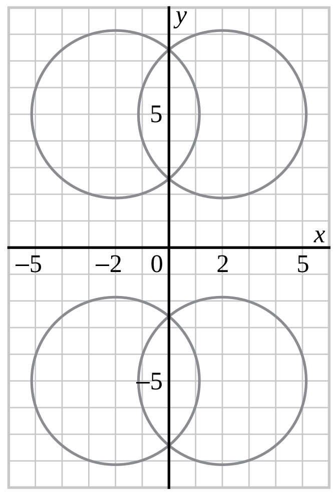
Selected Solutions — Chapter 9

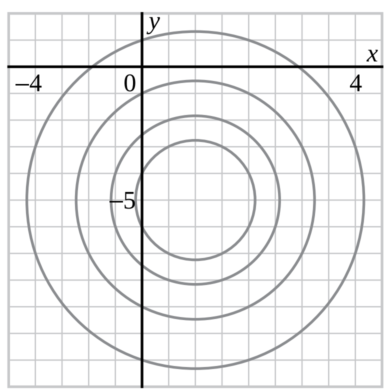
9.1 Exercises, page 525

3. b) Explanations may vary. For part iv:
I started with the standard form for the equation of a circle with centre (h, k) and radius r.
$(x - h)^2 + (y - k)^2 = r^2$
Then I substituted for h, k, and r the values given.
$(x + 2)^2 + (y - 6)^2 = 25$

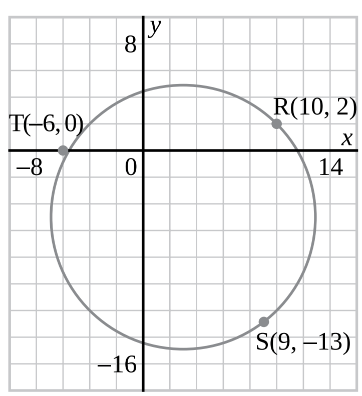
7. a) All the circles have the same radius, $\sqrt{10}$. The centres form a rectangle centred at the origin.



b) All the circles have the same centre (2, –5), but different radii: $\sqrt{5}$, $\sqrt{10}$, $2\sqrt{5}$, $2\sqrt{10}$

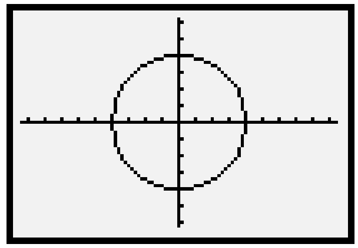


8. a)



Selected Solutions — Chapter 9

13. b) A circle, centre the origin, radius 4



15. b) Explanations may vary. For part i:

First I determined the equation of the circle, in the same way as I did in exercise 3:

$(x - 3)^2 + (y - 2)^2 = 25$

Then, since C is a point on the circle, I substituted its coordinates into the equation for the circle:

$(m - 3)^2 + (3 - 2)^2 = 25$

Then I solved the resulting equation for *m*:

$(m - 3)^2 = 24$

$m - 3 = \pm 2\sqrt{6}$

$m = 3 \pm 2\sqrt{6}$

18. Solve each system.

a) $(x - 3)^2 + (y + 1)^2 = 5$

Expand.

$x^2 - 6x + 9 + y^2 + 2y + 1 = 5$

Simplify.

$x^2 + y^2 - 6x + 2y + 5 = 0$ ①

$(x - 1)^2 + (y + 2)^2 = 32$

Expand.

$x^2 - 2x + 1 + y^2 + 4y + 4 = 32$

Simplify.

$x^2 + y^2 - 2x + 4y - 27 = 0$ ②

Subtract equation ② from equation ①.

$-4x - 2y + 32 = 0$

Solve for *y*.

$y = 16 - 2x$ ③

Substitute equation ③ into equation ①.

$x^2 + (16 - 2x)^2 - 6x + 2(16 - 2x) + 5 = 0$

Expand.

$x^2 + 256 - 64x + 4x^2 - 6x + 32 - 4x + 5 = 0$

Simplify.

$5x^2 - 74x + 293 = 0$ ④

If the circles intersect, equation ④ has real roots.

For real roots, $b^2 - 4ac \geq 0$

$74^2 - 4(5)(293) = -384 < 0$

The circles do not intersect.

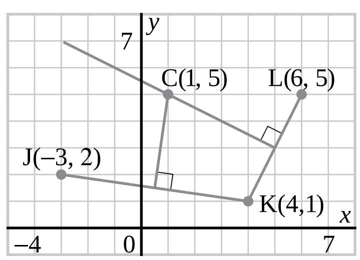
Alternatively, draw each graph to illustrate that the first circle lies inside the second circle.

b) $(x - 5)^2 + (y + 1)^2 = 8$
Expand.
$x^2 - 10x + 25 + y^2 + 2y + 1 = 8$
Simplify.
$x^2 + y^2 - 10x + 2y + 18 = 0$ ①
$(x + 3)^2 + (y + 7)^2 = 49$
Expand.
$x^2 + 6x + 9 + y^2 + 14y + 49 = 49$
Simplify.
$x^2 + y^2 + 6x + 14y + 9 = 0$ ②
Subtract equation ① from equation ②.
$16x + 12y - 9 = 0$
Solve for y.
$y = \frac{9 - 16x}{12}$ ③
Substitute equation ③ into equation ②.
$x^2 + \left(\frac{9 - 16x}{12}\right)^2 + 6x + 14\left(\frac{9 - 16x}{12}\right) + 9 = 0$
Expand and simplify.

$$x^2 + \frac{81 - 288x + 256x^2}{144} + 6x + \frac{126 - 224x}{12} + 9 = 0$$
$$144x^2 + 81 - 288x + 256x^2 + 864x + 1512 - 2688x + 1296 = 0$$
$$400x^2 - 2112x + 2889 = 0 \quad ④$$

If the circles intersect, equation ④ has real roots.
For real roots, $b^2 - 4ac \geq 0$
$(-2112)^2 - 4(400)(2889) = -161\ 856 < 0$
The circles do not intersect.

19. Plot the points J, K, and L on grid paper.



Connect JK and KL. JK and KL are chords of the circle.
Construct the perpendicular bisectors of JK and KL.
Determine the equation of each perpendicular bisector, and then determine their point of intersection.
KL has slope $\frac{5 - 1}{6 - 4}$, or 2.
Therefore, the perpendicular bisector has slope $-\frac{1}{2}$.
The midpoint of KL is $\left(\frac{6 + 4}{2}, \frac{5 + 1}{2}\right)$ or (5, 3).

The equation of the perpendicular bisector is $\frac{y-3}{x-5} = -\frac{1}{2}$
or $2y - 6 = -x + 5$
or $x + 2y = 11$ ①

JK has slope $\frac{2-1}{-3-4}$, or $\frac{1}{7}$
Therefore, the perpendicular bisector has slope –7.
The midpoint of JK is $\left(\frac{-3+4}{2}, \frac{2+1}{2}\right)$, or (0.5, 1.5).
The equation of the perpendicular bisector is $\frac{y-1.5}{x-0.5} = 7$
or $y - 1.5 = 7x - 3.5$
or $7x - y = 2$ ②

Solve equation ① and ② to determine where the bisectors intersect.
From ①, $x = 11 - 2y$
Substitute for x in ②.
$7(11 - 2y) - y = 2$
$77 - 14y - y = 2$
$15y = 75$
$y = 5$
Substitute $y = 5$ in ①
$x + 2(5) = 11$
$x = 1$
The point of intersection of the perpendicular bisectors is the centre of the circle, C(1, 5).
The distance from C to J, K, or L is the radius of the circle.
$r^2 = CK^2$
$= (1-4)^2 + (5-1)^2$
$= 25$
The equation of the circle is $(x-1)^2 + (y-5)^2 = 25$

Alternatively, use a method similar to that in exercise 18. Write the standard form for the equation of a circle. Then substitute each of the three sets of coordinates into the equation to obtain three equations involving h, k, and r. Subtract one equation from the other two to obtain two linear equations involving h and k. Then solve the two linear equations for h and k, and substitute the results back into one of the original equations to obtain r.

20. The centre of the small circle is the origin, so its equation is $x^2 + y^2 = r^2$. To find the radius r of the small circle, notice that if R is the radius of one of the big circles, then $r + R$ is the distance from the origin to the centre of one of the big circles. But from the diagram,
$r + R = \sqrt{5^2 + 5^2}$
$= 5\sqrt{2}$
and $R = 5$
Thus, $r + R = 5\sqrt{2}$
$r = 5\sqrt{2} - 5$
$r^2 = (5\sqrt{2} - 5)^2$
$= 50 - 50\sqrt{2} + 25$
$= 75 - 50\sqrt{2}$

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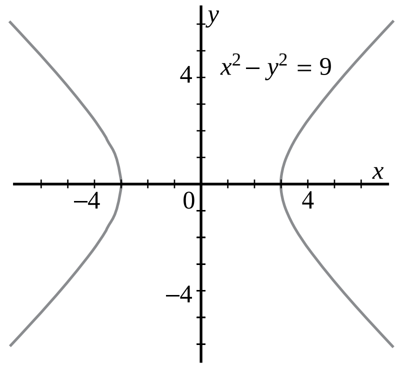
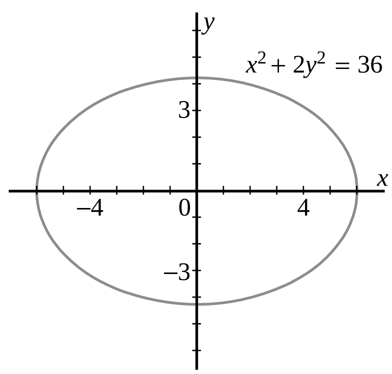
Thus, the equation of the small circle is
$x^2 + y^2 = 75 - 50\sqrt{2}$

21. a) $x^2 - y^2 = 9$
Solve for y.
$y^2 = x^2 - 9$
$y = \pm\sqrt{x^2 - 9}$
Input each equation separately.
From the calculator screen, the graph is a hyperbola.



b) $x^2 + 2y^2 = 36$
Solve for y.
$2y^2 = 36 - x^2$
$y^2 = \frac{36 - x^2}{2}$
$y = \pm\sqrt{\frac{36 - x^2}{2}}$
From the calculator screen, the graph is an ellipse.



c) $x^4 + y^4 = 625$
Solve for y.
$y^4 = 625 - x^4$
$y = \pm\sqrt[4]{625 - x^4}$
To input a fourth root, input
$y = \pm(625 - x^4)^{\frac{1}{4}}$
From the calculator screen, the graph approximates a square with "rounded" vertices.

