Selected Solutions — Chapter 7

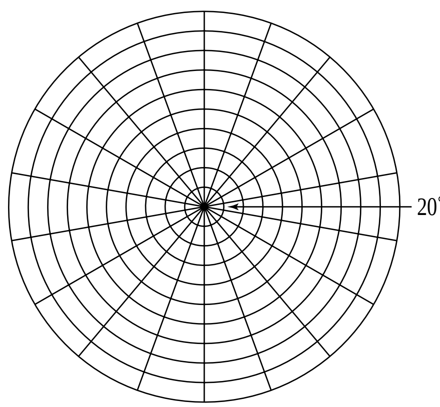
Mathematics File: Conjectures and Proof in Geometry, page 424

- Triangles EBD and ECD are congruent because 2 pairs of sides and the contained angles are equal. Specifically, BD = CD, ∠EDB = ∠EDC, and ED is a common side. Since the triangles are congruent, corresponding sides are equal; that is, EB = EC ①.
- Triangles EAF and EBF are congruent because one side is common and two pairs of corresponding angles are equal. Specifically, EF is common, ∠EFA = ∠EFB, ∠EAF = 90° – ∠ECD, ∠EBF = 90° – ∠EBD, and using ∠ECD = ∠EBD from above, ∠EAF = ∠EBF. The triangles are congruent by AAS. Since the triangles are congruent, EA = EB ②.
  Combining ① and ②, EB = EC = EA. Therefore, the midpoint of AC is equidistant from A, B and C.

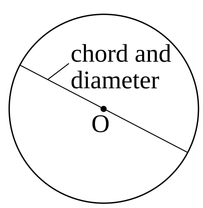
7.1 Exercises, page 428

3. **a)** The disk has 80 concentric circles divided into 18 sectors.

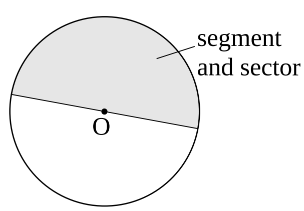
b)



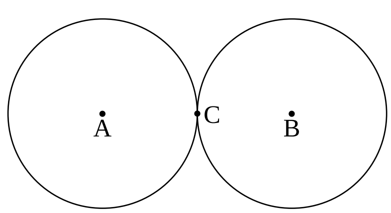
5. **a)**



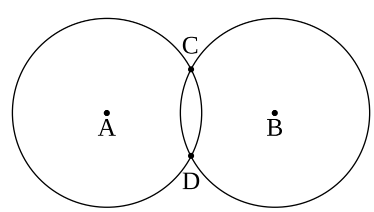
b)



6.

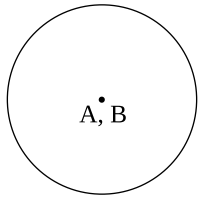


Intersect at one point



Intersect at two points

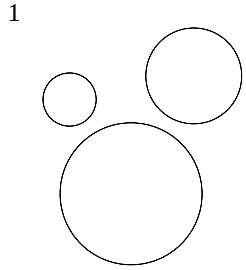
Selected Solutions — Chapter 7



Coincide: intersect at
an infinite number of points

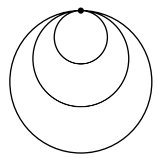
7. c) There are 7 possibilities to consider.

1



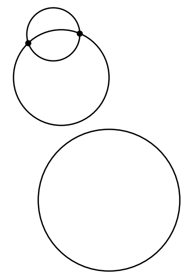
0 points of intersection

2



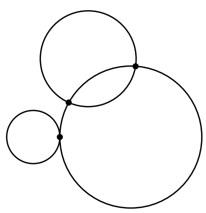
1 point of intersection

3



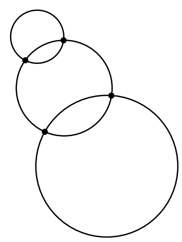
2 points of intersection

4



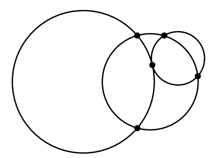
3 points of intersection

5



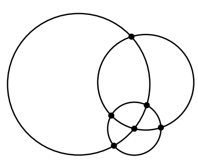
4 points of intersection

6



5 points of intersection

7

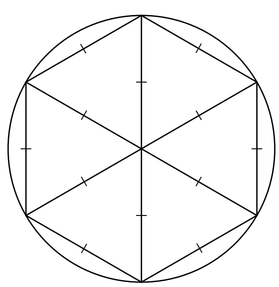


6 points of intersection

8. All diameters pass through the centre of the circle, by definition. The point of intersection is the centre.

10. b) Answers may vary. For part i:

i)



Selected Solutions — Chapter 7

Six triangles were drawn. The sides that were drawn are all radii of the circle; therefore, the six triangles are congruent by the SSS Congruence Theorem. The angle in each triangle at the centre of the circle is $360^\circ \div 6 = 60^\circ$. It was then calculated that the remaining angles must also be 60°; therefore, the triangles are equilateral. Thus, one side of the hexagon is 8.5 cm and the hexagon has perimeter 6×8.5 cm $= 51$ cm.

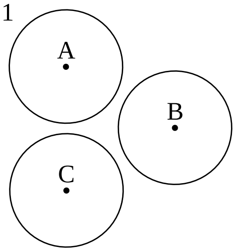
12. b) The length increases until the chord becomes a diameter, then decreases to 0, as the two points coincide.

14. a) Three equal circles intersect at an infinite number of points; when the three circles coincide

b) Three equal circles intersect at 0 points when drawn separately.

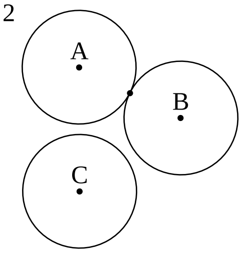
c) There are 8 possibilities to consider.

