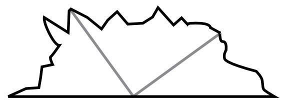
Selected Solutions — Chapter 6

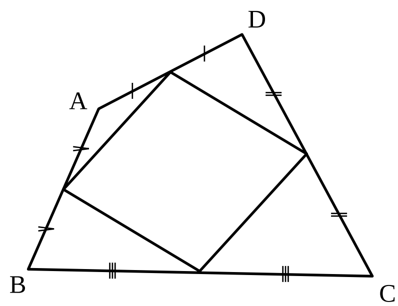
6.1 Exercises, page 371

4. Cut out a large triangle from a piece of paper. Cut off the three corners and put them side by side as shown.



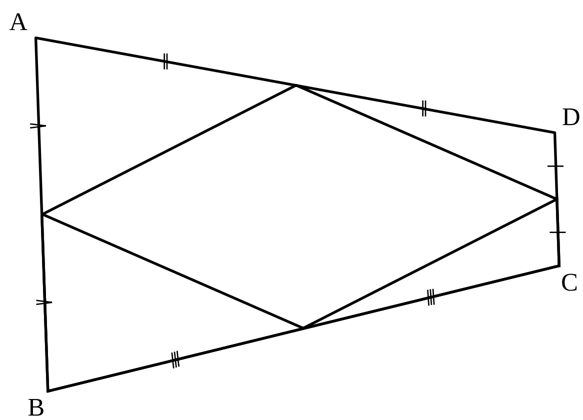
The angles make a straight angle.

8. a), b)



A parallelogram.

c)



The figure formed by joining the midpoints of the four sides of any quadrilateral is a parallelogram.

6.2 Exercises, page 375

5. e) Since ∠DAB, ∠BAC, and ∠CAE form a straight angle, their sum is 180˚. Angle DAB and ∠CAE are equal to ∠B and ∠C, respectively. Therefore, ∠BAC, ∠B and ∠C must also add to 180˚.

7. Both △ABC and △EDC have a 90˚ angle, ∠B and ∠D respectively. Angle ACB and ∠ECD are opposite angles and thus are equal. Since the 3 angles of each triangle must add to 180˚, the remaining angles, ∠A and ∠E must be equal.

8. In △ABC, ∠A, ∠B, and ∠ACB add to 180˚. Similarly, ∠ACD and ∠ACB add to 180˚ because they are supplementary angles. By comparing the two statements,

∠ACD = ∠A + ∠B.

Selected Solutions — Chapter 6

9. **a)** Let $2m$ and $2n$ represent two even numbers, where m and n are integers.
$2m + 2n = 2(m + n)$, which is divisible by 2, so
$2m + 2n$ is even.

Therefore, the sum of two even numbers is always even.

b) Let $2k + 1$ and $2l + 1$ represent two odd numbers, where k and l are integers.

$$\begin{aligned}(2k + 1)(2l + 1) &= 4kl + 2k + 2l + 1\\ &= 2(2kl + k + l) + 1\end{aligned}$$

$2(2kl + k + l)$ is an even number, so
$2(2kl + k + l) + 1$ is an odd number.

Therefore, the product of two odd numbers is always odd.

10. **a)** Let $2m$ and $2n$ represent two even numbers, where m and n are integers.

$$\begin{aligned}(2m)^2 - (2n)^2 &= 4m^2 - 4n^2\\ &= 4(m^2 - n^2), \text{ which is divisible by 4.}\end{aligned}$$

Therefore, the difference of the squares of two even numbers is always divisible by 4.

b) Let $2k + 1$ and $2l + 1$ represent two odd numbers, where k and l are integers.

$$\begin{aligned}(2k + 1)^2 - (2l + 1)^2 &= 4k^2 + 4k + 1 - 4l^2 - 4l - 1\\ &= 4k^2 + 4k - 4l^2 - 4l\\ &= 4(k^2 + k - l^2 - l), \text{ which is divisible by 4.}\end{aligned}$$

Therefore, the difference of the squares of two odd numbers is always divisible by 4

12. Since JKLM is a square: JK = KL ①
Since △JXK is equilateral: JK = XK ②
Since △KLY is equilateral: KL = YK ③
Comparing ① and ②: KL = XK ④
Comparing ③ and ④: YK = XK

Therefore, △KXY is isosceles.

Mathematical Modelling: Incredible Journeys, page 379

1. **a)** I would use pencil and ruler to draw a line on a piece of paper.

b) In the imaginary journey, the line would be magnified. The first few diagrams would be the magnification of the line, which would become a dot on the line, then to the atomic structure of the carbon atoms that comprise the graphite in the pencil.

2. **a)** A smooth lacquered table top is a good model for a plane.

b) In the imaginary journey approaching the plane, you would see the composition of the lacquer and the atoms of which it is made.

Selected Solutions — Chapter 6

6.3 Exercises, page 382

3. **b)** True. An odd number has the last digit 1, 3, 5, 7 or 9. If 1 is added, the last digit becomes 2, 4, 6, 8 or 0, which are the criteria for an even number.

d) True. An acute angle is less than 90°. If two angles that are less that 90° are added, the result is less than180°.

g) True. A square is a quadrilateral with four angles of 90° and four equal sides. A rectangle is a quadrilateral with four angles of 90°. Therefore, every square is a rectangle.

4. **b)** It works for triangles where the length of the hypotenuse is 1 greater than length of the given side. Let y represent the length of the given side, and $y + 1$ the length of the hypotenuse. Let x represent the unknown length.

By the Pythagorean Theorem,

$$\begin{aligned} x^2 &= (y+1)^2 - y^2 \\ &= y^2 + 2y + 1 - y^2 \\ &= 2y + 1 \\ &= y + y + 1 \end{aligned}$$

$x = \sqrt{y + y + 1}$, which is the square root of the sum of the two given lengths.