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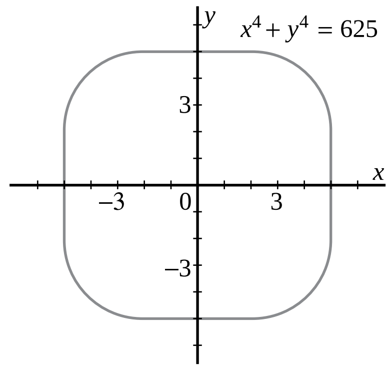
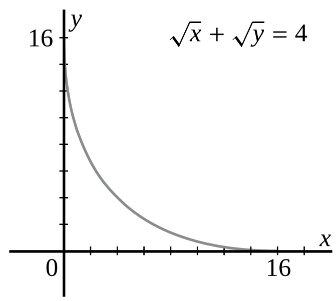


d) $\sqrt{x} + \sqrt{y} = 4$
Solve for y.
$\sqrt{y} = 4 - \sqrt{x}$
$y = (4 - \sqrt{x})^2$
From the calculator screen, the graph is a curve in the first quadrant.



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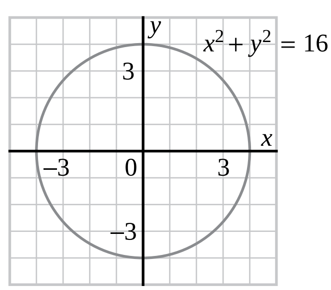
1. b) $\angle COD = \frac{360^\circ}{5}$, or 72°
Since $\triangle COD$ is isosceles, then ON is an altitude and bisects $\angle COD$. Hence $\angle CON = \frac{72^\circ}{2}$, or 36°.

9.2 Exercises, page 531

1.



c) The horizontal tangents are $y = \pm 4$, which are 8 units apart, that is, the tangents are separated by a diameter.

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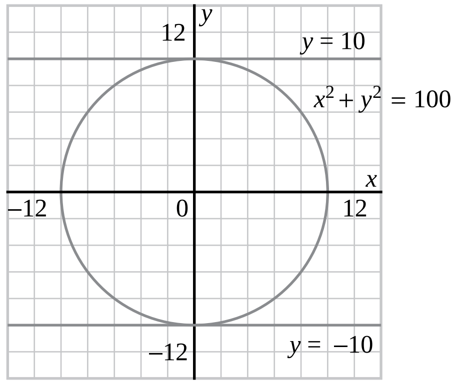
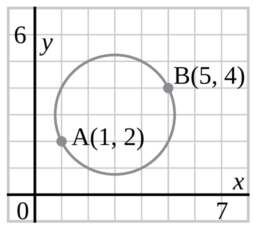
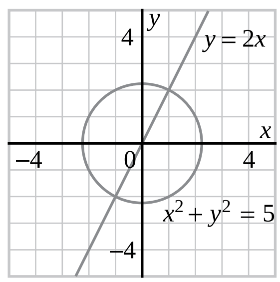
2.



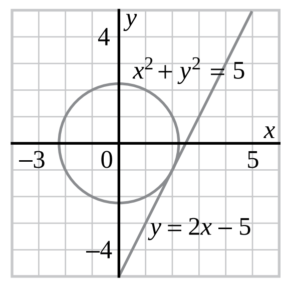
3. a)



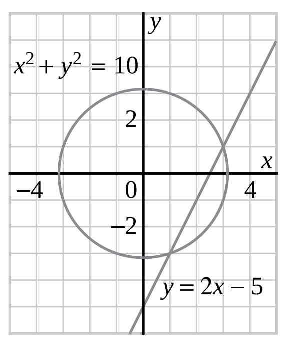
4. a) i)



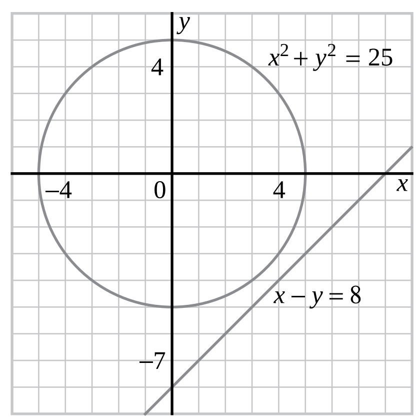
ii)



iii)



iv)



b) Explanations may vary. For part ii:
I labelled $y = 2x - 5$ equation ① and $x^2 + y^2 = 10$ equation ②.
I substituted for y from ① into equation ② to get $x^2 + (2x - 5)^2 = 10$.
I simplified this equation to get $5x^2 - 20x + 15 = 0$, or
$x^2 - 4x + 3 = 0$. This factors to $(x - 1)(x - 3) = 0$ or $x = 1$ and
$x = 3$. I substituted $x = 1$ and $x = 3$, in turn, into ① to get $y = -3$
and $y = 1$ respectively. The points of intersection are (3, 1)
and (1, –3).

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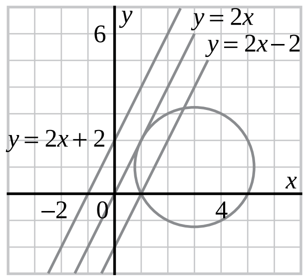
5. a), c)

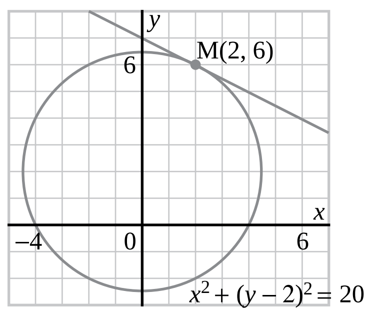


i) There are two points of intersection; the line intersects the circle in two points.

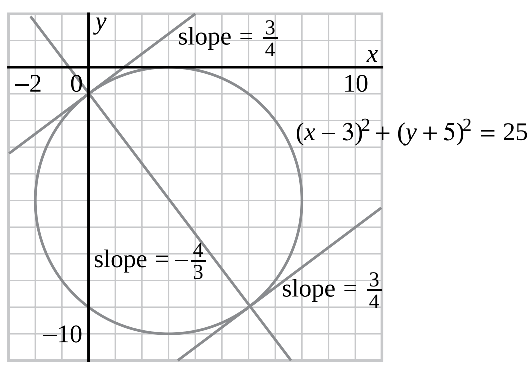
ii) There is one point of intersection; the line is tangent to the circle.

iii) There are no points of intersection; the line does not intersect the circle.

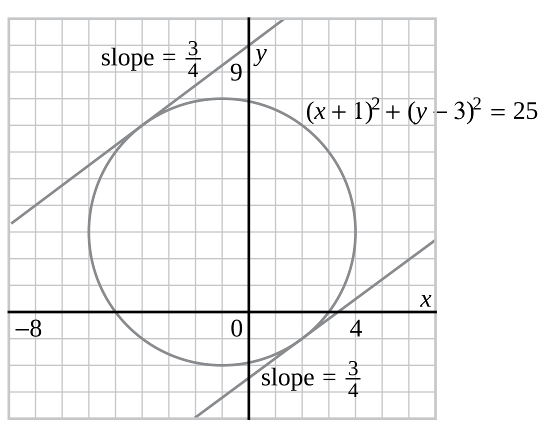
6. a), c)



7.



8.



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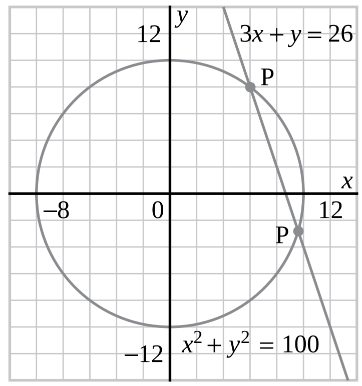
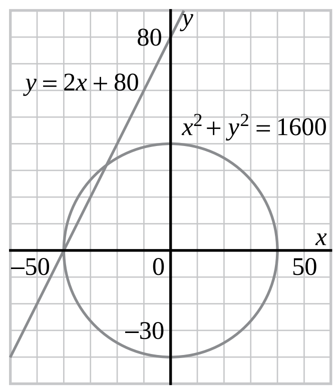
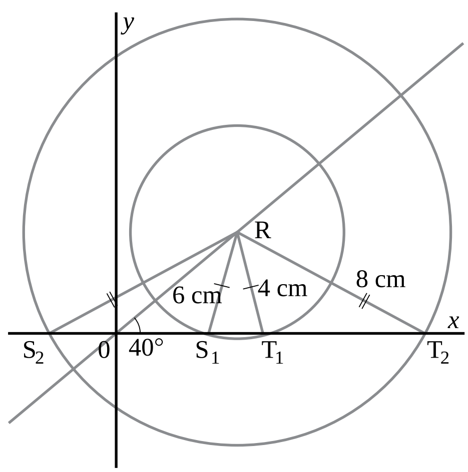
9.



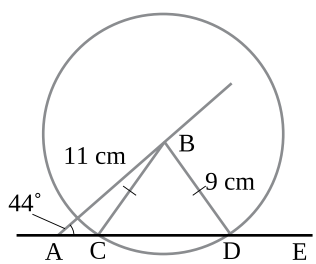
10.



11.



12.



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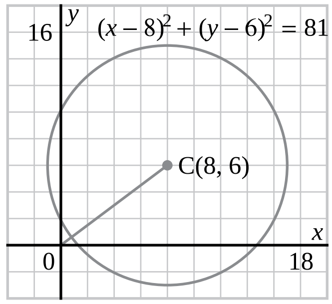
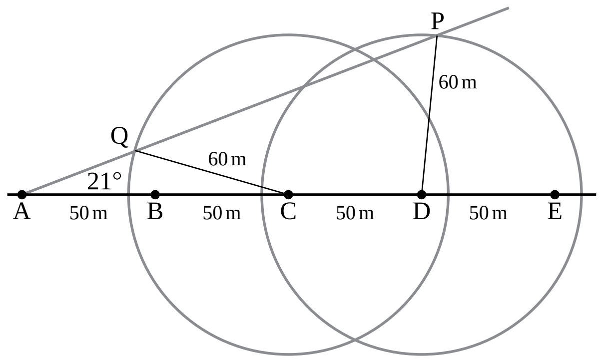
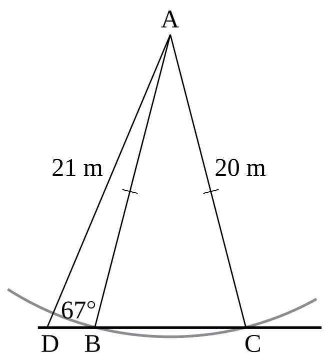
13.



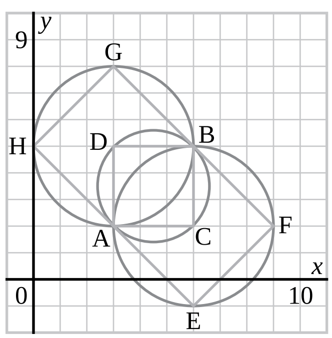
14.



15.



16.



There are 3 possible squares with vertices A and B.

Case 1: square ACBD

The centre of the circle is $\left(\frac{3+6}{2}, \frac{2+5}{2}\right)$ or (4.5, 3.5) and the radius is one-half the length of the diagonal of the square, which is $\frac{1}{2}\sqrt{(3-6)^2+(2-5)^2} = \frac{1}{2}\sqrt{18}$, or $\frac{3\sqrt{2}}{2}$. Thus, the equation of the circle is

$$(x-4.5)^2+(y-3.5)^2 = \left(\frac{3\sqrt{2}}{2}\right)^2$$
$$= 4.5$$

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Case 2: square ABFE

From the graph, the centre of the circle is (6, 2), and the radius is 3. Thus, the equation of the circle is $(x-6)^2+(y-2)^2=9$.