1



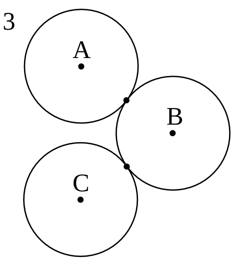
0 points of intersection

2



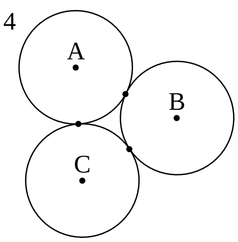
1 point of intersection

3



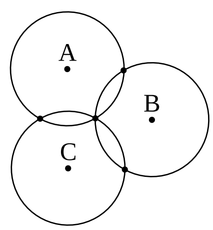
2 points of intersection

4



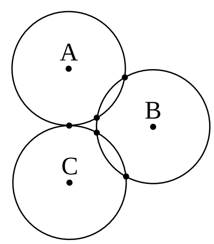
3 points of intersection

5



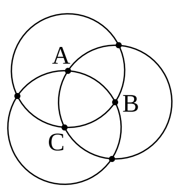
4 points of intersection

6



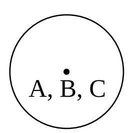
5 points of intersection

7



6 points of intersection

8

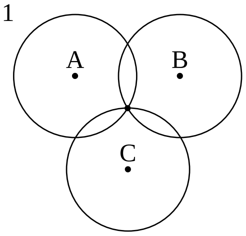


Infinite points of intersection

Selected Solutions — Chapter 7

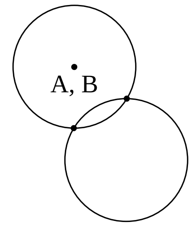
Alternatively, all three circles intersect in fewer situations as illustrate below.

1



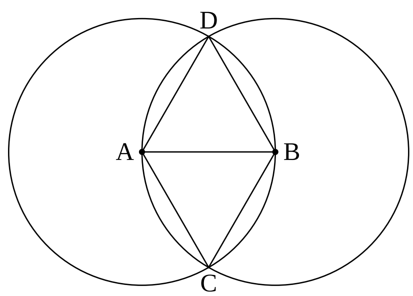
1 point of intersection

2 Two circles coincide.



2 points of intersection

15. a)



AD, AB, and AC are all radii of circle centre A, and thus equal to r. BD, BA, and BC are all radii of circle centre B, and thus equal to r.

In $\triangle ADB$, $AB = BD = DA$

Therefore, $\triangle ADB$ is equilateral.

In $\triangle ACB$, $AB = BC = CA$

Therefore, $\triangle ACB$ is equilateral.

Hence, $\angle CAB = \angle DAB = 60^\circ$

and $\angle CBA = \angle DBA = 60^\circ$

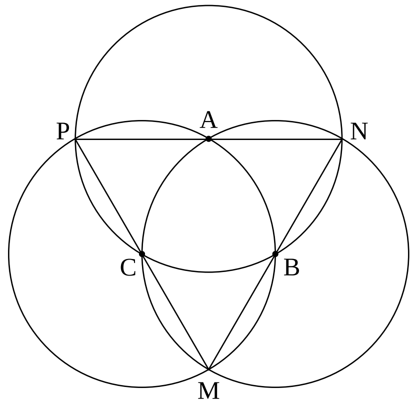
Thus, $\angle DAC = \angle DBC = 120^\circ$

The perimeter P is given by

$$\begin{aligned} P &= 2\left(2\pi r - \frac{120}{360}(2\pi r)\right) \\ &= 2\left(2\pi r - \frac{2}{3}\pi r\right) \\ &= \frac{8\pi r}{3} \end{aligned}$$

Selected Solutions — Chapter 7

b)



Use the method of part a.
The perimeter P is given by
P = 3(circumference of a circle) − (arc PCB + arc PAB + arc NAC + arc NB + arc BM + arc CM).
Recall from part a:
Arc PCB = arc PAB = arc NAC = $\frac{120}{360}(2\pi r)$
Similarly, from part a, ∠BAN = ∠BCM = ∠CBM = 60°
Therefore, arc BM = arc CM = arc NB = $\frac{60}{360}(2\pi r)$

$$P = 3(2\pi r) - \left(\frac{3}{3}(2\pi r) + \frac{3}{6}(2\pi r)\right)$$
$$= 6\pi r - 3\pi r$$
$$P = 3\pi r$$

Linking Ideas: Mathematics & Science
Circles in Space, page 431

1. Earth's circumference is a circle. The orbits of the planets, stars, and satellites are circles. The ring around Saturn is a circle.

2. Gravitational pull, which is equal in all directions, creates spheres and circular orbits.

3. Black holes, which are without light, occur in space.

Investigate, page 432

1. **c)** A line through the centre of a circle that bisects a chord (not a diameter) is perpendicular to the chord.

2. **c)** The perpendicular from the centre of a circle to a chord bisects the chord.

3. **c)** The perpendicular bisector of any chord contains the centre of the circle.

Selected Solutions — Chapter 7

7.2 Exercises, page 435

3. Answers may vary. For exercise 1a, the Pythagorean Theorem was applied, where the base of the triangle is $\frac{x}{2}$. That is, $5^2 = \left(\frac{x}{2}\right)^2 + 3^2$.

This equation was then solved for x, to get $25 = \frac{x^2}{4} + 9$, or $x^2 = 64$ and $x = 8$.

6. **d)** They lie on a semicircle with diameter PQ.

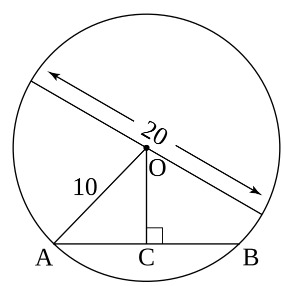
7. **a)** They lie on major arc PQ.

b) The circle would be larger.

c) The circle would be smaller.