8. The temperature at which water boils varies with altitude, air pressure, and other factors. Thus, the temperature at which water boils on top of Mount Everest is not 100°C.

9. **a)** Explore the function using a graphing calculator. Use the window settings shown in the second diagram. Since there are points plotted to the left of the y-axis, this disproves Bruce's statement. Using the table function provides a list of these points. A counterexample is when $x = -2,\ y = \frac{1}{4}$.

b) To disprove Lisa's conjecture, it is sufficient to find one counterexample, one point other than the fifteen she has found. Use the table function of the graphing calculator. Scroll down the table through the negative x-values until you find an x-value less than –4.7. For example, when $x = -5$, $y = -0.000\ 32$.

10. **a)** $x = -0.2, -0.4, -0.6, \ldots$

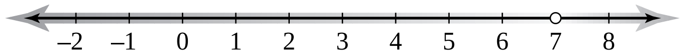
c) $y = x^x$ is defined for $x = -3,\ -2.96,\ -2.92, \ldots -0.04,\ 0.01,\ 0.02, \ldots$

d) $y = x^x$ is defined for $x > 0$.
When x is written to 2 decimal places, $y = x^x$ is defined for all values of x that are divisible by 4. $y = x^x$ cannot be determined for $x = 0$.
When x is written to 2 decimal places, $y = x^x$ is undefined for all negative values that cannot be divided by 4.

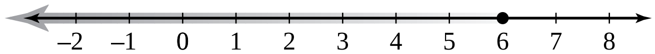
Selected Solutions — Chapter 6

6.4 Exercises, page 391

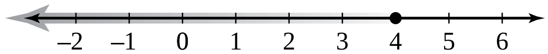
6. a)



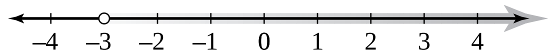
b)



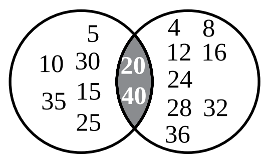
c)



d)

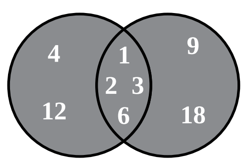


8. In exercise 7a, draw two overlapping circles. Put numbers that are multiples of 4 and multiples of 5 in the overlapping part. Put multiples of 5 that are not multiples of 4 on the left side. Put multiples of 4 that are not multiples of 5 on the right side. Shade in the overlapping part.



Multiples of 5 Multiples of 4

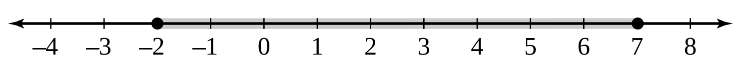
10. In exercise 9a, draw two overlapping circles. Put numbers that are factors of 12 and factors of 18 in the overlapping part. Put factors of 12 that are not factors of 18 on the left side. Put factors of 18 that are not factors of 12 on the right side. Shade in both circles.



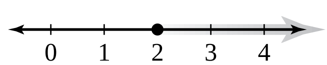
Factors of 12 Factors of 18

11. a)

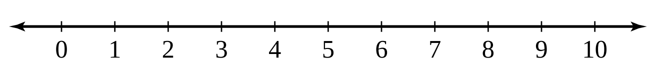
b)



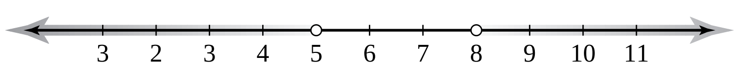
c)



d)

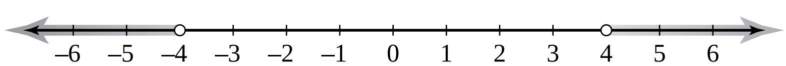


12. a)

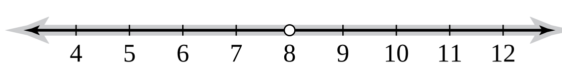


b)

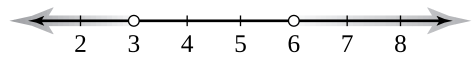
c)

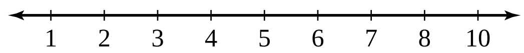


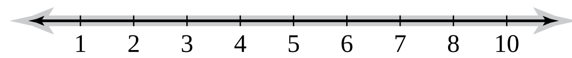
d)

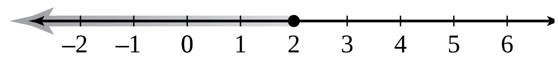


Selected Solutions — Chapter 6

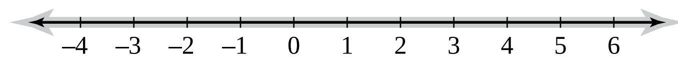
13. a)


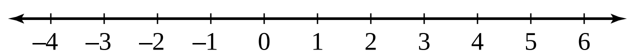
b)


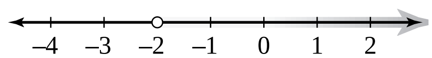
c)


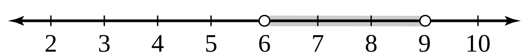
d)


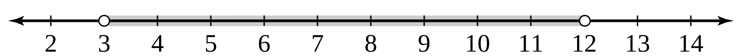
14. a)

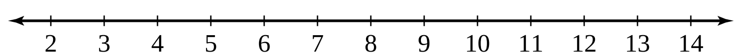

b)


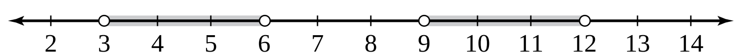
c)


d)


15. a)


b)


c)


d)


26. The wife's statement can be written, "My husband is a truth-teller or I am a liar." If the wife is telling the truth, then her husband is a truth-teller. If the wife is lying, then the negative of her statement is true; that is, "My husband is a liar and I am a truth-teller." This contradicts the assumption that she is lying. The conclusion is they are both truth-tellers.