Case 3: square ABGH

From the graph, the centre of the circle is (3, 5), and the radius is 3. Thus, the equation of the circle is $(x-3)^2+(y-5)^2=9$.

17. **a)** Find the equation of line PQ.

The slope is $\frac{10-4}{5-(-3)}=\frac{3}{4}$.

Thus, the equation has the form $y=\frac{3}{4}x+b$, where b is the y-intercept.

The point (5, 10) is on the line. Substitute these coordinates in the equation.

$$10=\frac{3}{4}(5)+b$$
$$b=\frac{25}{4}$$

The equation is $y=\frac{3}{4}x+\frac{25}{4}$.

Find the point of intersection of the line and the circle by substituting the equation of the line for y in the equation of the circle, $x^2+y^2=25$.

$$x^2+\left(\frac{3}{4}x+\frac{25}{4}\right)^2=25$$

Expand and simplify.

$$x^2+\frac{9}{16}x^2+\frac{150}{16}x+\frac{625}{16}=25$$
$$25x^2+150x+225=0$$
$$x^2+6x+9=0$$
$$(x+3)^2=0$$
$$x=-3$$

Since this equation has only one real root, the line touches the circle.

b) Find the equation of line PA.

The slope is $\frac{10-4}{5-3}=3$.

The equation has the form $y=3x+b$, where b is the y-intercept. (From the graph, the y-intercept appears to be –5, but we cannot assume it is.)

Substitute the coordinates of A(3, 4) to determine b.

$4=3(3)+b$

$b=-5$

The line has equation $y=3x-5$.

Find the point of intersection of the line and the circle. Substitute for y in the equation of the circle.

$x^2+(3x-5)^2=25$

Expand and simplify.

$$10x^2-30x=0$$
$$10x(x-3)=0$$
$$x=0 \text{ or } x=3$$

Substitute each value of x into the equation of the line to get $y = -5$ or $y = 4$.
The points of intersection are (0, –5) and (3, 4). The coordinates of B are (0, –5).

c) $PQ^2 = (5 - (-3))^2 + (10 - 4)^2$
$= 64 + 36$
$= 100$

$PA = \sqrt{(5-3)^2 + (10-4)^2}$
$= \sqrt{40}$

$PB = \sqrt{(5-0)^2 + (10-(-5))^2}$
$= \sqrt{250}$

$PA \times PB = \sqrt{40 \times 250}$
$= \sqrt{10\ 000}$
$= 100$
$= PQ^2$

18. a) Find the equation of line AP.
The slope is $\frac{4-(-1)}{-3-2} = -1$.
The equation of the line has the form $y = -x + b$, where b is the y-intercept.
Substitute the coordinates of P(2, –1) to determine b.
$-1 = -(2) + b$
$b = 1$
The equation of AP and AB is $y = -x + 1$.
Find the points of intersection of the line and circle. Substitute for y in the equation of the circle, $x^2 + y^2 = 25$
$x^2 + (-x + 1)^2 = 25$
$2x^2 - 2x - 24 = 0$
$x^2 - x - 12 = 0$
$(x + 3)(x - 4) = 0$
$x = -3$ or $x = 4$
Substitute each value of x in the straight line equation to calculate y.
$y = 4$ or $y = -3$
The points of intersection are A(–3, 4) and B(4, –3).
Find the equation of line CP.
The slope is $\frac{-5-(-1)}{0-2} = 2$.
The equation of the line has the form $y = 2x + b$.
Substitute the coordinates of P(2, –1) to determine b.
$-1 = 2(2) + b$
$b = -5$
The equation of CP and CD is $y = 2x - 5$.
Find the points of intersection of the line and the circle. Substitute for y in the equation of the circle.

Selected Solutions — Chapter 9

$x^2 + (2x - 5)^2 = 25$

$5x^2 - 20x = 0$

$5x(x - 4) = 0$

$x = 0$ or $x = 4$

Substitute each value of x in the straight line equation to calculate y.

$y = -5$ or $y = 3$

The points of intersection are C(0, –5) and D(4, 3).

b) $PA = \sqrt{(2 - (-3))^2 + (-1 - 4)^2}$

$= \sqrt{50}$

$PB = \sqrt{(2 - 4)^2 + (-1 - (-3))^2}$

$= \sqrt{8}$

$PA \times PB = \sqrt{50 \times 8}$

$= \sqrt{400}$

$= 20$

$PC = \sqrt{(2 - 0)^2 + (-1 - (-5))^2}$

$= \sqrt{20}$

$PD = \sqrt{(2 - 4)^2 + (-1 - 3)^2}$

$= \sqrt{20}$

$PC \times PD = \sqrt{20} \times \sqrt{20}$

$= 20$

$= PA \times PB$

19. Use the Sine Law in △AEB to find ∠AEB.

$\frac{\sin E}{35} = \frac{\sin 38^\circ}{23}$

$\sin E = \frac{35 \sin 38^\circ}{23}$

$\doteq 0.936\ 876$

$\angle E \doteq 69.5^\circ$

But $\angle E$ might be $180^\circ - 69.5^\circ = 110.5^\circ$.

Consider each case.

Case 1: When $\angle E = 69.5^\circ$, $\angle A = 180^\circ - 69.5^\circ - 38^\circ$

$= 72.5^\circ$

Then use the Sine Law to calculate EB.

$\frac{EB}{\sin 72.5^\circ} = \frac{23}{\sin 38^\circ}$

$EB = \frac{23 \sin 72.5^\circ}{\sin 38^\circ}$

$EB \doteq 35.63$

Use the Cosine Law in △EBC to calculate EC, or b.

$b^2 = c^2 + e^2 - 2ce \cos B$

$b^2 = 35.63^2 + 18^2 - 2(35.63)(18) \cos 44^\circ$

$b^2 = 670.81$

$b \doteq 25.9$

Case 2: In △AEB, when $\angle E = 110.5^\circ$,

$\angle A = 180^\circ - 110.5^\circ - 38^\circ$

$= 31.5^\circ$

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Then use the Sine Law to calculate EB.

$$\frac{EB}{\sin 31.5^\circ} = \frac{23}{\sin 38^\circ}$$

$$EB = \frac{23 \sin 31.5^\circ}{\sin 38^\circ}$$

$$EB \doteq 19.52$$

Use the Cosine Law in $\triangle EBC$ to calculate EC.

$$b^2 = c^2 + e^2 - 2ce \cos B$$

$$= 19.52^2 + 18^2 - 2(19.52)(18) \cos 44^\circ$$

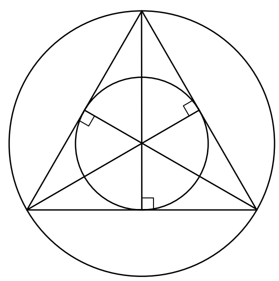
$$b^2 \doteq 199.53$$

$$b \doteq 14.1$$

To the nearest metre, EC is 14 m or 26 m.

Linking Ideas: Mathematics & Art
Crop Circles, page 535

1.–3.



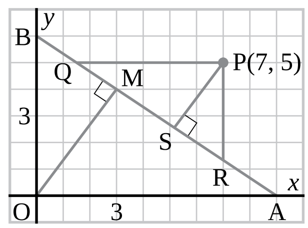
4. Explanations may vary. Drive a stake into the ground. Attach a 50 m string to the dowel, and hold the string taut. Walk around to form the outer circle. Move the stake to the circumference. Mark a point on the circumference that is 50 m from the stake in a straight line. Move to the other side of the stake. Mark a point on the circumference that is 50 m in a straight line in the opposite direction. These two points are two vertices of the equilateral triangle. Move the stake to one vertex, mark a point on the circumference 50 m from the stake. Move the stake to this marked point, then mark a point on the circumference, 50 m farther. This is the third vertex. Move in a straight line between two vertices, then the next two, and the last two to form the equilateral triangle. The radius of the inner circle is 25 m. Fold the string in half, move the stake to the centre of the first circle, and walk around to form the inner circle.

Investigate, page 536

1. b) The shortest distance from a point to a line is measured along the perpendicular from the point to the line. The distance from any other point on the line to the origin is greater than the length of OM.

Selected Solutions — Chapter 9

2.



3. To find OM.

Since OA = 9 and OB = 6, I used trigonometry in △OAB.

$\tan A = \frac{6}{9}$, then $\angle A \doteq 33.7°$

In △OMA, I know ∠A and the length of OA, so I used trigonometry to find OM.

$\sin A = \frac{OM}{9}$, or $OM \doteq 9 \sin 33.7°$, to give $OM \doteq 4.99$

To find PS.

Since PQ and AO are horizontal, the lines are parallel, and $\angle PQR = \angle QAO \doteq 33.7°$.

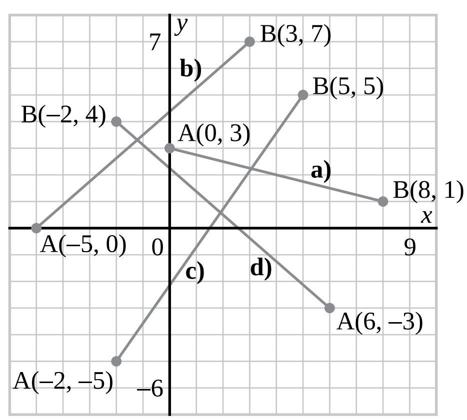
To find the length of PQ, I calculated the x-coordinate of Q by substituting $y = 5$ in $2x + 3y = 18$ to get $x = \frac{3}{2}$.

Then $PQ = 7 - \frac{3}{2}$, or $\frac{11}{2}$.

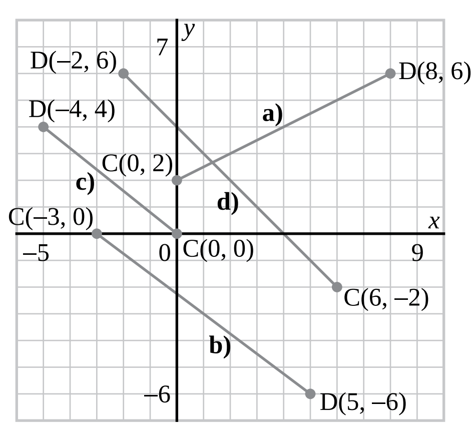
In △PQS, $\sin Q = \frac{PS}{PQ}$ or $\sin 33.7° \doteq \frac{PS}{5.5}$, which gives $PS \doteq 3.05$

9.3 Exercises, page 540

3.

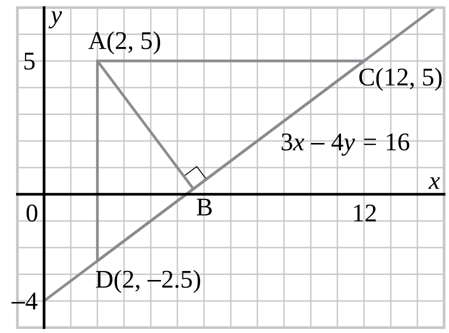


4.