9. **b)** Answers may vary.

i)



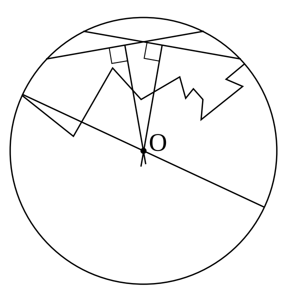
A perpendicular was dropped from O to the chord AB. The midpoint of AB was labelled C and $\triangle OBC$ was formed. Using the Pythagorean Theorem, $OC^2 = OA^2 - AC^2$ with $OA = 10$ and $AC = 8$. Then $OC^2 = 36$, and $OC = 6$. The chord is 6 units from the centre.

10. Draw the perpendicular bisector of each chord. Where the bisectors intersect is the centre of the circle.

11. **a)** Find the centre of the circle using the method of exercise 10. The radius is the distance from the centre to one end of one chord.

b) If the chords are parallel, the method doesn't work, since the perpendicular bisectors coincide.

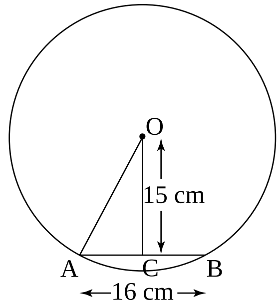
12. **a)**



b) Draw two chords in the diagram, and find the centre of the circle using the method of exercise 10. The radius is the distance from the centre to any part on the arc.

Selected Solutions — Chapter 7

19. Answers may vary. For exercise 18, draw a diagram.

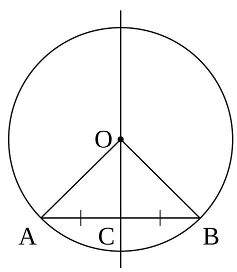


To find OA, the radius, the Pythagorean Theorem was applied; that is, $OA^2 = OC^2 + AC^2$ with OC = 15, AC = 8. OA was calculated to be 17. The diameter is twice the radius. Therefore, the diameter is 34 cm.

24. **c)** If two chords in a circle have the same length, they are equidistant from the centre.

26. **c)** If two chords in a circle have different lengths, they are not equidistant from the centre.

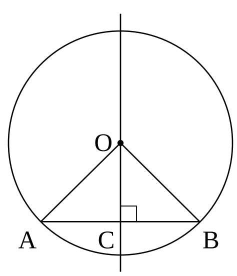
28. **a)**



b) The two triangles OAC and OBC are congruent because the 3 pairs of corresponding sides are equal (SSS). Specifically, OA and OB are radii, AC equals BC, and OC is common to both triangles.

c) Since triangles OAC and OBC are congruent, the corresponding angles are equal; that is, $\angle OCA$ equals $\angle OCB$. In addition, $\angle OCA$ and $\angle OCB$ are supplementary angles; therefore, each angle is 90°. Therefore, OC is perpendicular to AB.

29. **a)**



b) To conclude that two triangles are congruent, we need to know that 3 pairs of corresponding sides are equal or two pairs of corresponding sides and their contained angle are equal.

In this case, we know that OA and OB are radii, OC is common to both triangles and $\angle OCB$ equals $\angle OCA$. This is not sufficient to use the congruent axioms; therefore, we may not conclude that the triangles are congruent in the usual way.

Selected Solutions — Chapter 7

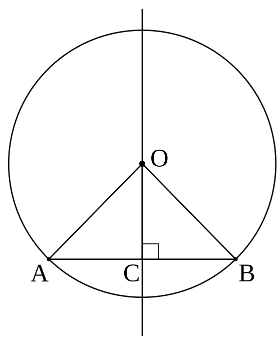
c) Lengths AC and BC may be found using Pythagorean Theorem. That is, $AC^2 = OA^2 - OC^2$ and $BC^2 = OB^2 - OC^2$. Since OA and OB are radii and equal, AC and BC are equal. Therefore, C is the midpoint of AB.

7.3 Exercises, page 444

1. b) In $\triangle OAC$ and $\triangle OBC$, corresponding angles are equal.
OA = OB (radii)
AC = BC
OC is common.
Therefore, $\triangle OAC \cong \triangle OBC$ (SSS)

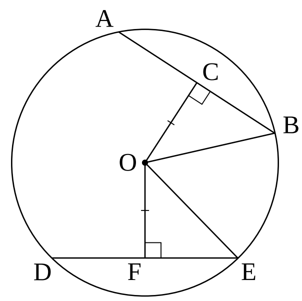
c) Since $\triangle OAC \cong \triangle OBC$,
$\angle OCA = \angle OCB$
But $\angle ACB$ is a straight angle.
Hence, $\angle OCA = \angle OCB = 90^\circ$.

2. Chord AB is drawn in a circle centre O. The perpendicular from O meets AB at C. Join OA and OB. We need to prove that AC = CB.



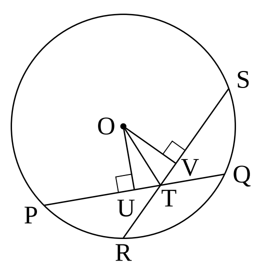
Use the Pythagorean Theorem. Since $\triangle OAC$ and $\triangle OBC$ are right triangles,
$AC^2 = OA^2 - OC^2$
$BC^2 = OB^2 - OC^2$
But OA = OB because they are radii.
$BC^2 = OA^2 - OC^2$
Since expressions for AC^2 and BC^2 are equal, $AC^2 = BC^2$.
AC = BC
Therefore, C is the midpoint of AB.
To prove Corollary 3 of the Chord Perpendicular Bisector Theorem, two right triangles were constructed and compared. Two pairs of sides are equal and using the Pythagorean Theorem, the third sides, AC and BC were proven equal.