27. a) The prisoner states that, "I will be shot." The judge could conclude that the prisoner's statement is false, in which case the prisoner will be hanged and the judge was correct. The judge could also conclude that the prisoner's statement is true and so the prisoner will be shot. The prisoner should not make this statement.

b) The prisoner states that, "I will be hanged." The judge could conclude that the prisoner's statement is false, in which case the prisoner should be hanged, which is a contradiction. If the statement is true, the prisoner should be shot, which is also a contradiction. The judge must let him go and therefore, this is the statement the prisoner should make.

6.5 Exercises, page 398

12. Part c is true. The contrapositive of a true statement is always true.

14. b) It is too long, and not as forceful.

Selected Solutions — Chapter 6

16. If the person is telling the truth, then:
Only one of them is a liar or both of them are liars.
They can't both be liars, because that contradicts the assumption that the person is telling the truth. Hence, the person who is not speaking is the liar.
If the person is lying, then the negation of the statement is that neither of them is a liar. This contradicts the assumption that she is lying.
The conclusion is that the person speaking is a truth-teller, and the other person is a liar.

17. a) If you ask "Which road leads to safety?", the liar will say the road that leads to danger and the truth-teller will say the road that leads to safety. But you don't know which is which, so this question doesn't help.

b) If you ask "Which road would your friend tell me leads to safety?" the liar will say the road that leads to danger, and the truth-teller will say the road that leads to danger. In this case, you should take the other road. This question works.

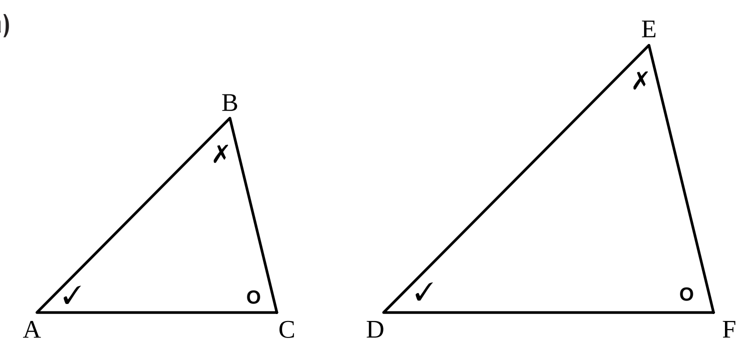
c) If you ask "If I asked you if the road on the right leads to safety, would you say yes?" the liar would say yes if the road does lead to safety, and no if it leads to danger. The truth-teller would say yes if it does lead to safety, and no if it leads to danger. Thus, if the person said yes, you'd take the road on the right. If the person said no, you'd take the road on the right. This question also works.

Linking Ideas: Mathematics & Sports
Logical Connectives in Sports Rules, page 401

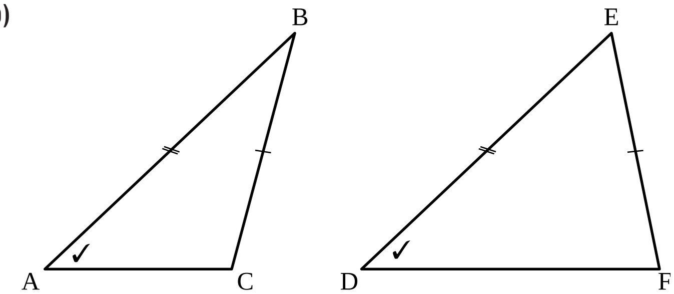
2.
- In baseball, if there are 3 outs, then the other team is up to bat or if it is the bottom of the ninth inning, then the game is over.
- If the batter is hit by the pitcher or the pitcher throws 4 balls, then the batter walks to first base.
- Disputes are discussed between the captains and the umpires, not the team members.

6.6 Exercises, page 406

4. a)



b)



5. a) In △PQS and △RQS, there are 3 pairs of sides that are equal. That is, PQ = RQ, QS is common, and PS = RS. Since all 3 corresponding sides are equal, then the two triangles are congruent, (SSS).

b) Triangle PQS and △RQS are congruent; therefore, the corresponding angles are equal. That is, ∠P = ∠R.

6. a) In △ABR and △CBR, AB = CB, ∠ABR = ∠CBR, and BR is common. Since two pairs of corresponding sides are equal, and the contained angles are equal, then the two triangles are congruent, (SAS).

b) Triangle ABR and △CBR are congruent; therefore, the corresponding angles are equal. That is, ∠A = ∠C.

7. a) In △ABD and △CBD, ∠ADB = ∠CDB, DB is common, and ∠ABD = ∠CBD. Since two angles and the contained side of △ABD are equal to two angles and the contained side of △CBD, then the two triangles are congruent, (ASA).

b) Triangle ABD and △CBD are congruent; therefore, corresponding sides are equal. That is, AB = CB.

8. In △AED and △BEC, AE = BE, ∠DEA = ∠CEB (opposite angles), and EA = EB. Since two sides and the contained angle of △AED are equal to two sides and the contained angle of △BEC, then △AED and △BEC are congruent (SAS). Since the two triangles are congruent, the corresponding sides and corresponding angles are equal; that is, AD = BC and ∠D = ∠C.

9. In △ACB and △DCE, ∠ABC = ∠DEC, BC = EC and ∠BCA = ∠ECD (opposite angles). Since two angles and the contained side of △ACB are equal to two angles and the contained side of △DCE, then △ACB and △DCE are congruent (ASA). Since the two triangles are congruent, the corresponding sides and corresponding angles are equal; that is, CA = CD and ∠A = ∠D.