Selected Solutions — Chapter 9

5. b) Explanations may vary. For part i:



I graphed the line and the point A. The slope of the line is $\frac{3}{4}$, and the line passes through the point (0, –4). Then I constructed a perpendicular from A to intersect the line at B. I need to find the length of AB.

I constructed AC and AD as shown in the diagram. Then I found the coordinates of C and D using the equation of the line.

When $x = 2$, $3(2) - 4y = 16$, or $y = -2.5$

When $y = 5$, $3x - 4(5) = 16$, or $x = 12$

Then I found the lengths of AC and AD.

AD = 5 + 2.5, or 7.5, AC = 12 – 2, or 10

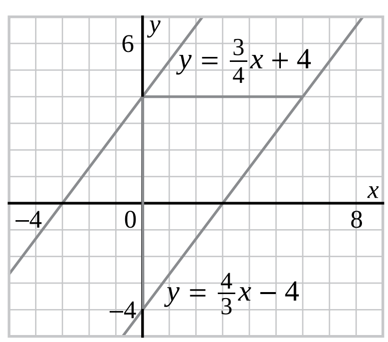
Then I used trigonometry in △ADC to find the measure of ∠DCA.

$$\tan \angle DCA = \frac{AD}{AC} = 0.75$$
$$\angle DCA \doteq 36.87^\circ$$

Then I used trigonometry in △ABC to find the length of AB.

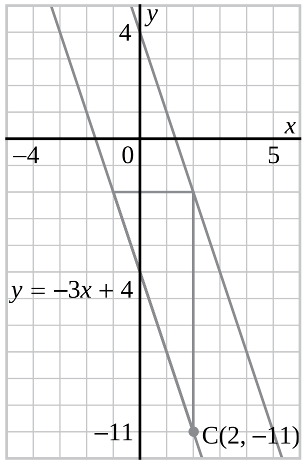
$$\sin \angle DCA = \frac{AB}{10}$$
$$AB = 10 \sin \angle DCA$$
$$\doteq 10 \sin 36.87^\circ$$
$$\doteq 6$$

6.



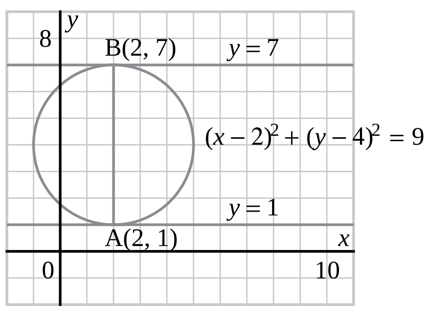
Selected Solutions — Chapter 9

7.



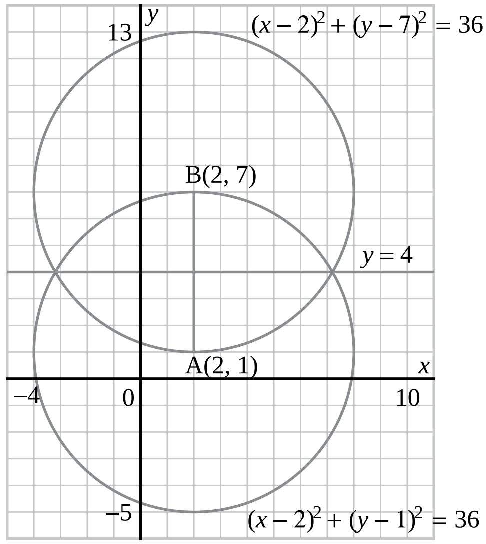
10. Explanations may vary. For part b:
From the diagram, A has coordinates (2, 1) and B has coordinates (8, 6). To divide the line into 3 parts, the vertical distance and the horizontal distance between A and B are divided into 3 parts. Between A and B, the vertical distance is 6 – 1 = 5. Divide the distance into 3 parts. Each part is $\frac{5}{3}$. Between A and B, the horizontal distance is 8 – 2 = 6. Divide the distance into 3 parts. Each part is $\frac{6}{3} = 2$. The coordinates of the points are $\left(2 + 2,\ 1 + \frac{5}{3}\right)$ and $\left(8 - 2,\ 6 - \frac{5}{3}\right)$, or $\left(4,\ \frac{8}{3}\right)$ and $\left(6,\ \frac{13}{3}\right)$.

11.



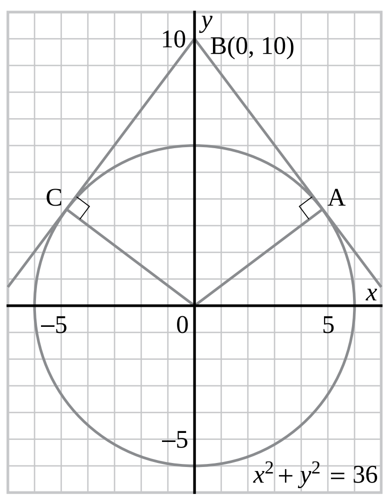
a) C can be anywhere on the two horizontal lines or the circle, except points A and B.

12.



a) C can be anywhere on the horizontal line except the point of intersection of the two lines, anywhere on the top circle except point A and the diametrically opposite point, or anywhere on the bottom circle except point B and the diametrically opposite point.

13.



Plot the point B(0, 10). Draw the circle $x^2 + y^2 = 36$, to represent all points that are 6 units from the origin. Draw 2 tangents from (0, 10) to the circle. Draw radii to the points of tangency. From the Pythagorean Theorem in △OAB (or △OAC), the distance from B to the points of tangency A and C is 8 units.

Let the slope of the tangent be m. Then the equation of the tangent is $y = mx + 10$.

Substitute for y in the equation of the circle, $x^2 + y^2 = 36$.

$$x^2 + (mx + 10)^2 = 36$$

$$x^2 + m^2x^2 + 20mx + 100 - 36 = 0$$

$$(1 + m^2)x^2 + 20mx + 64 = 0$$

For the line to be a tangent to the circle, the above equation must have equal roots; that is, $b^2 - 4ac = 0$

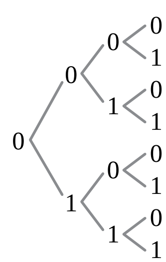
$$(20m)^2 - 4(1 + m^2)64 = 0$$
$$400m^2 - 256 - 256m^2 = 0$$
$$144m^2 = 256$$
$$m^2 = \frac{256}{144}$$
$$m = \pm\frac{16}{12}$$
$$m = \pm\frac{4}{3}$$

When $m = \frac{4}{3}$, AC is the tangent and its equation is $y = \frac{4}{3}x + 10$.
When $m = -\frac{4}{3}$, AB is the tangent and its equation is $y = -\frac{4}{3}x + 10$.
But AB is perpendicular to OB, so its slope is $-\frac{4}{3}$.
The equation of AB is $4x + 3y - 30 = 0$.

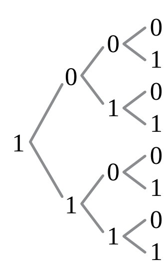
14. The three equations in exercise 11 are $y = 7$, $y = 1$, and $(x - 2)^2 + (y - 4)^2 = 9$.
Write each equation with its right side zero.
$y - 7 = 0$, $y - 1 = 0$, and $(x - 2)^2 + (y - 4)^2 - 9 = 0$
Hence, a single equation that represents all possible positions of C is
$(y - 7)(y - 1)((x - 2)^2 + (y - 4)^2 - 9) = 0$
Any point on any of the 3 lines in exercise 11 has coordinates that make one of the 3 factors above equal to 0.

Mathematical Modelling: What Comes after the Cube?, page 542

1.



(0, 0, 0, 0)
(0, 0, 0, 1)
(0, 0, 1, 0)
(0, 0, 1, 1)
(0, 1, 0, 0)
(0, 1, 0, 1)
(0, 1, 1, 0)
(0, 1, 1, 1)



(1, 0, 0, 0)
(1, 0, 0, 1)
(1, 0, 1, 0)
(1, 0, 1, 1)
(1, 1, 0, 0)
(1, 1, 0, 1)
(1, 1, 1, 0)
(1, 1, 1, 1)

2. Each coordinate quadruplet in exercise 1 is a "vertex" of the hypercube, so the hypercube has 16 vertices.

3. a) The length of the hypotenuse of the red triangle is $\sqrt{1^2 + 1^2} = \sqrt{2}$. Thus, the length of the hypotenuse of the yellow triangle (which is the length of the main diagonal of the unit cube) is $\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$.

b) By analogy with the results of part a, the length of the main diagonal of the unit hypercube is $\sqrt{1^2 + 1^2 + 1^2 + 1^2} = \sqrt{4} = 2$.

Selected Solutions — Chapter 9

4. a) Each vertex has 4 edges attached to it, and there are 16 vertices, so at first thought one might say that there are 64 edges. However, note that in this calculation, each edge has been counted twice, since each edge joins two vertices. Thus, there are 32 edges.

b), c)

Figure	point	segment	square	cube	hypercube	5-cube	6-cube
Dimensions	0	1	2	3	4	5	6
Number of vertices	1	2	4	8	16	32	64
Number of edges	0	1	4	12	32	80	192

The number of vertices is 2^n, where n is the dimensionality of the figure. Thus, the number of vertices increases by a factor of 2 across the second row of the table. For the last row, use the same reasoning as in part a. Thus, the number of edges is $\frac{n \times 2^n}{2} = n2^{n-1}$. Thus, to get from one number to the next in the last row of the table (starting from 1), divide by the current dimension and then multiply by the next dimension and also multiply by 2.

5. Each model contains two cubes. In one model, one cube is inside another. In the other model, one cube "slides" to combine with the other cube. In both models, there are line segments joining corresponding vertices.

6. a) Visualize folding the net to form the cube. A cube has 12 edges. In the net, 5 edges have already been joined. That leaves 7 pairs of edges to be joined as the net is folded.

b) Count the number of exposed faces of cubes and divide by 2 (just as with the net of the cube there are 14 exposed edges, leading to 7 pairs of edges joined together). There are $(5 + 5 + 5 + 5 + 5 + 4 + 5)$ exposed faces, which is 34. Then, $\frac{34}{2} = 17$.

Investigate, page 545

1. b) The slopes of opposite sides are equal, so opposite sides are parallel.

2. a) The lengths of opposite sides are equal.

b) The lengths of opposite sides are equal.

c) The lengths of opposite sides of a parallelogram are equal.

3. a) The midpoints coincide.

b) The midpoints coincide.

c) The diagonals of a parallelogram bisect each other.

Selected Solutions — Chapter 9

9.4 Exercises, page 547

1. a) $AB^2 = (-1-1)^2 + (3-7)^2$
$= 20$
$BC^2 = (1-5)^2 + (7-5)^2$
$= 20$
$AC^2 = (-1-5)^2 + (3-5)^2$
$= 40$
Thus, $AC^2 = AB^2 + BC^2$, and $\triangle ABC$ is a right triangle by the Pythagorean Theorem.

b) $\triangle ABC$ is an isosceles triangle because AB = BC, as was shown in part a.

c) M has coordinates (0, 5) and N has coordinates (2, 4).
slope MN $= -\frac{1}{2}$
slope BC $= -\frac{1}{2}$
Thus, MN is parallel to BC because they have the same slope.