3. If two chords of a circle are equidistant from the centre, then the chords are equal. Draw a circle, centre O, with two chords AB and DE, so that the chords are the same distance OC and OF, respectively, from the centre of the circle. Join OB and OE. We want to prove that AB = DE.



Use the Pythagorean Theorem.
In $\triangle OCB$: $CB^2 = OB^2 - OC^2$
In $\triangle OFE$: $FE^2 = OE^2 - OF^2$
But we are given $OC = OF$, and we know $OB = OE$ because they are both radii.
Hence, $CB^2 = FE^2$, or $CB = FE$
From Corollary 3 of the Chord Perpendicular Bisector Theorem,
$AB = 2CB$
$DE = 2FE$
But $CB = FE$, therefore $AB = DE$

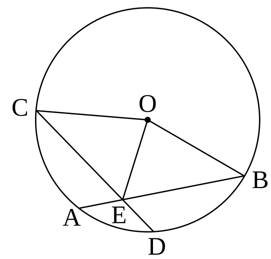
4.



Construct perpendiculars from O to U on PQ, and from O to V on RS. Connect O to T.
$OU = OV$ (Two Chords Theorem) ①
Use the Pythagorean Theorem in $\triangle OUT$ and $\triangle OVT$.
$UT^2 = OT^2 - OU^2$
$VT^2 = OT^2 - OV^2$
$= OT^2 - OU^2$ (using ①)
Hence, $UT = VT$ ②
$PT = PU + UT$ ③
$PU = \frac{1}{2}PQ$ (Corollary 3)
But $PQ = RS$
Thus, $PU = \frac{1}{2}RS$
So, $PU = SV$ ④
Hence, $PT = SV + VT$ (using ②, ③, and ④)
$PT = ST$ ⑤
Since $PQ = RS$
$PT + QT = ST + RT$ ⑥
Substituting ⑤ into ⑥,
$ST + QT = ST + RT$
Which simplifies to $QT = RT$.
Therefore, $PT = ST$ and $QT = RT$ is proven.

Selected Solutions — Chapter 7

5.



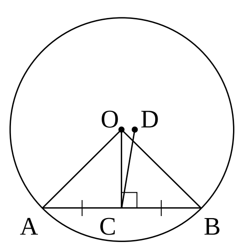
Connect OC and OB.
In △OCE and △OBE,
OC = OB (radii)
CE = BE (from exercise 4)
OE is common.
Thus, △OCE ≅ △OBE (SSS)
Since corresponding angles are equal, ∠OEC = ∠OEB
Hence, OE bisects ∠CEB.

6. OC is the perpendicular bisector of AB.
Hence, AE = BE

In △AEC and △BEC,
AE = BC
∠AEC = ∠BEC = 90°
CE is common
Therefore, △AEC ≅ △BEC (SAS)
Since corresponding sides are equal, CA = CB
Therefore, △ABC is isosceles.

7. In △BDO and △CEO,
OB = OC
∠DOB = ∠EOC (opposite angles)
DO = EO (radii)
Therefore △BDO ≅ △CEO (SAS)
Since the triangles are congruent, ∠BDO = ∠CEO ①
Also, in △DOE, OD = OE (radii)
Therefore, △DOE is isosceles and ∠OED = ∠ODE ②
In △ADE, ∠ADE = ∠BDO + ∠ODE ③
By substituting ① and ② into ③
∠ADE = ∠CEO + ∠OED
= ∠AED
Since ∠ADE = ∠AED, △ADE is isosceles.

8. a)

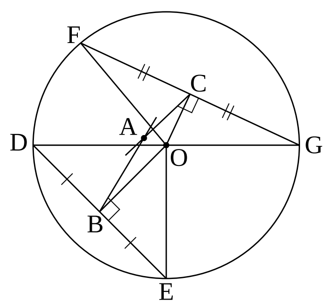


In circle, centre O, chord AB is drawn with perpendicular bisector DC. Assume the perpendicular bisector does not contain the

Selected Solutions — Chapter 7

centre. Then it must contain another point, D, that is not the centre.
Connect OC, OA, and OB.
In △OCA and △OCB,
OC is common.
OA = OB (radii)
AC = BC
Hence, △OCA ≅ △OCB (SSS)
Since corresponding angles are equal
∠OCA = ∠OCB
But ∠OCA + ∠OCB = 180°
Therefore, ∠OCA = ∠OCB = 90°
But ∠DCA = 90°
Hence, DC must be parallel to OC. But this is impossible, since they intersect. Hence, the assumption is incorrect. The perpendicular bisector of the chord does contain the centre of the circle.

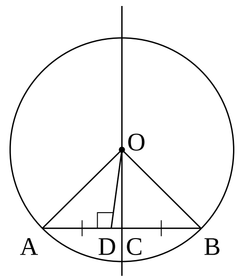
b)



In a circle, centre O, two non-parallel chords FG and DE are drawn. Their perpendicular bisectors are AC and AD. Assume the perpendicular bisectors of the chords intersect at some point A that is not the centre.
Connect OF, OG, OD, OE.
In △OCG and △OCF,
OG = OF (radii)
GC = FC
OC is common.
Hence, △OCG ≅ △OCF (SSS)
Since corresponding angles are equal, ∠OCG = ∠OCF
But ∠OCG + ∠OCF = 180°
Therefore, ∠OCG = ∠OCF = 90°
But ∠ACF = 90°
Thus, AC is parallel to OC. But this is impossible, since they intersect. Therefore, the assumption must be incorrect. Hence, the centre of a circle is the point of intersection of the perpendicular bisectors of any two non-parallel chords.

Selected Solutions — Chapter 7

c)



In a circle, centre O, line OC bisects chord AB. Assume OC is not perpendicular to AB. Then there must be a point D on the chord such that OD is perpendicular to AB.