10. In △ACP and △ABP, AP is common, ∠PAC = ∠PAB, and AC = AB. Since two sides and the contained angle of △ACP are equal to two sides and the contained angle of △ABP, then △ACP and △ABP are congruent (SAS). Since the triangles are congruent, corresponding sides are equal; that is, PC = PB.

In △PCB, two sides are equal; therefore, the triangle is isosceles.

Selected Solutions — Chapter 6

11. In $\triangle OAB$ and $\triangle ODC$,
$AB = DC$
$OA = OD$ (radii)
$OB = OC$ (radii)
Therefore, $\triangle OAB \cong \triangle ODC$ (SSS)
Since the triangles are congruent, $\angle AOB = \angle DOC$.

12. By the Opposite Angles Theorem, $\angle PRQ = \angle TRS$
Since PQ || ST, the alternate angles are equal,
$\angle RPQ = \angle RTS$
In $\triangle PQR$ and $\triangle TSR$,
$\angle RPQ = \angle RTS$
$PR = TR$
$PRQ = \angle TRS$
Therefore, $\triangle PQR \cong \triangle TSR$ (ASA)
Since the triangles are congruent, $QR = SR$ and R is the midpoint of QS.

13. In $\triangle ADC$ and $\triangle ABC$,
$\angle ADC = \angle ABC$
$DC = BC$
$\angle ACD = \angle ACB$
Therefore, $\triangle ADC \cong \triangle ABC$ (ASA)
Since the triangles are congruent, $AB = AD$

14. Draw line segment BD.
In $\triangle BAD$ and $\triangle BCD$,
$BA = BC$
$AD = CD$
$BD = BD$
Therefore, $\triangle BAD \cong \triangle BCD$ (SSS)
Since the triangles are congruent, $\angle ABD = \angle CBD$ and BD bisects $\angle ABC$.

15. In $\triangle QPS$ and $\triangle QRS$,
$QP = QR$
$PS = RS$
$QS = QS$
Therefore, $\triangle QPS \cong \triangle QRS$ (SSS)
Since the triangles are congruent, $\angle PQS = \angle RQS$

In $\triangle QPT$ and $\triangle QRT$,
$QP = QR$
$\angle PQT = \angle RQT$
$QT = QT$
Therefore, $\triangle QPT \cong \triangle QRT$ (SAS)
Since the triangles are congruent, $PT = TR$

16. By the Opposite Angles Theorem, $\angle ECD = \angle BCA$
Since $\angle B = \angle E$, and $\angle BCA = \angle ECD$, then by the Angles in a Triangle Theorem, $\angle A = \angle D$.

In $\triangle ABC$ and $\triangle DEC$,

Selected Solutions — Chapter 6

$\angle A = \angle D$
$AB = DE$
$\angle B = \angle E$
Therefore, $\triangle ABC \cong \triangle DEC$ (ASA)
Since the triangles are congruent, $CE = CB$ ①
and $AC = DC$ ②

Now, $AE = AC + CE$ ③

Substitute ① and ② into ③
$AE = DC + CB$
$= DB$
Therefore, $AE = DB$ is proved.

17. In $\triangle KLM$ and $\triangle NML$,
$\angle KML = \angle NLM$
$LM = ML$
$\angle KLM = \angle NML$
Therefore, $\triangle KLM \cong \triangle NML$ (ASA)

Since the triangles are congruent, $KL = NM$.

18. In $\triangle DAE$ and $\triangle BAC$,
$AE = AC$
$\angle A = \angle A$
$DA = BA$
Therefore, $\triangle DAE \cong \triangle BAC$ (SAS)
Since the triangles are congruent, $DE = BC$.

19. In $\triangle ABC$, $AB = AC$
Therefore, $\triangle ABC$ is isosceles and $\angle ABC = \angle ACB$ ①
Since $AB \parallel DE$, $\angle ABC$ and $\angle BCE$ are alternate angles and equal.
That is, $\angle ABC = \angle BCE$ ②
From ① and ②, $\angle ACB = \angle BCE$

Therefore, BC bisects $\angle ACE$.

20. In $\angle ABC$, $AB = AC$
Therefore, $\triangle ABC$ is isosceles and $\angle ABC = \angle ACB$ ①
Since $AE \parallel BC$, $\angle ACB$ and $\angle EAC$ are alternate angles and $\angle ABC$ and $\angle DAE$ are congruent angles.
That is, $\angle ACB = \angle EAC$ ②

$\angle ACB = \angle DAE$ ③
From ① and ③
$\angle ACB = \angle DAE$ ④
From ② and ④
$\angle DAE = \angle EAC$

Therefore, AE bisects $\angle CAD$.

21. In $\triangle ABC$, $AB = AC$
Therefore, $\triangle ABC$ is isosceles
and $\angle ABC = \angle ACB$
Since EBC and BCD are straight lines, and
$\angle ABC = \angle ACB$, then $\angle ABE = \angle ACD$

Selected Solutions — Chapter 6

In $\triangle ABE$ and $\triangle ACD$,
$AB = AC$
$\angle ABE = \angle ACD$
$BE = CD$
Therefore, $\triangle ABE \cong \triangle ACD$ (SAS)
Since the triangles are congruent, AE = AD.

22. In $\triangle QTR$, QT = QR
Therefore, $\triangle QTR$ is isosceles and
$\angle QRT = \angle QTR$
Since QRS and PTR are straight lines, and
$\angle QRT = \angle QTR$, we know that $\angle PTQ = \angle TRS$.
In $\triangle QPT$ and $\triangle STR$,
$QT = SR$
$\angle PTQ = \angle TRS$
$TP = RT$
Therefore, $\triangle QPT \cong \triangle STR$ (SAS)
Since the triangles are congruent, PQ = TS.

