal expansion by a factor of $\frac{1}{0.139}$, and a reflection in the y-axis of the graph of $y = e^x$.

10. This is a growth model, so $A = Pe^{\frac{in}{100}}$, where A is the amount accumulated, P is the principal, i is the interest rate in percent, and n is the number of years. For the amount to double, $A = 2P$.

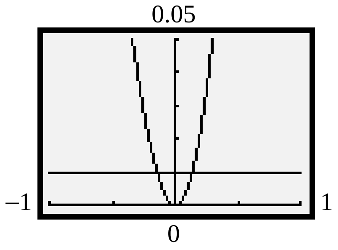
$$2P = Pe^{\frac{in}{100}}$$
$$2 = e^{\frac{in}{100}}$$
$$\ln 2 = \frac{in}{100}$$
$$n \doteq \frac{69.3}{i}$$

Thus, the number of years is about $\frac{70}{i}$.

Problem Solving, page 144

1. a) Graph $y = e^x$ on a graphing calculator, and zoom in until it looks like a straight line. One point on the line is (0, 1). Graph lines of the form $y = mx + 1$ until you find the one that corresponds to $y = e^x$. The line $y = x + 1$ seems to work.

b) Find x so that $|x + 1 - e^x| < 0.01$.
Graph $y = |x + 1 - e^x|$ and $y = 0.01$.



Find values of x for which $y = |x + 1 - e^x|$ is below $y = 0.01$. Find the points of intersection of $y = |x + 1 - e^x|$ and $y = 0.01$. They are $(-0.1448, 0.01)$ and $(0.1382, 0.01)$. Thus, for $-0.1448 < x < 0.1382$, the y-coordinates differ by less than 0.01.

2. **a)** Graph $y = e^x$ on a graphing calculator, and zoom in until it looks like half a parabola. One point on the parabola is $(0, 1)$, so try parabolas of the form $y = ax^2 + bx + 1$. Trace to find two other points on the parabola. One is $(0.5, 1.649)$, another is $(-0.5, 0.607)$. Substitute these into the equation to find a and b.

$$1.649 = 0.25a + 0.5b + 1$$
$$0.607 = 0.25a - 0.5b + 1$$

Subtract the equations.

$$1.042 = b$$

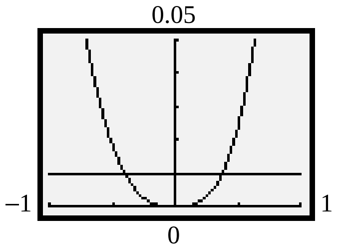
Add the equations.

$$\begin{aligned} 2.256 &= 0.5a + 2 \\ 0.5a &= 0.256 \\ a &= 0.512 \end{aligned}$$

The equation of the parabola is about $\frac{1}{2}x^2 + x + 1$.

b) Find x so that $|\frac{1}{2}x^2 + x + 1 - e^x| < 0.01$.

Graph $y = |\frac{1}{2}x^2 + x + 1 - e^x|$ and $y = 0.01$.



Find values of x for which $y = |\frac{1}{2}x^2 + x + 1 - e^x|$ is below $y = 0.01$. Find the points of intersection of $y = |\frac{1}{2}x^2 + x + 1 - e^x|$ and $y = 0.01$. They are $(0.379, 0.01)$ and $(-0.405, 0.01)$. Thus, for $-0.405 < x < 0.379$, the y-coordinates differ by less than 0.01.

3. Use $y = x + 1$ or $y = \frac{1}{2}x^2 + x + 1$ as an approximation to $y = e^x$ when x is close to 0. Note that the parabola is a better approximation since it's range of x-values more than doubles the range of the line.

Selected Solutions — Chapter 2

4. **a)** $2! = 2 \times 1 = 2$ **b)** $3! = 3 \times 2! = 6$

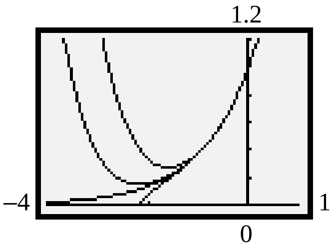
c) $5! = 5 \times 4 \times 3! = 120$ **d)** $6! = 6 \times 5! = 720$

e) $7! = 7 \times 6! = 5040$

5. $e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \cdots$

6. **a)** For the 3 functions, the greater the number of terms, the closer the approximation will be to the actual graph of $y = ex$.

b)



7. **a)** $e^0 = 1$

b) $e^1 = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \cdots \doteq 2.718$

c) $e^{-1} = 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \frac{1}{720} - \cdots \doteq 0.368$

d) $e^2 = 1 + 2 + \frac{1}{2} \times 4 + \frac{1}{6} \times 8 + \frac{1}{24} \times 16 + \cdots \doteq 7.389$

e) $e^{-2} = 1 - 2 + \frac{1}{2} \times 4 - \frac{1}{6} \times 8 + \frac{1}{24} \times 16 - \cdots \doteq 0.135$

8. **a)** $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \frac{1}{6!}x^6 + \cdots$

b) Rewrite the first two terms of the sequence above so that they conform to the rest and observe.

$e^x = \frac{1}{0!}x^0 + \frac{1}{1!}x^1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \cdots$

Since division by zero is prohibited, 0! must equal 1, making the first term equal to 1. The second term, x, has coefficient 1 and thus, $1! = 1$.