In right triangles △ODB and △ODA,

OB = OA (radii) ①

Use the Pythagorean Theorem.

$AD^2 = OA^2 - OD^2$

$BD^2 = OB^2 - OD^2$

$BD^2 = OA^2 - OD^2$ (using ①)

Hence, AD = BD ②

AD = AC – DC ③

BD = BC + DC ④

By substituting ③ and ④ into ②,

AC – DC = BC + DC

But, AC = BC

AC – DC = AC + DC

2DC = 0

DC = 0

This is impossible, so the assumption must be incorrect. Thus, OC is perpendicular to AB.

Mathematical Modelling: How Can We Map Earth's Surface?, page 446

1. No. Whatever the measures for the length and width of the rectangle, all points on Earth will still have the same position relative to each other.

2. The length of the rectangle represents the 360° around the Equator. The lines of longitude are 15° apart. The quotient of 360° and 15° is 24, so 23 vertical lines are drawn to produce 24 spaces horizontally. Similarly, the width of the rectangle represents the 180° from the North Pole to the South Pole. The lines of latitude are 15° apart. The quotient of 180° and 15° is 12, so 11 horizontal lines are drawn to produce 12 spaces vertically.

3. The North and South poles are single points on Earth but are represented as straight lines on the rectangular map.

4. The regions of Earth near the two poles are the most distorted. They are stretched out along the top and bottom of the rectangular map.

Selected Solutions — Chapter 7

5. **b)** It was winter in Canada when these images were made. The South Pole is always illuminated and the North Pole is always in darkness during Canada's winter.

   c) The North Pole would be illuminated all day and the South Pole would be in constant darkness.

6. Toronto is located near the middle of the day region on Earth. Therefore, it is about noon.

7. The illuminated area would shift left across the image of Earth.

Investigate, page 448

1. **c)** The angle inscribed in a semicircle is a right angle.

2. **c)** Inscribed angles subtended by the same arc of a circle are congruent.

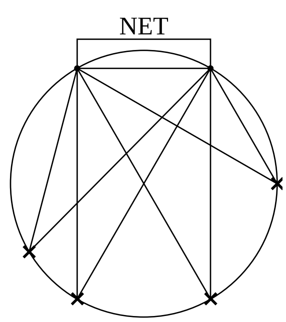
3. **c)** The measure of the central angle is twice the measure of the inscribed angle subtended by the same arc. This appears to be true whether the central angle is less than or greater that 180˚

7.4 Exercises, page 452

3. Answers may vary.
   For exercise 1a, Angle Property 3 was used. Since the measure of the central angle is twice the measure of the inscribed angle,
   $x = 120^\circ \div 2 = 60^\circ$.

4. **a)** The shooting angles are different. The angles subtended by the net at the blue line differ for different points along the blue line.

   b) To have shooting angles the same, the posts of the net must be on a chord of a circle and the players shoot from the circumference of the circle. (Angle Property 2)

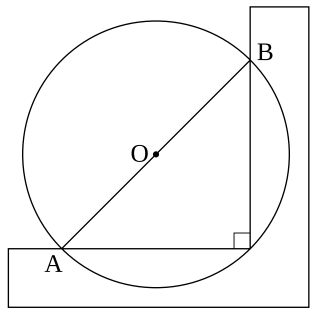


7. **b)** When P is on the minor arc AB, the measure of ∠P is different than in part a. From Investigate exercise 3c, ∠P is $\frac{1}{2}$ of reflex angle AOB. In this case, ∠P = 150˚

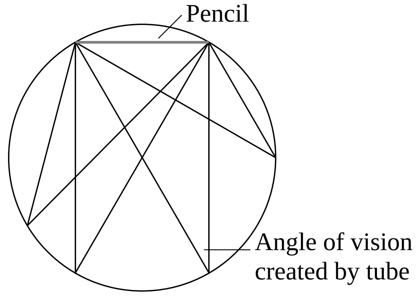
8. **b)** Answers may vary.

   iii) Angle Property 3 was used to find the value for *x*. The central angle subtended by the same arc is 90˚, so $x = 45^\circ$. It was then observed that *z* also equals 45˚, since *x* and *z* are equal angles in an isosceles triangle. The Angles in a Triangle Theorem was then used to find y; that is, $y = 180^\circ - 45^\circ - 45^\circ = 90^\circ$.

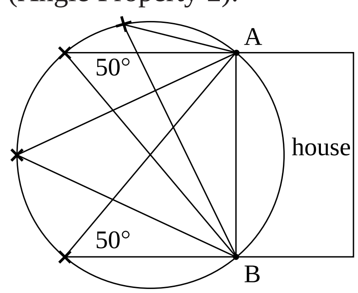
11. The carpenter must position the square so that the right angle is on the circumference of the circle, as in the diagram. A line is drawn between A and B, and its length is measured. The centre of the line segment is the centre of the circle (Angle Property 1).



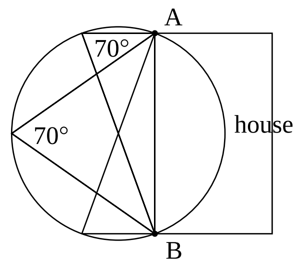
12. The tube enforces an angle of vision, which is constant throughout the movement. By Angle Property 2, the pencil is on a chord and the points on the floor form the major arc of a circle.



13. **a)** Take the picture from anywhere on major arc AB (Angle Property 2).



b) Take the picture from anywhere on major arc AB. (Angle Property 2).



Selected Solutions — Chapter 7

c) The major arc in part a is part of a larger circle than in part b. In addition, the front of the house in part a is farther from the centre of the circle than in part b.