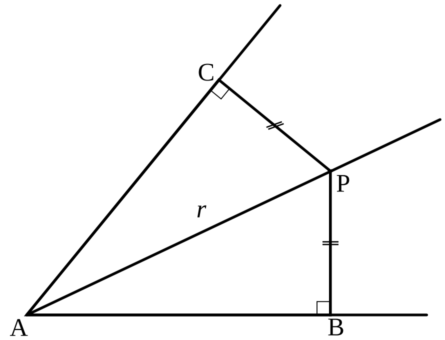
23. a) In $\triangle PAC$ and $\triangle PBC$,
$AC = BC$
$PA = PB$
$PC = PC$
Therefore, $\triangle PAC \cong \triangle PBC$ (SSS)
Since the triangles are congruent, $\angle PCA = \angle PCB$
Since ACB is a straight line, $\angle PCA = PCB = 90^{\circ}$
Therefore, PC is perpendicular to AB.

b) The converse of the Perpendicular Bisector Theorem states that any point that is equidistant from the endpoints of a line segment is on the perpendicular bisector of the segment. In the example above, C is the midpoint of AB and P is a point equidistant from A and B. It was proved that PC is the perpendicular bisector of AB, which proves the converse of the Perpendicular Bisector Theorem.

24. We are required to prove that EP = FP.
Since $\angle E = \angle F$, and $\angle EBP = \angle FBP$, then, by the Angles in a Triangle Theorem, $\angle EPB = \angle FPB$.
In $\triangle PBE$ and $\triangle PBF$,
$\angle EBP = \angle FBP$
$BP = BP$
$\angle EPB = \angle FPB$
Therefore, $\triangle PBE \cong \triangle PBF$ (ASA)
Since the triangles are congruent, EP = FP.

25. Converse of the Angle Bisector Theorem:
Any point that is equidistant from the arms of an angle is on the bisector of the angle.
In the diagram, ray AP is drawn on $\angle CAB$, so that point P is equidistant from AC and AB. We want to prove that $\angle PAC = \angle PAB$.

Selected Solutions — Chapter 6

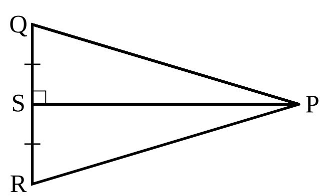


Let x represent the length of CP and BP.
Let r represent the length of AP.
Use the Pythagorean Theorem.

In △ACP : $AC^2 = r^2 - x^2$ ①
In △ABP : $AB^2 = r^2 - x^2$ ②
From ① and ②
AC = AB or AC = AB

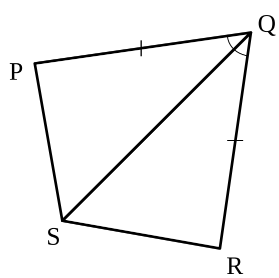
In △PAC and △PAB,
PC = PB
AC = AB
PA = PA
Therefore, △PAC ≅ △PAB (SSS)
Since the triangles are congruent, ∠PAC = ∠PAB
Therefore, PA is the bisector of ∠BAC.

26.



In △PQS and △PRS,
∠PSQ = ∠PSR
SQ = SR
PS = PS
Therefore, △PSQ ≅ △PRS (SAS)
Since the triangles are congruent, PQ = PR
Therefore, △PQR is isosceles.

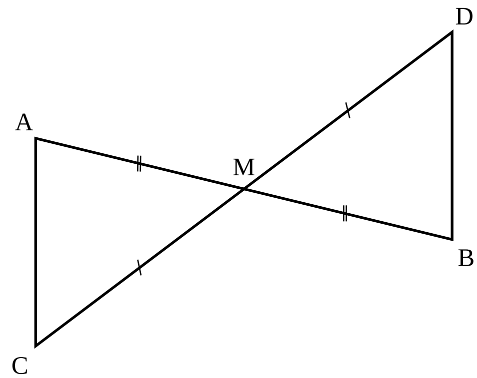
27.



Selected Solutions — Chapter 6

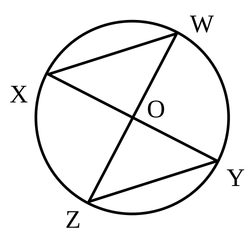
In △PQS and △RQS,
∠PQS = ∠RQS
PQ = RQ
QS = QS
Therefore, △PQS ≅ △RQS (SAS)
Since the triangles are congruent, PS = RS

28.



From the Opposite Angles Theorem,
∠AMC = ∠BMD
In △AMC and △BMD,
AM = BM
∠AMC = ∠BMD
CM = DM
Therefore, △AMC ≅ △BMD (SAS)
Since the triangles are congruent, AC = BD

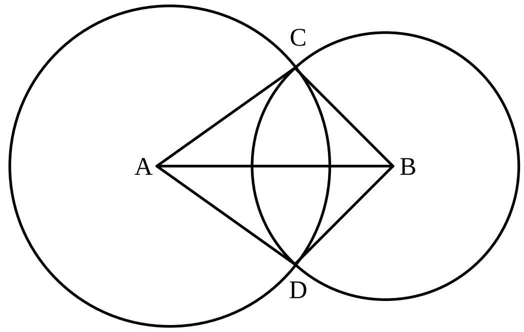
29.



By the Opposite Angles Theorem,
∠XOW = ∠ZOY
XO = OY = OW = OZ since they are all radii.
In △OXW and △OZY,
OX = OZ
∠XOW = ∠ZOY
OW = OY
Therefore, △OXW ≅ △OZY (SAS)
Since the triangles are congruent, XW = ZY

Selected Solutions — Chapter 6

30.