9.
$$\begin{aligned}10^x &= e^{\ln 10x}\\ &\doteq e^{2.3026x}\\ &= \sum_{i=1}^{\infty} \frac{1}{(i-1)!}(2.3026x)^{i-1}\end{aligned}$$

Investigate, page 146

6. All the methods could be used except the one in exercise 3. 7^{2x} could be written as a power of 2, but it would be approximate.

Selected Solutions — Chapter 2

2.11 Exercises, page 149

4. Answers may vary. Graph $y = 8^{x+1}$ and $y = 64^{x-1}$ and find their point of intersection; or graph $y = 8^{x+1} - 64^{x-1}$ and find the x-intercept.

2.12 Exercises, page 153

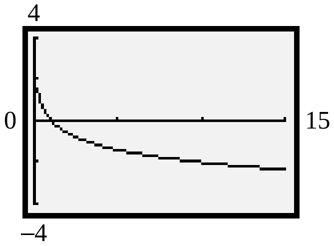
7. a) When $a = 10$ and $x = 5$:

$$\begin{aligned}\text{Left side} &= \log \frac{1}{25}\\ &= \log 0.04\\ &\doteq -1.397\ 94\\ \text{Right side} &= -2 \log 5\\ &\doteq -1.397\ 94\end{aligned}$$

Since Left side = Right side, the identity is verified.

c) Graph the functions $y = \log_a \frac{1}{x^2}$ and $y = -2 \log_z x$.

The graphs appear to be identical.



d) To prove that $\log_a \frac{1}{x^2} = -2 \log_a x$, for $x > 0$, prove the left side is equal to the right side.

$$\begin{aligned}\text{Left side} &= \log_a \frac{1}{x^2}\\ &= \log_a (x^2)^{-1}\\ &= \log_a x^{-2}\\ &= -2 \log_a x\end{aligned}$$

Hence Left side = Right side

8. a)

$$\begin{aligned}\text{Left side} &= \log (x-1) + \log (x-2)\\ &= \log (x-1)(x-2)\\ &= \log (x^2 - 3x + 2)\end{aligned}$$

Hence Left side = Right side

$x > 2$

b)

$$\begin{aligned}\text{Left side} &= \log x + \log (x+3)\\ &= \log x(x+3)\\ &= \log (x^2 + 3x)\end{aligned}$$

Hence Left side = Right side

$x > 0$

c)

$$\begin{aligned}\text{Left side} &= \log (x-5) + \log (x+5)\\ &= \log (x-5)(x+5)\\ &= \log (x^2 - 25)\end{aligned}$$

Hence Left side = Right side

$x > 5$

Selected Solutions — Chapter 2

9. **a)** Let $x = \log_b c$

$$b^x = c$$

Take the logarithm base a of each side.

$$x \log_a b = \log_a c$$

$$x = \frac{\log_a c}{\log_a b}$$

$$\text{Left side} = (\log_a b)\left(\frac{\log_a c}{\log_a b}\right)$$

$$= \log_a c$$

Hence Left side = Right side

b) From part a, $(\log_a b)(\log_b c) = \log_a c$.

$$\text{Left side} = (\log_a b)(\log_b c)(\log_c d)$$

$$= (\log_a c)(\log_c d)$$

Using the same pattern from part a,

$$\text{Left side} = \log_a d$$

$$= \text{Right side}$$

10. **a)** **i)** Use the results from exercise 9.

$$\text{Left side} = \frac{1}{\log_3 10} + \frac{1}{\log_4 10}$$

$$= \frac{1}{\frac{\log 10}{\log 3}} + \frac{1}{\frac{\log 10}{\log 4}}$$

$$= \log 3 + \log 4$$

$$= \log (3 \times 4)$$

$$= \log 12$$

$$\text{Right side} = \frac{1}{\log_{12} 10}$$

$$= \frac{1}{\frac{\log 10}{\log 12}}$$

$$= \log 12$$

Hence Left side = Right side

ii) $$\text{Left side} = \frac{1}{\log_3 x} + \frac{1}{\log_4 x}$$

$$= \frac{1}{\frac{\log x}{\log 3}} + \frac{1}{\frac{\log x}{\log 4}}$$

$$= \frac{\log 3}{\log x} + \frac{\log 4}{\log x}$$

$$= \frac{\log 3 + \log 4}{\log x}$$

$$= \frac{\log (3 \times 4)}{\log x}$$

$$= \frac{\log 12}{\log x}$$

$$= \frac{1}{\frac{\log x}{\log 12}}$$

$$= \frac{1}{\log_{12} x}$$

Hence Left side = Right side

Selected Solutions — Chapter 2

10. b) Prove that $\dfrac{1}{\log_a x} + \dfrac{1}{\log_b x} = \dfrac{1}{\log_{ab} x}$.

$$\begin{aligned}
\text{Left side} &= \frac{1}{\log_a x} + \frac{1}{\log_b x} \\
&= \frac{1}{\frac{\log x}{\log a}} + \frac{1}{\frac{\log x}{\log b}} \\
&= \frac{\log a}{\log x} + \frac{\log b}{\log x} \\
&= \frac{\log ab}{\log x} \\
&= \frac{1}{\frac{\log x}{\log ab}} \\
&= \frac{1}{\log_{ab} x}
\end{aligned}$$

Hence Left side = Right side