14. a) $\angle ROS = 2\angle RQS$ (Angle Property 3)
$= 2(45^\circ)$
$= 90^\circ$
Thus, RO is perpendicular to SO.

b) i) SOQ will be a diameter of the circle and OQR will form a right isosceles triangle.

ii) QOR will be a diameter of the circle and SOQ will form a right isosceles triangle.

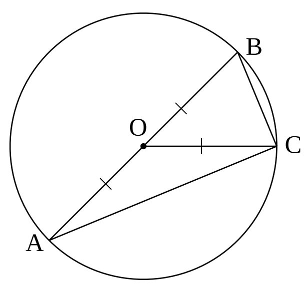
iii) $\triangle ROS$ is a right isosceles triangle.

iv) Concave quadrilateral OSQR is formed with reflex $\angle SOR$, and $SO = OR$.

c) ORQS is a quadrilateral with $\angle SQR = 135^\circ$ (Angle Property 3 with the central angle being the reflex angle, which measures 270°).

15. c) The triangles formed when two chords intersect in a circle are similar.

16. a)



b) In $\triangle OAC$, $OA = OC$ (radii)
Therefore, $\triangle OAC$ is isosceles.
Therefore, $\angle OAC = \angle OCA$

c) In $\triangle OBC$, $OB = OC$
Therefore, $\triangle OBC$ is isosceles.
Therefore, $\angle OCB = \angle OBC$

d) The four angles are interior angles of $\triangle ABC$; therefore, they add to 180°.

e) $\angle OAC + \angle OCA + \angle OCB + \angle OBC = 180^\circ$
But from parts b and c:
$\angle OAC = \angle OCA$ and $\angle OCB = \angle OBC$
Therefore, $\angle OCA + \angle OCA + \angle OCB + \angle OCB = 180^\circ$
$2\angle OCA + 2\angle OCB = 180^\circ$
Divide by 2. $\angle OCA + \angle OCB = 90^\circ$
But $\angle OCA + \angle OCB = \angle ACB$
Therefore, $\angle ACB = 90^\circ$

Selected Solutions — Chapter 7

Problem Solving: Paradoxes and Proof, page 457

1. The meaning of the contents on the page is referring to the page itself. The page is not blank because there is a sentence on it.
2. The meaning of the words refers to the act of writing them. The person should have planned ahead to ensure there was enough room.
3. Sentences 1 and 2 are false. Sentence 3 refers to the truthfulness of itself. If sentence 3 is true, then sentence 3 is false. However, if sentence 3 is false then sentence 3 is true. The sentence is neither true nor false.
4. Suppose the sentence in the box on the left is true. Then the sentence in the box on the right must be true. This implies that the sentence in the box on the left is false, contradicting the assumption. Now suppose that the sentence in the box on the left is false. This implies that the sentence in the box on the right is false. If that is so, the sentence in the box on the left is true, again contradicting the assumption. Therefore, the sentences are neither true nor false.
5. If all rules have exceptions, then this rule has an exception. But this implies that there is a rule that has no exceptions. There is a contradiction; thus, the question itself is "undecidable."
6. Suppose that the barber shaves himself. This contradicts the barber's rule that he shaves only those men who do not shave themselves. Suppose that the barber does not shave himself. Then he is a man who does not shave himself. Therefore, he must shave himself. This again is a contradiction. The question is "undecidable."
7. If the "undecidable" statement is true (that is, no counterexamples have been found), it is possible that this statement could not be proved either true or false.

7.5 Exercises, page 460

1. In $\triangle PAD$ and $\triangle PCB$,
   By Corollary 1 of the Angles in a Circle Theorem,
   $\angle ADP = \angle CBP$
   $\angle DAP = \angle BCP$
   From the Angles in a Triangle Theorem, since two pairs of corresponding angles are equal, the third pair of angles must be equal.
   Therefore, $\triangle PAD \sim \triangle PCB$
2. In $\triangle PAD$ and $\triangle PCB$, $\angle APD$ and $\angle CPB$ are common.
   From Corollary 1 of the Angles in a Circle Theorem,
   $\angle PAD = \angle PCB$
   From the Angles in a Triangle Theorem, since two pairs of corresponding angles are equal, the third pair of angles must be equal.
   Therefore, $\triangle PAD \sim \triangle PCB$

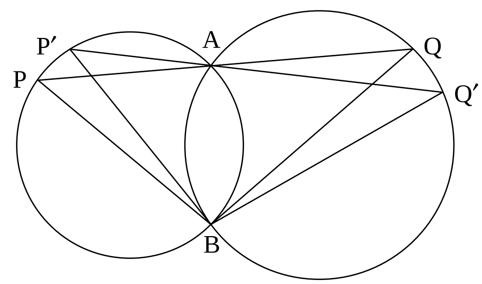
Selected Solutions — Chapter 7

3. The perpendicular is constructed by constructing an angle inscribed in a semicircle. From Corollary 2 of the Angles in a Circle Theorem, the angle inscribed in a semicircle is a right angle.