Since AC and AD are radii, AC = AD
Since BC and BD are radii, BC = BD
Join AB.

In △ACB and △ADB,

AC = AD
CB = DB
AB = AB
Therefore, △ACB ≅ △ADB (SSS)
Since the triangles are congruent, ∠ACB = ∠ADB

Problem Solving: Logic Puzzles, page 410

1. Cutting a log into 3 pieces requires 2 cuts, or $2.50 per cut. Cutting a log into 6 pieces requires 5 cuts.
$5 \times 2.50 = 12.50$
She should charge $12.50.

2. Three socks will either be all the same colour, or 2 of one colour and 1 of the other colour. So, 3 socks is the least number.

3. *Case 1:* If the first statement is correct, then the second statement is also correct, which is impossible because only one statement is correct.
Case 2: If the second statement is correct, then the other two statements must be incorrect. If the first statement is incorrect, then the third statement is correct, which is impossible because only one statement is correct.
Thus, the third statement is correct. Hence, the first statement is incorrect, and the treasure is not in the red chest. The second statement is also incorrect, which means the treasure is in the second chest.

4. All 3 statements are incorrect. Thus, from the first statement, the treasure is in the red chest. From the second statement, the treasure is not in the blue chest. From the third statement, the treasure is not in the yellow chest. Therefore, the treasure is in the red chest.

5. Pick a fruit from the third box.
Case 1: If it is an orange, then the third box should be labelled "oranges." Then the first box must be labelled "apples and oranges," and the second box must be labelled "apples."

Selected Solutions — Chapter 6

Case 2: If it is an apple, then the third box should be labelled "apples." Then the first box must be labelled "oranges," and the second box must be labelled "apples and oranges."

6. Make a table to keep track of the information.

	King	Laird	Port
Ken			
Louise			
Max			

Port's first name is not Max. So put an ✘ in row 3, column 3. Laird is Max's uncle. So we know that Max and Laird are different people, and Laird is a man. Put an ✘ in row 3, column 2, and row 2, column 2. Now we can put a ✓ in row 1, column 2. This means we can put ✘s in row 1, columns 1 and 3. We can now also put a ✓ in row 3, column 1. Thus, we can put an ✘ in row 2, column 1. Thus, we can put a ✓ in row 2, column 3.

	King	Laird	Port
Ken	✗	✓	✗
Louise	✗	✗	✓
Max	✓	✗	✗

Thus, the names are Max King, Louise Port, and Ken Laird.

7. Since Ms. Vrentzos has no brothers or sisters, her mother's daughter is herself. Therefore, she is that woman's mother.

8. Two cards need to be picked up and turned over, the green card and the card with a square. If the green card has a circle on the other side and the card with the square is not green on the other, you may conclude that every green card has a circle on the other side.

It should be noted that it is irrelevant what shape is on the yellow card and which colour is on the card with a circle to make the preceding statement.

9. If the wife is a liar, she would say she was a truth-teller. This contradicts what the husband said, so the husband is a liar.
If the wife is a truth-teller, she would say she was a truth-teller. This contradicts what the husband said, so the husband is a liar.

10. If she is a truth-teller, the statement is a contradiction. Thus, she is a liar. Since she is lying, her husband must be a truth-teller.

11. **a)** No. Since Andy's hat is red, my hat could be red or white.

b) Yes. If Andy can't tell, he must see a red hat on me, so my hat is red.

c) The roles are reversed. If the teacher asked me, I couldn't tell, because there are 2 red hats. Since I can't tell, Andy knows he has a red hat.

d) Yes. If my hat is white, then Andy knows his is red.

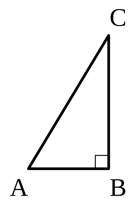
Selected Solutions — Chapter 6

e) No. If my hat is red, then Andy's hat is either red or white, so Andy can't say his is white.

6.7 Exercises, page 414

1. With the indirect proof, it will be assumed that PQ = PR. This implies that $\angle Q = \angle R$, which contradicts the given information. Therefore, the assumption is incorrect, and $PQ \neq PR$.

2. a)



Either $\angle A = \angle B = 90^\circ$ or $\angle A \neq \angle B = 90^\circ$
Assume $\angle A = \angle B = 90^\circ$ ①

In $\triangle ABC$, by the Angles in a Triangle Theorem,
$\angle A + \angle B + \angle C = 180^\circ$ ②

Comparing ① and ②, $\angle C = 0^\circ$
This contradicts the given information; that is, ABC is a triangle.
The assumption that $\angle A = \angle B = 90^\circ$ is incorrect.
Therefore, $\angle A \neq \angle B$, and a triangle cannot have two right angles.

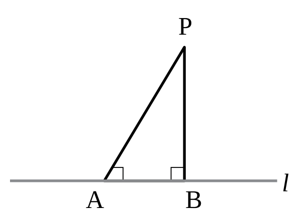
b) Either a triangle can have two obtuse angles, or it cannot.
Assume $\triangle ABC$ has two obtuse angles, $\angle A$ and $\angle B$.

In $\triangle ABC$, $\angle A > 90^\circ$
$\angle B > 90^\circ$

Therefore, $\angle A + \angle B > 180^\circ$

This contradicts the given information; that is, if ABC is a triangle, $\angle A + \angle B + \angle C = 180^\circ$
The assumption that a triangle can have two obtuse angles is incorrect. Therefore, a triangle cannot have two obtuse angles.

3.



Either one perpendicular line can be drawn from point P to the line *l* or more than one line can be drawn.
Assume 2 perpendicular lines can be drawn as illustrated above, and PAB is a triangle.