2. a) Slope AB $= \frac{2}{3}$, slope CD $= \frac{2}{3}$
Hence, AB is parallel to CD, since they have the same slope.

b) $BC = \sqrt{(-2-5)^2 + (1-(-6))^2}$
$= \sqrt{98}$
$DA = \sqrt{(2-(-5))^2 + (-8-(-1))^2}$
$= \sqrt{98}$
Thus, BC = DA

3. a) $PQ = \sqrt{(-3+1)^2 + (4+2)^2}$, or $2\sqrt{10}$
$PR = \sqrt{(-3-3)^2 + (4-2)^2}$, or $2\sqrt{10}$
Thus, $\triangle PQR$ is isosceles because PQ = PR.

b) M has coordinates (–2, 1), and N has coordinates (0, 3).
$RM = \sqrt{3+2)^2 + (2-1)^2} = \sqrt{26}$
$QN = \sqrt{(-1-0)^2 + (-2-3)^2} = \sqrt{26}$
Thus, RM = QN

4. The coordinates of D are (2, 5) and the coordinates of E are (4.5, 2.5).
$DE = \sqrt{(2-4.5)^2 + (5-2.5)^2}$
$= \sqrt{12.5}$
$AC = \sqrt{(-1-4)^2 + (3-(-2))^2}$
$= \sqrt{50}$
$= 2\sqrt{12.5}$
Thus, $DE = \frac{1}{2}AC$

5. d) Yes. The centre of the circle is (3, –7), which is on the line $x = 3$.

Selected Solutions — Chapter 9

6. a) For AB to be a chord, A and B must lie on the circle.
Substitute the coordinates of each point in the circle equation.
For A: L.S. $= 6^2 + 2^2$
$= 40$
$=$ R.S.
For B: L.S. $= 2^2 + (-6)^2$
$= 40$
$=$ R.S.

c) The centre of the circle is (0, 0), and $y = -\frac{1}{2}x$ passes through (0, 0).

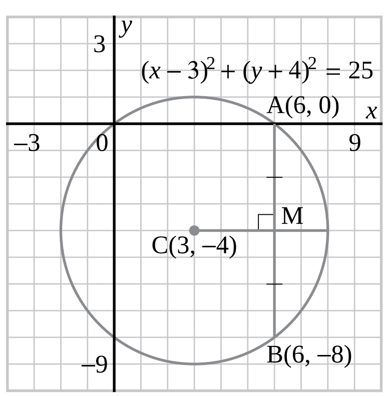
7. a) For MN to be a chord, M and N must lie on the circle.
Substitute the coordinates of each point in the circle equation.
For M: L.S. $= 8^2 + 6^2$
$= 100$
$=$ R.S.
For N: L.S. $= (-6)^2 + 8^2$
$= 100$
$=$ R.S.

c) The slope of MN $= -\frac{1}{7}$, and the slope of $y = 7x$ is 7. Thus, the line is perpendicular to the chord.

8. a) For PQ to be a chord, P and Q must lie on the circle.
Substitute the coordinates of each point in the circle equation.
For P: L.S. $= 2^2 + 5^2$
$= 29$
$=$ R.S.
For Q: L.S. $= 5^2 + (-2)^2$
$= 29$
$=$ R.S.

c) The midpoint of PQ is $\left(\frac{7}{2}, \frac{3}{2}\right)$.
To verify that this point is on the line $y = \frac{3}{7}x$, substitute the coordinates of the midpoint in the equation.
L.S. $= \frac{3}{2}$
R.S. $= \frac{3}{7}\left(\frac{7}{2}\right)$
$= \frac{3}{2}$
$=$ L.S.

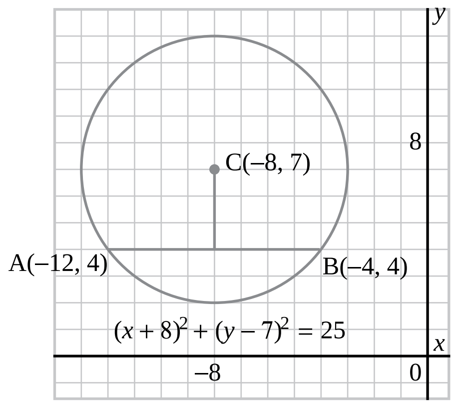
9.



Selected Solutions — Chapter 9

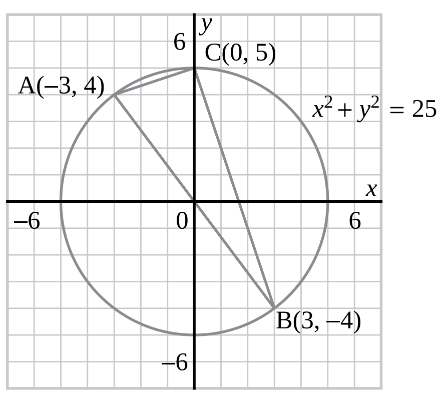
Chords may vary. Draw the circle $(x - 3)^2 + (y + 4)^2 = 25$ and a chord. From the diagram, choose chord with endpoints A(6, 0) and B(6, –8). Find the midpoint of the chord M(6, –4). Connect the centre of the circle and the midpoint of the chord. The slope of CM is 0. The slope of the chord is undefined. The line connecting the centre to the midpoint is perpendicular to the chord.

10.



Chords may vary. Draw the circle $(x + 8)^2 + (y - 7)^2 = 25$. Choose any chord, such as AB with coordinates A(–12, 4) and B(–4, 4). The perpendicular from the centre of the circle to the chord has equation $x = -8$. The midpoint of the chord has coordinates M(–8, 4). Since the midpoint lies on the line $x = -8$, the perpendicular to the chord bisects the chord.

11.



Diameters and chords may vary. Draw the circle $x^2 + y^2 = 25$. Choose any diameter, such as AB, with coordinates A(–3, 4) and B(3, –4). Choose any point on the circumference, such as C(0, 5).

$AC^2 = (-3 - 0)^2 + (4 - 5)^2$, or 10

$BC^2 = (3 - 0)^2 + (-4 - 5)^2$, or 90

$AB^2 = (-3 - 3)^2 + (4 - (-4))^2$, or 100

Thus, $AC^2 + BC^2 = AB^2$. Hence, by the Pythagorean Theorem, $\angle ACB = 90^\circ$.

Selected Solutions — Chapter 9

12. d) $A = \sqrt{\left(\frac{13+\sqrt{41}}{2}\right)\left(\frac{13+\sqrt{41}}{2} - \sqrt{41}\right)\left(\frac{13+\sqrt{41}}{2} - 5\right)\left(\frac{13+\sqrt{41}}{2} - 8\right)}$

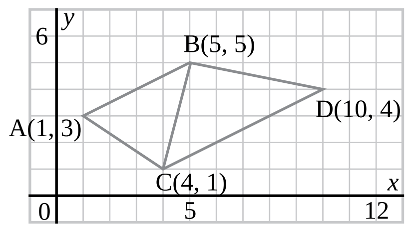
$= \sqrt{\left(\frac{13+\sqrt{41}}{2}\right)\left(\frac{13-\sqrt{41}}{2}\right)\left(\frac{3+\sqrt{41}}{2}\right)\left(\frac{-3+\sqrt{41}}{2}\right)}$

$= \sqrt{\left(\frac{169-41}{4}\right)\left(\frac{-9+41}{4}\right)}$

$= \sqrt{(32)(8)}$

$= 16$

13. a)



$\text{slope AB} = \frac{5-3}{5-1}$

$= \frac{1}{2}$

$\text{slope CD} = \frac{4-1}{10-4}$

$= \frac{1}{2}$

Thus, AB is parallel to CD because slope AB = slope CD.

d) $\cos \angle BCD = \frac{BC^2 + CD^2 - BD^2}{2(BC)(CD)}$

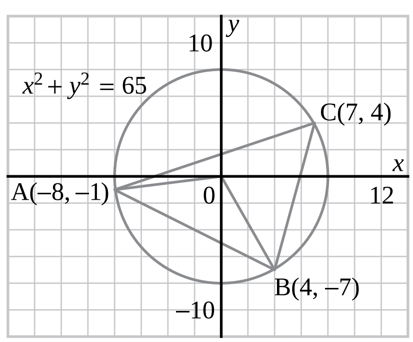
$= \frac{17 + 45 - 26}{2(\sqrt{17})(3\sqrt{5})}$

$= \frac{6}{\sqrt{85}}$

$\angle BCD \doteq 49.4^\circ$

Hence, $\angle ABC = \angle BCD$

14. a)



Substitute the coordinates of each point in the equation $x^2 + y^2 = 65$.

A(–8, –1): $\text{L.S.} = (-8)^2 + (-1)^2$

$= 64 + 1$

$= 65$

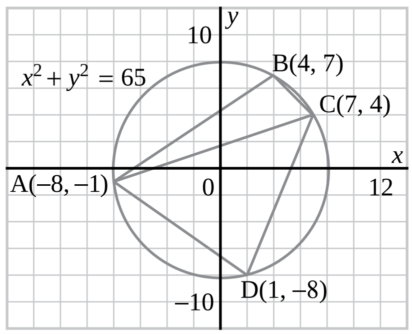
$= \text{R.S.}$

$B(4, -7)$: L.S. $= 4^2 + (-7)^2$
$= 16 + 49$
$= 65$
$=$ R.S.

$C(7, 4)$: L.S. $= 7^2 + 4^2$
$= 49 + 16$
$= 65$
$=$ R.S.

e) From parts c and d, $\angle AOB = 2\angle ACB$

15.



Draw $x^2 + y^2 = 65$. Use the points from exercise 14, and one more; point D is chosen by symmetry: A(–8, –1), B(4, 7), C(7, 4), and D(1, –8). All the points are on the circle. Join AC. Calculate the length of each side in $\triangle ABC$.

$$AB = \sqrt{(-8-4)^2 + (-1-7)^2} = 4\sqrt{13}$$

$$BC = \sqrt{(4-7)^2 + (7-4)^2} = 3\sqrt{2}$$

$$AC = \sqrt{(-8-7)^2 + (-1-4)^2} = 5\sqrt{10}$$

Use the cosine formula in $\triangle ABC$ to calculate the measure of $\angle B$.

$$\cos\angle ABC = \frac{AB^2 + BC^2 - AC^2}{2(AB)(BC)} = \frac{208 + 18 - 250}{2(4\sqrt{13})(3\sqrt{2})} = -\frac{1}{\sqrt{26}}$$

$\angle ABC \doteq 101.3°$

Calculate the lengths of the other two sides in $\triangle ADC$.

$$AD = \sqrt{(-8-1)^2 + (-1+8)^2} = \sqrt{130}$$

$$CD = \sqrt{(1-7)^2 + (-8-4)^2} = 6\sqrt{5}$$

Use the cosine formula in $\triangle ADC$ to calculate the measure of $\angle D$.

$$\cos\angle ADC = \frac{AD^2 + DC^2 - AC^2}{2(AD)(DC)}$$
$$= \frac{130 + 180 - 250}{2(\sqrt{130})(6\sqrt{5})}$$
$$= \frac{1}{\sqrt{26}}$$

$\angle ADC \doteq 78.7^\circ$

$\angle ABC + \angle ADC = 180^\circ$

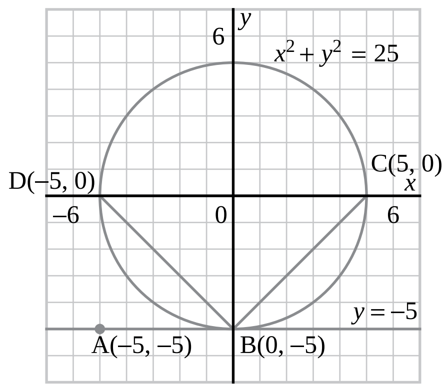
Since the sum of the angles in a quadrilateral is 360°, then

$\angle ABC + \angle ADC + \angle DAB + \angle DCB = 360^\circ$

Therefore, $\angle DAB + \angle DCB = 180^\circ$

Thus, the opposite angles of a cyclic quadrilateral are supplementary.