4. From Corollary 2 Semicircle Theorem,
BA ⊥ AC ①
By the Chord Perpendicular Bisector Theorem,
OE ⊥ AC ②
OD ⊥ AB ③
The quadrilateral formed by OD, AB, AC, and OE has three right angles. Since the sum of the angles in a quadrilateral is 360˚, the 4th angle in the quadrilateral must be 90˚. Hence OD ⊥ OE

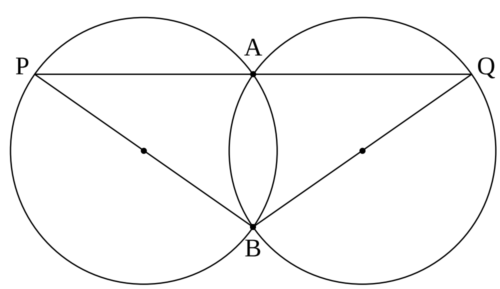
5. From Corollary 1 of the Angles in a Circle Theorem:
Arc PS subtends equal angles at the circumference
∠SRT = ∠TQP
Arc RQ subtends equal angles at the circumference
∠RST = ∠TPQ
Since PQ is parallel to SR, alternate angles between these parallel lines are equal:
∠SRT = ∠TPQ
∠RST = ∠TQP

6. a)



Construct a line P′AQ′.
We need to prove that ∠PBQ = ∠P′BQ′.
In △BPQ and △BP′Q′
From Corollary 1 of the Angles in a Circle Theorem:
Arc AB subtends equal angles at the circumference.
For the circle on the left: ∠BPQ = ∠BP′Q′
For the circle on the right: ∠BQP = ∠BQ′P′
From the Angles in a Triangle Theorem, since two pairs of corresponding angles are equal, the third pair of angles must be equal: ∠PBQ = ∠P′BQ′
That is, the measure of ∠PBQ is constant.

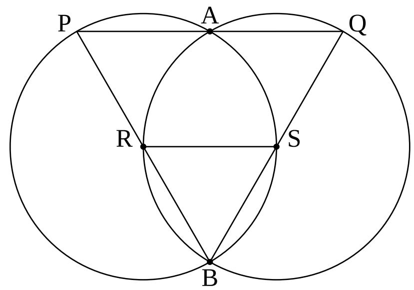
b)



Selected Solutions — Chapter 7

When the circles have equal radii, they are congruent circles.
Each has an inscribed angle subtended by the same arc (arc AB).
Therefore, the inscribed angles ∠BPQ and ∠BQP are equal.
△BPQ is isosceles.

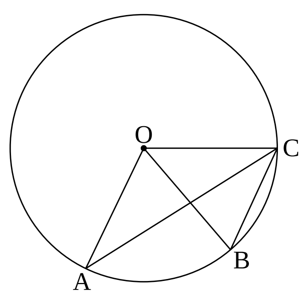
c)



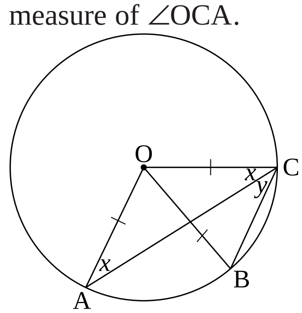
When each circle passes through the centre of the other, △PQB is equilateral.
Since ∠PBQ is constant and ∠BPQ = ∠BQP, we may take the special case, where BQ and BP are diameters.
In △BRS, BR = RS = SB (radii)
Therefore, ∠RBS = 60°
In △BPQ, ∠PBQ = 60°
∠BPQ = ∠BQP and
Therefore, ∠PBQ = ∠BPQ = ∠BQP = 60° (Angles in a Triangle Theorem)
Therefore, △PQB is equilateral.

7. P can be anywhere on major arc AB of the left-hand circle, and Q can be anywhere on major arc AB of the right-hand circle.

8. a)



b) We must prove that ∠AOB = 2∠ACB.
Since OA, OB, and OC are radii,
OA = OB = OC
Hence, △AOC and △BOC are isosceles.
Let y represent the measure of ∠ACB and let x represent the measure of ∠OCA.



Since $\triangle AOC$ is isosceles,
$\angle OAC = \angle OCA = x$
Triangle BOC is also isosceles. Therefore,
$\angle OBC = \angle OCB = x + y$ ①
We want the measure of $\angle AOB$ in terms of x and y.
$\angle AOB = \angle AOC - \angle BOC$ ②
From the Angles in a Triangle Theorem.
$\angle AOC = 180^\circ - 2x$ ③
and $\angle BOC = 180^\circ - 2(x + y)$ ④
Substitute ③ and ④ into ②.

$$\begin{aligned}\angle AOB &= (180^\circ - 2x) - (180^\circ - 2(x + y))\\ &= 2y\\ &= 2\angle ACB\end{aligned}$$

Therefore, the Angles in a Circle Theorem is proven.

Linking Ideas: Mathematics & Technology
Dynamic Circle Designs, page 463

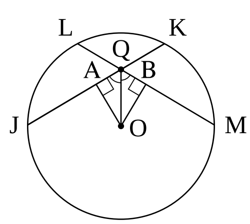
2. All circles have diameter 2.3 cm. The original circle is 2.3 cm in diameter and radius of 1.15 cm. The other circles are created with the same radius, since the centres are on the circumference of the original circle and the centre of the original circle is on the circumference of the new circles.

3. **a)** Similar to the first or second design on page 462.
b) Similar to the second design on page 463.
c) Similar to the first design on page 463.

7 Review, page 464

4. Explanations may vary. For exercise 1a, the Pythagorean Theorem was used. That is, $13^2 = 5^2 + \left(\frac{x}{2}\right)^2$. This was then solved for x to get $x = 24$.

5. Construct perpendiculars from O to A on JK and from O to B on LM.



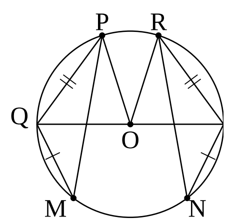
In $\triangle OAQ$ and $\triangle OBQ$
$\angle OAQ = \angle OBQ = 90^\circ$
$\angle OQA = \angle OQB$
By the Angles in a Triangle Theorem, with 2 pairs of corresponding angles equal in the triangles, the third pair of angles is equal.
$\angle AOQ = \angle BOQ$
OQ is common.
Therefore, $\triangle OAQ \cong \triangle OBQ$ (ASA)
Since the triangles are congruent, OA = OB. That is, the chords are equidistant from the centre.

Selected Solutions — Chapter 7

Two chords equidistant from the centre have the same length (converse of the Two Chords Theorem).
Therefore, JK = LM is proven.