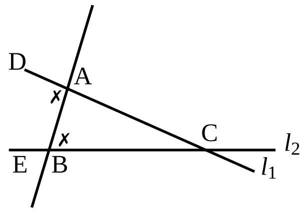
In $\triangle PAB$, $\angle A = 90^\circ$ (assumption)
$\angle B = 90^\circ$ (assumption)

By the Angles in a Triangle Theorem, $\angle P = 0^\circ$.

This contradicts the given information that PAB is a triangle.
The assumption that 2 perpendicular lines can be drawn is incorrect.
Therefore, only 1 perpendicular line can be drawn.

Selected Solutions — Chapter 6

4.



In the diagram, the alternate angles are equal, that is,
∠BAD = ∠ABC.
Either l_1 and l_2 are parallel or they are not.
Assume the lines are not parallel. Then, l_1 must intersect l_2 at some point C.
∠BAD = ∠ABC + ∠BCA   (Exterior Angle Theorem)
This contradicts the given information that ∠BAD = ∠ABC.
The assumption that the lines are not parallel must be incorrect.
Therefore, the lines must be parallel.

5. Either n is even or n is odd
Assume that n is even.
If n is even, n can be written in the form $2k$, where k is an integer.
$n^2 = (2k)^2$
$= 4k^2$, which is even
This is a contradiction, because we are given n^2 is odd.
The assumption is incorrect and hence n must be odd.

6. Either n is even or n is odd.
Assume that n is odd.
Since n is odd, n can be written in the form $2k + 1$, where k is an integer.
$n^2 = (2k + 1)^2$
$= 4k^2 + 4k + 1$
$= 4(k^2 + k) + 1$, which is odd
This is a contradiction, because we are given n^2 is even.
The assumption is incorrect. Therefore, n must be even.

7. Either both m and n are odd or at least one of them is even.
Assume one of m or n is even
Case 1: Assume that m is even and n may be even or odd.
Then m can be written in the form $2k$, where k is an integer.
$mn = 2kn$, which is even
This is a contradiction, because we are given mn is odd.
The assumption is incorrect; m must be odd.
Case 2: Assume that n is even and m may be even or odd.
Then n can be written in the form $2j$, where j is an integer.
$mn = 2mj$, which is even
This is a contradiction, because we are given mn is odd.
The assumption is incorrect; n must be odd.
Combining Case 1 and Case 2, the assumption that at least one of m or n is even is incorrect. Therefore, both m and n are odd.

Selected Solutions — Chapter 6

8. Either $AB \neq AC$ or $AB = AC$
Assume $AB = AC$

In $\triangle ABC$, $AB = AC$ (assumption)
Therefore, $\triangle ABC$ is isosceles and $\angle B = \angle C$

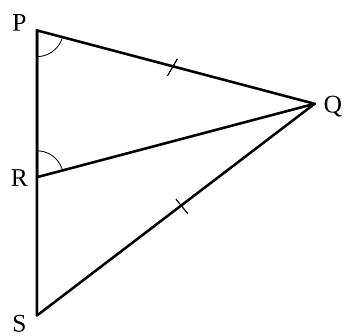
In $\triangle ABM$ and $\triangle ACM$,
$AB = AC$ (assumption)
$\angle B = \angle C$
$BM = CM$
Therefore, $\triangle ABM \cong \triangle ACM$ (SAS)
Since the triangles are congruent, $\angle AMB = \angle AMC$
Since BMC is a straight line, $\angle AMB = \angle AMC = 90°$
This contradicts the given information; that is, $\angle AMC = 60°$.
The assumption $AB = AC$ is incorrect. Therefore, $AB \neq AC$.

9. Either $PQ \neq PR$ or $PQ = PR$
Assume $PQ = PR$

In $\triangle PQR$, $PQ = PR$
Therefore, $\triangle PQR$ is isosceles and $\angle Q = \angle R$
We are given $\angle PSR = 90°$.
Since $\angle QSR$ is a straight angle, then $\angle PSQ = 90°$.
From the Angles in a Triangle Theorem in each of $\triangle PQS$ and $\triangle PRS$, $\angle QPS = \angle RPS$.

In $\triangle PQS$ and $\triangle PRS$,
$\angle PSQ = \angle PSR$
$PS = PS$
$\angle QPS = \angle RPS$
Therefore, $\triangle PQS \cong \triangle PRS$ (ASA)
Since the triangles are congruent, $QS = RS$
This contradicts the given information; that is, $QS \neq RS$
The assumption $PQ = PR$ is incorrect. Therefore, $PQ \neq PR$

10. Either $PQ = RQ$ or $PQ \neq RQ$.
Assume $PQ \neq RQ$.
Then there is a point S on PR extended such that $PQ = SQ$.



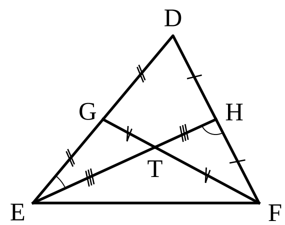
By the Isosceles Triangle Theorem,
$\angle QSP = \angle QPS$ ①
$\angle QPS = \angle QRP$ (given) ②

Compare ① and ②.
$\angle QSP = \angle QRP$.
Thus, since these are corresponding angles, QS is parallel to QR.

Selected Solutions — Chapter 6

This is impossible, because QS intersects QR.
Hence, the assumption that PQ ≠ RQ is incorrect.
Therefore, PQ = RQ.

11. Either the medians bisect each other or they do not.
Assume the medians bisect each other at T; that is,
GT = TF and ET = TH.