16.



Mark points A(–5, –5), B(0, –5), C(5, 0), and D(–5, 0).

Show that $\angle DBA = \angle DCB$.

In right △ADB,

$$DB = \sqrt{5^2 + 5^2}$$
$$= 5\sqrt{2}$$

AB = 5

AD = 5

$$\tan\angle DBA = \frac{5}{5}$$
$$= 1$$

$\angle DBA = 45^\circ$

In △DBC,

CD = 10

$$CB = \sqrt{5^2 + 5^2}$$
$$= 5\sqrt{2}$$

$$\cos\angle DCB = \frac{BC^2 + CD^2 - BD^2}{2(BC)(CD)}$$
$$= \frac{50 + 100 - 50}{2(5\sqrt{2})(10)}$$
$$= \frac{1}{\sqrt{2}}$$

$\angle DCB = 45^\circ$

Thus, $\angle DBA = \angle DCB$

The angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord.

Selected Solutions — Chapter 9

9.5 Exercises, page 553

1. The diagonals are AC and BD. To prove that the diagonals are perpendicular, prove that their slopes are negative reciprocals.

$$\begin{aligned}\text{slope AC} &= \frac{a}{a}\\ &= 1\end{aligned}$$

$$\begin{aligned}\text{slope BD} &= \frac{0-a}{a-0}\\ &= -\frac{a}{a}\\ &= -1\end{aligned}$$

The slopes are negative reciprocals, so the diagonals are perpendicular.

2. The coordinates of M are (a, b). Prove that MA = MB = MC.

$$\begin{aligned}MC^2 &= a^2 + b^2\\ MA^2 &= (2a - a)^2 + (0 - b)^2\\ &= a^2 + b^2\\ MB^2 &= (a - 0)^2 + (b - 2b)^2\\ &= a^2 + b^2\\ MA^2 &= MB^2 = MC^2\end{aligned}$$

Thus, MA = MB = MC

3. Place the midpoint of AB at the origin of a coordinate grid. Thus, AB lies along the x-axis, and the perpendicular bisector is the y-axis. Let the coordinates of P be $(0, p)$. Point A is the same distance from the origin as B, so let their coordinates be $(-a,\ 0)$ and $(a, 0)$ respectively.

$$\begin{aligned}PA &= \sqrt{(0-(-a))^2 + (p-0)^2}\\ &= \sqrt{a^2 + p^2}\\ PB &= \sqrt{(0-a)^2 + (p-0)^2}\\ &= \sqrt{a^2 + p^2}\end{aligned}$$

Thus, PA = PB

4. a) Place A at the origin of a coordinate grid. Let the coordinates of B be $(b, 0)$. Let the coordinates of D be (m, n). Since DC is parallel to AB, C has the same y-coordinate as D. Since DC = AB, the x-coordinate of C is $m + b$. Thus, the coordinates of C are $(m + b,\ n)$.

$$\begin{aligned}AD &= \sqrt{m^2 + n^2}\\ BC &= \sqrt{(b-(m+b))^2 + (0-n)^2}\\ &= \sqrt{m^2 + n^2}\end{aligned}$$

Thus, AD = BC

To show that AD is parallel to BC, show that the two line segments have the same slope.

$$\text{slope AD} = \frac{n-0}{m-0} = \frac{n}{m}$$

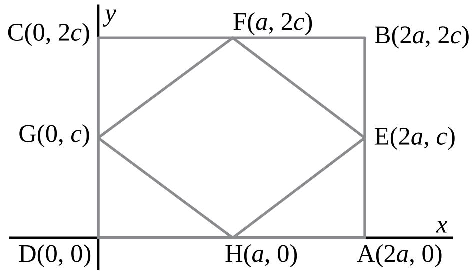
$$\text{slope BC} = \frac{n-0}{m+b-b} = \frac{n}{m}$$

Thus, AD and BC have the same slope, so they are parallel.

Selected Solutions — Chapter 9

b) A quadrilateral in which two sides are parallel and equal is a parallelogram.

5.



Draw and label rectangle ABCD as shown. Let E, F, G, and H be the midpoints of AB, BC, CD, and DA respectively. The coordinates of the midpoints are E$(2a, c)$, F$(a, 2c)$, G$(0, c)$, and H$(a, 0)$. Show that EF = FG = GH = HE.

$\text{EF} = \sqrt{a^2 + c^2}$

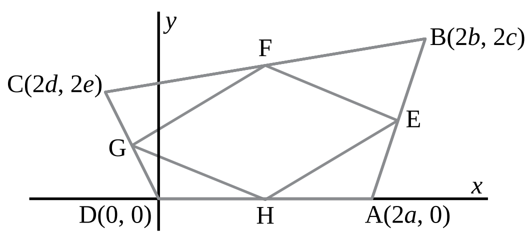
$\text{FG} = \sqrt{a^2 + c^2}$

$\text{GH} = \sqrt{a^2 + c^2}$

$\text{HE} = \sqrt{a^2 + c^2}$

Thus, EFGH is a rhombus because EF = FG = GH = HE.

6.



Draw and label ABCD as shown. Let E, F, G, and H be the midpoints of AB, BC, CD, and DA respectively. The coordinates of the midpoints are E$(a + b, c)$, F$(b + d, c + e)$, G(d, e), and H$(a, 0)$. Show that EF is parallel to HG and GF is parallel to HE.

$\text{slope EF} = \dfrac{c + e - c}{b + d - (a + b)} = \dfrac{e}{d - a}$

$\text{slope HG} = \dfrac{e - 0}{d - a} = \dfrac{e}{d - a}$

Thus, EF is parallel to HG, because slope EF = slope HG.

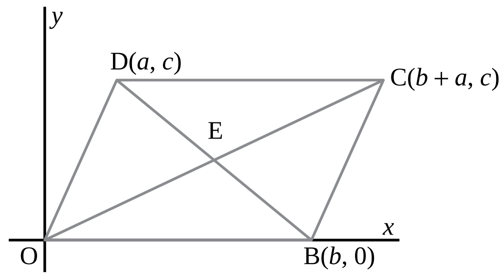
$\text{slope GF} = \dfrac{c + e - e}{b + d - d} = \dfrac{c}{b}$

$\text{slope HE} = \dfrac{c - 0}{a + b - a} = \dfrac{c}{b}$

Thus, GF is parallel to HE, because slope GF = slope HE.
Hence, EFGH is a parallelogram.

Selected Solutions — Chapter 9

7.



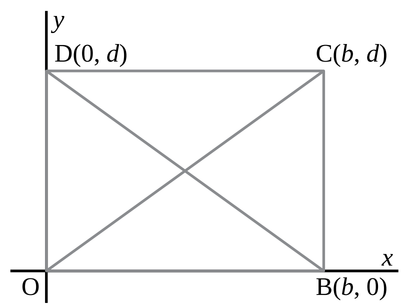
Let the coordinates of D be (a, c), and the coordinates of B be $(b, 0)$.
Then, since OBCD is a parallelogram, the coordinates of C are $(b + a, c)$.
Show that the midpoints of DB and OC are the same point.

Midpoint of DB is $\left(\frac{a+b}{2}, \frac{c+0}{2}\right)$ or $\left(\frac{a+b}{2}, \frac{c}{2}\right)$

Midpoint of OC is $\left(\frac{b+a}{2}, \frac{c}{2}\right)$.

Since the midpoints are the same, the diagonals bisect each other.

8.



Draw and label rectangle OBCD as shown.
Prove that OC = BD.

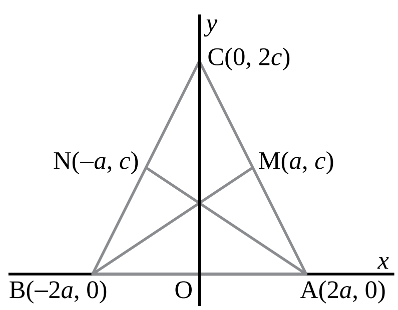
$OC = \sqrt{b^2 + d^2}$

$BD = \sqrt{(b - 0)^2 + (0 - d)^2}$

$= \sqrt{b^2 + d^2}$

Thus, OC = BD

9.



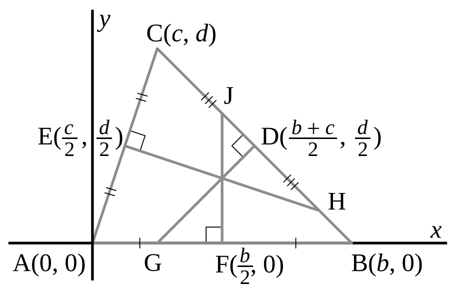
Draw and label △ABC as shown. Let M and N be the midpoints of AC and BC respectively. Show that AN = BM.

$$\text{AN} = \sqrt{(2a + a)^2 + (0 - c)^2}$$
$$= \sqrt{9a^2 + c^2}$$
$$\text{BM} = \sqrt{(-2a - a)^2 + (0 - c)^2}$$
$$= \sqrt{9a^2 + c^2}$$

Thus, AN = BM

10. d) BN and CM intersect on the y-axis. AO coincides with the y-axis, and is the altitude from A. Thus, the three altitudes intersect at a common point. They are concurrent.

11.



Draw and label △ABC as shown. The midpoints of the sides have the coordinates shown. Find the equations of the perpendicular bisectors DG, EH, and FJ.

DB: $\text{slope} = -\dfrac{1}{\text{slope BC}}$

$$= -\frac{1}{\left(\frac{d}{c-b}\right)}$$
$$= \frac{b-c}{d}$$

The equation of DG is $y = \dfrac{b-c}{d}x + r$, where r is the y-intercept.

Substitute the coordinates of D to determine r.

$$\frac{d}{2} = \frac{b-c}{d}\left(\frac{b+c}{2}\right) + r$$
$$r = \frac{c^2 + d^2 - b^2}{2d}$$

The equation of DG is $y = \dfrac{2(b-c)x + c^2 + d^2 - b^2}{2d}$.