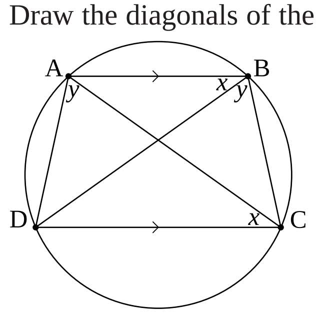
9. Join OD.
Then OB = OD = OA because they are radii.
In △OAD and △OAB,
OD = OB
OA is common.
AD = AB
Therefore, △OAD ≅ △OAB (SSS)
Since the triangles are congruent, corresponding angles are equal.
∠OAD = ∠OAB ①
Since △OAB is isosceles, ∠OAB = ∠OBA ②
Comparing ① and ②, ∠OAD = ∠OBA

10. Join OP, OR, OS, and OQ.



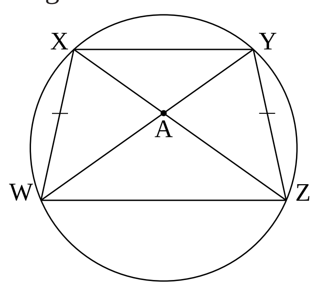
OP = OR = OS = OQ because they are all radii.
In △QOP and △SOR,
QO = OS
OP = OR
QP = RS
Therefore, △QOP ≅ △SOR (SSS)
Since triangles are congruent, corresponding angles are equal:
∠QOP = ∠SOR ①
From the Angles in a Circle Theorem, ∠QMP = $\frac{1}{2}$∠QOP
Substitute for ∠QOP from ①.
∠QMP = $\frac{1}{2}$∠ROS ②
From the Angles in a Circle Theorem,
∠ROS = 2∠RNS ③
Substitute for ∠ROS from ③ into ②.
∠QMP = $\frac{1}{2}$(2∠RNS)
∠QMP = ∠RNS

12. Trapezoid ABCD is inscribed in a circle where AB is parallel to DC.
Draw the diagonals of the trapezoid.



By the Angles in a Circle Theorem, Corollary 1,
$\angle DBA = \angle DCA = x$
$\angle DBC = \angle DAC = y$
By the Parallel Lines Theorem, alternate angles
$\angle BDC = \angle DBA = x$
In $\triangle ACD$ and $\triangle BDC$,
$\angle DAC = \angle CBD = y$
$\angle ACD = \angle BDC = x$
Therefore, by the Angles in a Triangle Theorem,
$\angle ADC = \angle BCD = 180^\circ - x - y$
DC is common.
Therefore, $\triangle ACD \cong \triangle BDC$ (ASA)
Since the triangles are congruent, corresponding sides are equal:
AD = BC
That is, at least one pair of opposite sides is equal.

13. Quadrilateral WXYZ is inscribed in a circle, with WX = YZ. The diagonals intersect at A.



By the Angles in a Circle Theorem, Corollary 1,
$\angle WXZ = \angle WYZ$
$\angle XWY = \angle YZX$
In $\triangle XAW$ and $\triangle YAZ$,
$\angle WXA = \angle ZYA$
WX = ZY
$\angle XWA = \angle YZA$
Therefore, $\triangle XAW \cong \triangle YAZ$ (ASA)
Since the triangles are congruent, corresponding sides are equal:
WA = ZA ①
AX = AY ②
Adding ① and ②
WA + AY = ZA + AX
which simplifies to WY = ZA
Therefore, the diagonals are equal.

7 Cumulative Review, page 466

3. Explanations may vary. For part a:
The formula for the accumulated amount is $A = P(1 + i)^n$. Since interest is $6\frac{1}{4}$% compounded monthly, $i = \frac{0.0625}{12}$, $P = 2100$, $n = 20$. These three values were substituted into the formula to get $A = 2100\left(1 + \frac{0.0625}{12}\right)^{20}$ and with a calculator it was evaluated to be $2329.92.

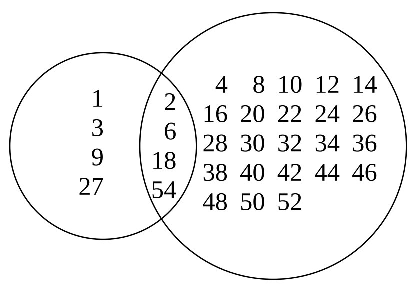
Selected Solutions — Chapter 7

10. Explanations may vary. For part a:
The factor theorem was first used to find $x - 1$ as a factor. Then long division was used to rewrite $x^3 - 13x + 12 = 0$ as $(x - 1)(x^2 + x - 12) = 0$, which was further factored to $(x - 1)(x + 4)(x - 3) = 0$. The solutions for x are now found by equating each factor to zero; that is, $x = 1$ or $x = -4$ or $x = 3$.

12. Explanations may vary. For part a:
To find the reciprocal function, put 1 in the numerator and $f(x)$ in the denominator; that is, $y = \frac{1}{\sqrt{x^2 - 5}}$. To find the restrictions, equate the denominator to zero and solve for x. That is, $x^2 = 5$, $x = \pm\sqrt{5}$.
Therefore, the reciprocal function is $y = \frac{1}{\sqrt{x^2 - 5}}$, $x \neq \pm\sqrt{5}$.

15. a) The discriminant of the equation was found. That is, $b^2 - 4ac = 36^2 - 4(2)(-25)$. This value is greater than 0. Therefore, there are 2 different real roots.

24. Explanations may vary. For part a: Draw 2 circles that overlap. In the right circle, write the multiples of 2 up to 54. In the left circle, write the factors of 54. In the overlapping region, write the multiples of 2 that are also factors of 54. Label each circle with a description of the numbers it contains.



Factors of 54 Multiples of 2