EH: $\text{slope} = -\dfrac{1}{\text{slope AC}}$

$$= -\frac{1}{\left(\frac{d}{c}\right)}$$
$$= \frac{-c}{d}$$

The equation of EM is $y = \dfrac{-c}{d}x + s$, where s is the y-intercept.

Substitute the coordinates of E to determine s.

$$\frac{d}{2} = \frac{-c}{d}\left(\frac{c}{2}\right) + s$$
$$s = \frac{c^2 + d^2}{2d}$$

The equation of EM is $y = \dfrac{-2cx + c^2 + d^2}{2d}$.

The equation of FJ is $x = \dfrac{b}{2}$.

Find the point of intersection of FJ and EH. The x-coordinate is $\dfrac{b}{2}$.

Substitute $x = \dfrac{b}{2}$ in the equation for EH.

$$y = \frac{-2c\left(\frac{b}{2}\right) + c^2 + d^2}{2d}$$
$$= \frac{-bc + c^2 + d^2}{2d}$$

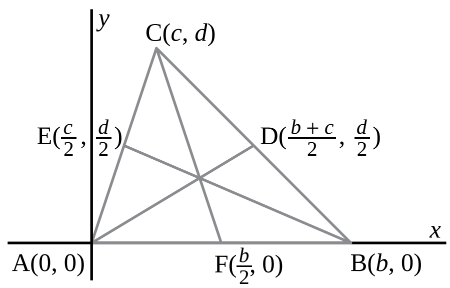
If the point $\left(\frac{b}{2}, \frac{-bc + c^2 + d^2}{2d}\right)$ lies on DG, the perpendicular bisectors are concurrent. Substitute these coordinates in the equation for DG.

$$\text{L.S.} = \frac{-bc + c^2 + d^2}{2d}$$

$$\text{R.S.} = \frac{2(b - c)\left(\frac{b}{2}\right) + c^2 + d^2 - b^2}{2d}$$
$$= \frac{-bc + c^2 + d^2}{2d}$$

Since L.S. = R.S., the perpendicular bisectors are concurrent.

12.



Draw and label △ABC as shown. The midpoints of the sides have the coordinates shown. Find the equations of the medians AD, BE, and CF.

AD: slope $= \frac{d}{b + c}$, y-intercept = 0

equation: $y = \frac{dx}{b + c}$

BE: slope $= \frac{\frac{d}{2}}{\frac{c}{2} - b} = \frac{d}{c - 2b}$

The equation of BE is $y = \frac{dx}{c - 2b} + r$, where r is the y-intercept.

Substitute the coordinates of B to determine r.

$$0 = \frac{db}{c - 2b} + r$$
$$r = \frac{-db}{c - 2b}$$

The equation of BE is $y = \frac{dx}{c - 2b} - \frac{db}{c - 2b}$ ①

CF: slope $= \frac{d}{c - \frac{b}{2}}$
$$= \frac{2d}{2c - b}$$

The equation of CF is $y = \frac{2dx}{2c - b} + s$, where s is the y-intercept.

Substitute the coordinates of F to determine s.

$$0 = \frac{2d\left(\frac{b}{2}\right)}{2c - b} + s$$
$$s = \frac{-db}{2c - b}$$

The equation of CF is $y = \frac{2dx}{2c - b} - \frac{db}{2c - b}$. ②

Selected Solutions — Chapter 9

Solve equations ① and ②. Substitute for y from ① into ②.

$$\frac{dx}{c-2b} - \frac{db}{c-2b} = \frac{2dx}{2c-b} - \frac{db}{2c-b}$$
$$dx(2c-b) - db(2c-b) = 2dx(c-2b) - db(c-2b)$$
$$dx(2c-b) - 2dx(c-2b) = 2bcd - b^2d - bcd + 2b^2d$$
$$x(2cd - bd - 2cd + 4bd) = b^2d + bcd$$
$$x(3bd) = bd(b+c)$$
$$x = \frac{b+c}{3}$$

Find y by substituting for x in the equation for BE.

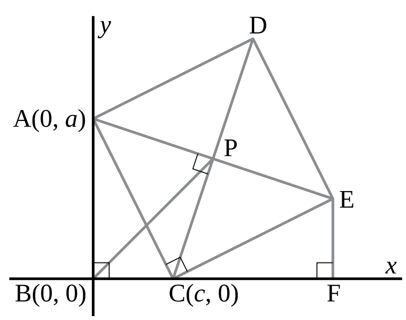
$$y = \frac{d\left(\frac{b+c}{3}\right)}{c-2b} - \frac{db}{c-2b}$$
$$y = \frac{db + dc - 3db}{3(c-2b)}$$
$$y = \frac{dc - 2db}{3(c-2b)}$$
$$y = \frac{d(c-2b)}{3(c-2b)}$$
$$y = \frac{d}{3}$$

If the point $\left(\frac{c+b}{3}, \frac{d}{3}\right)$ lies on AD, the medians are concurrent.
Substitute these coordinates in the equation for AD.

$$\text{L.S.} = \frac{d}{3}$$
$$\text{R.S.} = \frac{d}{b+c}\left(\frac{c+b}{3}\right)$$
$$= \frac{d}{3}$$

Since L.S. = R.S., the medians are concurrent.

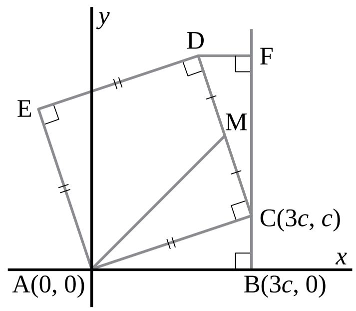
13.



Label the vertices as shown.
Draw the diagonals to intersect at P.
Let the coordinates of A and C be $(0, a)$ and $(c, 0)$ respectively. Draw a perpendicular from E to F on the x-axis.
From the Angles in a Triangle Theorem, $\angle BAC + \angle BCA = 90^\circ$
Since BCF is a straight angle and $\angle ACE = 90^\circ$, then
$\angle BCA + \angle FCE = 90^\circ$
Thus, $\angle BAC = \angle FCE$
But $\angle ABC = \angle CFE = 90^\circ$
Since the sides of a square are equal, AC = CE
Thus, $\triangle ABC \cong \triangle CFE$ (AAS)
Since the triangles are congruent, corresponding sides are equal.
Thus, AB = CF and BC = FE
This gives the coordinates of E as $(a + c, c)$.

Since P is the centre of the square, P is the midpoint of AE. The coordinates of P are thus $\left(\frac{a+c}{2}, \frac{a+c}{2}\right)$. Thus, P is on the line $y = x$, which means that PB makes an angle of 45° with the x-axis. Thus, $\angle PBC = 45°$.

14.

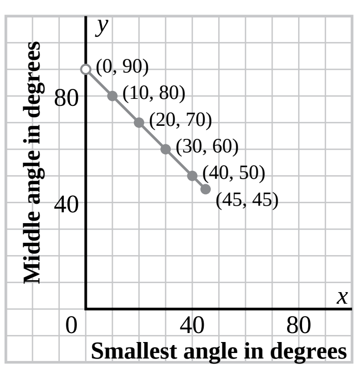


Draw the diagram on a coordinate grid with A at the origin and AB along the x-axis. Extend BC, and draw a perpendicular from D to meet BC extended at F. Let the coordinates of B be $(3c, 0)$. Then the coordinates of C are $(3c, c)$, since AB = 3BC.
Since the sides of a square are equal, AC = CD
$\angle ABC = \angle CFD = 90°$
From the Angles in a Triangle Theorem, $\angle CAB + \angle ACB = 90°$
Since FCB is a straight angle, and $\angle DCA = 90°$,
$\angle ACB + \angle DCF = 90°$
Thus, $\angle CAB = \angle DCF$
Hence, $\triangle CAB \cong \triangle DCF$ (AAS)
Since the triangles are congruent, corresponding sides are equal.
Thus, AB = CF and BC = FD
Therefore, the coordinates of D are $(2c, 4c)$. Thus, the coordinates of M, the midpoint of DC, are $\left(\frac{2c+3c}{2}, \frac{4c+c}{2}\right)$, or $(2.5c, 2.5c)$. Thus, M is on the line $y = x$, which means MA makes an angle of 45° with the x-axis. Thus, $\angle MAB = 45°$.

Problem Solving: Creative Problem Posing, page 555

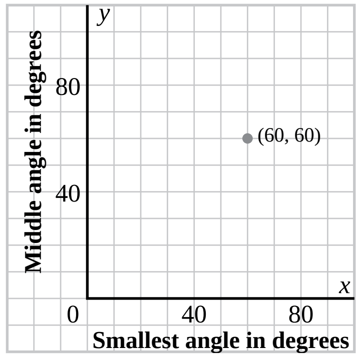
1. b) The ordered pair represents the two smallest angles of the triangle. This works in all cases. If two angles are equal, the point represents an isosceles triangle. In the case of the equilateral triangle, there is only one point, (60, 60).

2. Answers may vary. These points lie on the line $x + y = 90$, $0 < x \le y$.



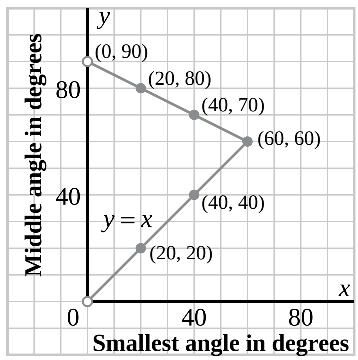
Selected Solutions — Chapter 9

3. a)



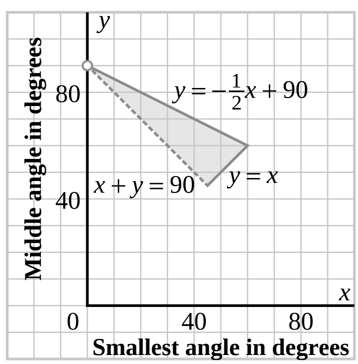
There is only one point, (60, 60).

b)



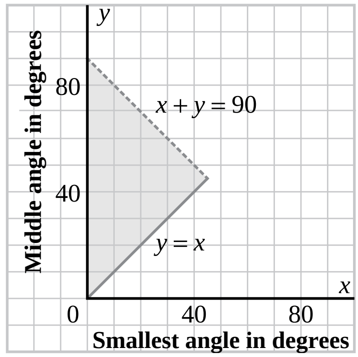
When x and y are the equal angles, the points are on the line $y = x$, for $0 < x \leq 60$.
When x is the vertex and y is one equal angle, the points are on the line $y = 90 - \frac{1}{2}x$ for $0 < x \leq 60$.

c)



The graph is a region bounded by $y = x$, $x + y = 90$ and $y = -\frac{1}{2}x + 90$.

d)

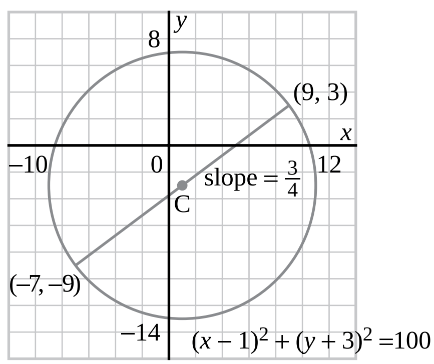


The graph is a region bounded by $x = 0$, $y = x$, and $x + y = 90$.

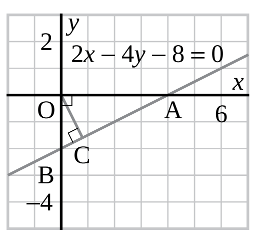
9 Review, page 558

3. Explanations may vary: For part e, I first wrote the standard equation of a circle $(x-h)^2 + (y-k)^2 = r^2$. Then I substituted the given values of h, k, and r, and simplified the equation:
$(x-(-1))^2 + (y-(-3))^2 = (2\sqrt{2})^2$ to get $(x+1)^2 + (y+3)^2 = 8$.

6. a)



8. Explanations may vary. For part c: I graphed the line $2x - 4y - 8 = 0$ using the x-intercept 4 and y-intercept -2.



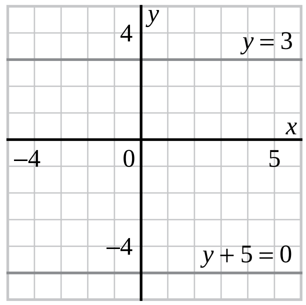
In the diagram, I need to find OC. Since the shortest distance from the origin to the line is a segment that is perpendicular to the line, $\angle OCA = 90^\circ$. I know the lengths of the line segments OA and OB. I used trigonometry in $\triangle OAB$ to find the measure of $\angle OBC$. Then I used trigonometry in triangle $\triangle OBC$ to find OC.

$$\tan \angle OBC = \frac{OA}{OB}$$
$$= 2$$
$$\angle OBC \doteq 63.4^\circ$$

$$\sin \angle OBC = \frac{OC}{2}$$
$$OC \doteq 2 \sin 63.4°$$
$$\doteq 1.79$$

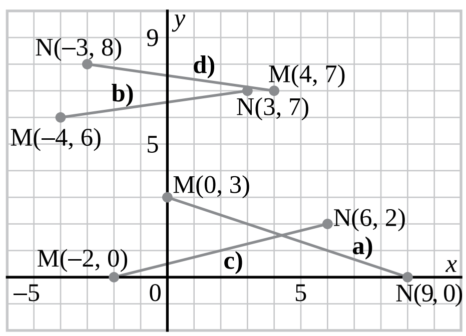
The shortest distance is 1.79 units.

10. Explanations may vary. For part b:



I know that the distance between the lines is measured along a line that is perpendicular to the parallel lines. But such a line is vertical, so I used the *y*-axis. Thus, the distance between the parallel lines is the distance between their *y*-intercepts, which is 8 units.

11.



12. a) The shortest distance from the circle to the line is the difference between the perpendicular distance from the centre of the circle to the line, and the radius of the circle.

13. a) Determine the slope of each side of the triangle.

$$\text{slope AB} = \frac{5 - 1}{1 - (-3)}$$
$$= 1$$
$$\text{slope BC} = \frac{8 - 5}{-2 - 1}$$
$$= -1$$

We do not need to determine the slope of the third side.
Since the slopes of two sides are negative reciprocals, the sides are perpendicular, and the triangle is a right triangle.

Selected Solutions — Chapter 9

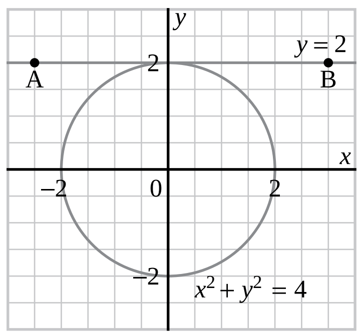
b) Determine the lengths of the sides.

$$\begin{aligned} AB &= \sqrt{(-3-1)^2 + (1-5)^2} \\ &= \sqrt{32} \\ BC &= \sqrt{(1-(-2))^2 + (5-8)^2} \\ &= \sqrt{18} \\ AC &= \sqrt{(-3-(-2))^2 + (1-8)^2} \\ &= \sqrt{50} \end{aligned}$$

Since no two sides are equal, the triangle is not isosceles.

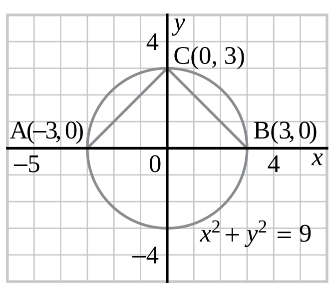
c) Since $AC^2 = 50$ and $AB^2 + BC^2 = 32 + 18$, or 50, then $AC^2 = AB^2 + BC^2$ and the Pythagorean Theorem is verified.

14.



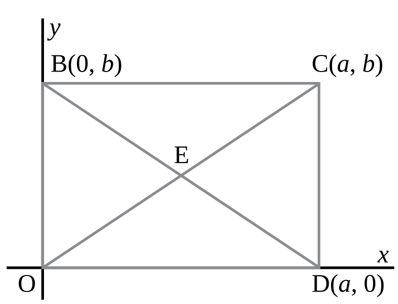
Graph $x^2 + y^2 = 4$ and construct a tangent $y = 2$ as shown.
Slope AB = 0, and slope OC is undefined. Thus, AB is perpendicular to OC.

15.



Graph $x^2 + y^2 = 9$, a diameter AB, and a point C(0, 3) on the circle.
Slope AC = 1, and slope BC = −1. Thus, the slopes are negative reciprocals, and AC is perpendicular to BC. Thus, $\angle ACB = 90^\circ$.

16.



If the diagonals bisect each other, they have the same midpoint.

Midpoint of AB has coordinates $\left(\frac{0+a}{2}, \frac{b+0}{2}\right)$, or $\left(\frac{a}{2}, \frac{b}{2}\right)$.

Midpoint of OC has coordinates $\left(\frac{a+0}{2}, \frac{b+0}{2}\right)$, or $\left(\frac{a}{2}, \frac{b}{2}\right)$.

Since the midpoints are the same, the diagonals bisect each other.

9 Cumulative Review, page 560

5. Explanations may vary. For part a:

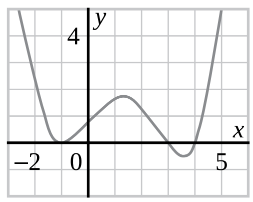
$y = 4x^2 - 9$

I interchanged x and y and then solved the resulting equation for y.

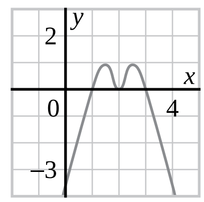
$$x = 4y^2 - 9$$
$$x + 9 = 4y^2$$
$$y = \pm\sqrt{\frac{x+9}{4}}$$

6. Graphs may vary.

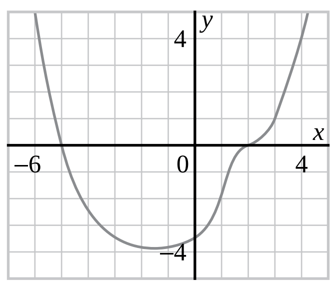
a)



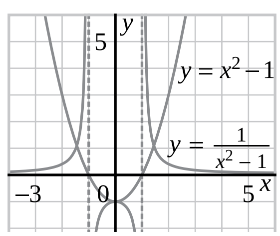
b)



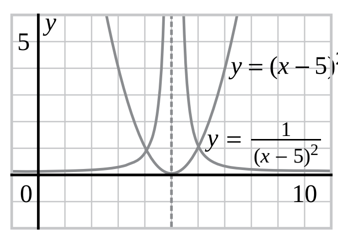
c)



7. **a)**



b)



10. Explanations may vary. For part a:

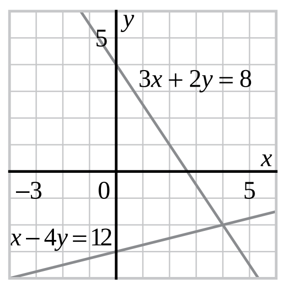
I used the quadratic formula. I substituted $a = 5$, $b = 11$, and $c = -12$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-11 \pm \sqrt{121 - 4(5)(-12)}}{2(5)}$$
$$= \frac{-11 \pm \sqrt{361}}{10}$$
$$= \frac{-11 \pm 19}{10}$$
$$= 0.8 \text{ or } -3$$

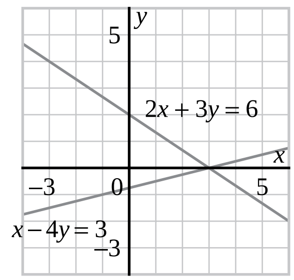
The roots are 0.8 and –3.

Selected Solutions — Chapter 9

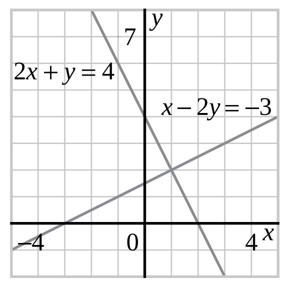
15. a)



b)

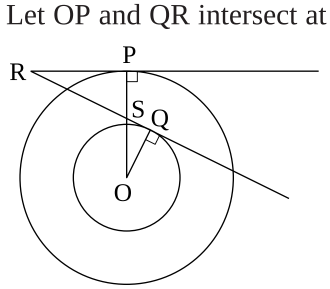


c)



19. In △ODE, since OD and OE are radii,
∠ODE = ∠OED ①
By the Parallel Lines Theorem, alternate angles,
∠FOE = ∠OED ②
Since ∠AED is a straight angle,
∠OED + ∠OEA = 180° ③
In parallelogram ABOD, cointerior angles are supplementary.
∠ODE + ∠BAE = 180° ④
∠ABF + ∠BAE = 180° ⑤
Comparing equations ④ and ⑤,
∠ODE = ∠ABF
Comparing this with equation ①,
∠OED = ∠ABF
Using this and equation ③,
∠ABF + ∠OEA = 180° ⑥
Comparing equations ① and ②,
∠ODE = ∠FOE
Using this with equation ④,
∠FOE + ∠BAE = 180° ⑦
Equations ⑥ and ⑦ show that the opposite angles of quadrilateral BOEA are supplementary.
Therefore, by the converse of the Cyclic Quadrilateral Theorem, quadrilateral BOEA is cyclic.

20. Let OP and QR intersect at point S.



Selected Solutions — Chapter 9

Since P and Q are points of tangency,
$\angle RPS = 90^\circ$ and $\angle OQS = 90^\circ$
In $\triangle RPS$ and $\triangle OQS$,
$\angle RPS = \angle OQS$
$\angle PSR = \angle QSO$ (opposite angles)
Since 2 pairs of corresponding angles are equal, the third pair of angles must be equal.
$\angle PRS = \angle QOS$
Thus, $\triangle RPS \sim \triangle OQS$.
But, $\angle PRS$ is $\angle PRQ$ and $\angle QOS$ is $\angle POQ$.
Therefore, $\angle PRQ = \angle POQ$