By the Opposite Angles Theorem, ∠GTE = ∠FTH

In △GTE and △FTH,

GT = FT

∠GTE = ∠FTH

ET = HT

Therefore, △GTE ≅ △FTH (SAS)

Since the triangles are congruent, ∠GET = ∠FHT

Thus, DE is parallel to HF and hence to DF (by the converse of the Parallel Lines Theorem (proved in exercise 4))

This contradicts the given information that DEF is a triangle.

The assumption that the medians bisect each other is incorrect.
Therefore, the medians cannot bisect each other.

12. Assume that Anjanee is not the oldest.

Case 1: Assume from youngest to oldest is Anjanee, Blair, Concetta.
Then statements 1, 2, 3, and 4 are false; that is a contradiction since only one statement is true.

Case 2: Assume from youngest to oldest is Anjanee, Concetta, and Blair. Then statements 1 and 2 are true, and statements 3 and 4 are false; that is a contradiction since only one statement is true.

Case 3: Assume from youngest to oldest is Blair, Anjanee, and Concetta. Then statements 1, 2, 3, and 4 are false; that is a contradiction since only one statement is true.

Case 4: Assume from youngest to oldest is Concetta, Anjanee, and Blair. Then statements 1 and 2 are false, and statements 3 and 4 are true; that is a contradiction since only one statement is true.

Since all possibilities contradict the fact that only one statement is true, Anjanee must be the oldest.

Selected Solutions — Chapter 6

Linking Ideas: Mathematics & History
Measuring Earth's Circumference, page 417

3. a) The distance from Syene to Alexandria is $\frac{1}{48}$th the circumference of Earth. Therefore, the circumference of Earth is (48 × 5000) stades or 240 000 stades.

b) 240 000 stades × 157 m/stade = 37 680 km
The Earth's circumference is 37 680 km or 37 680 000 m.

4. a) The distance from the equator to the North Pole is one quarter the circumference of Earth. Therefore, the circumference is 40 000 km.

b) The difference was 40 000 km – 37 680 km; a difference of 2320 km.

5. He may have asked a friend in advance to measure a shadow at a certain time.

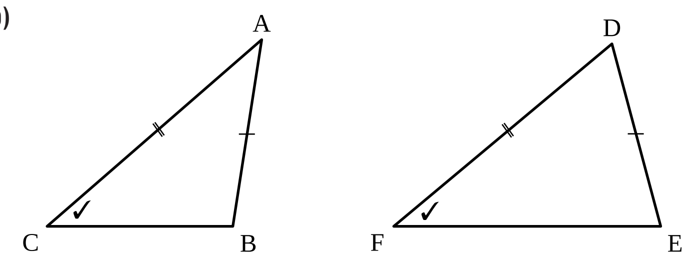
6 Review, page 418

3. The sum of ∠ABD and ∠CBD is a straight angle, whose measure is 180˚. Since ∠ABD and ∠CBD are equal, they must each equal 90˚. Specifically, ∠ABD = 90˚

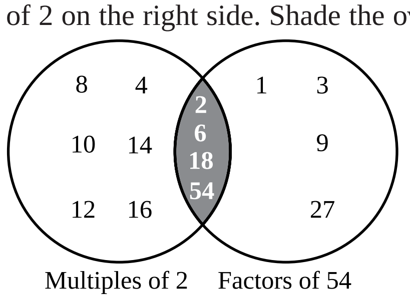
4. PQRS is a square, which means the sides are equal. Specifically, PS = QP. In addition, △PST is equilateral; therefore, its sides are equal. Most importantly, PS = PT. Compare the two equations.
QP = PT.

With two sides equal, △PQT is isosceles.

5. b)



8. Explanations may vary. For exercise 7a:
Draw two overlapping circles. Write the numbers that satisfy the statement in the overlapping part. Write multiples of 2 that are not factors of 54 on the left side, and factors of 54 that are not multiples of 2 on the right side. Shade the overlapping part.



Selected Solutions — Chapter 6

12. In △JLM and △LJK,

JM = LK

ML = KJ

JL = LJ

Therefore, △JLM ≅ △LJK (SSS)

Since the triangles are congruent, ∠M = ∠K.

13. In △ABD and △ACD,

AB = AC

BD = CD

AD = AD

Therefore, △ABD ≅ △ACD (SSS)

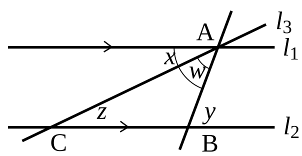
Since the triangles are congruent, ∠BAD = ∠CAD.

Therefore, AD bisects ∠A.

14. Assume the alternate angles are not equal: $x \neq y$.

Then there must be another line l_3 through A such that the alternate angles are equal; $w = y$.

l_3 is not parallel to l_2 and meets l_2 at C.

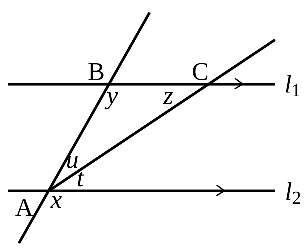


According to the Exterior Angle Theorem, $y = z + w$.

This contradicts the information that $w = y$.

Hence, the assumption that the alternate angles are not equal is incorrect. Therefore, the alternate angles must be equal.

15. Construct a diagram. Lines l_1 and l_2 are parallel. x and y are corresponding angles.



Either $x = y$ or $x \neq y$.

Assume $x \neq y$.

In △ABC, $u + y + z = 180°$ ①

Since z and t are alternate angles, $z = t$ ②

Substitute for z from ② into ①.

$u + y + t = 180°$ ③

Since the angles u, t, and x create a straight angle,

$x + t + u = 180°$ ④

Compare ③ and ④.

$x = y$

This contradicts the assumption.

The assumption $x \neq y$ is incorrect. Therefore, $x = y$.

Selected Solutions — Chapter 6

6 Cumulative Review, page 420

3. Explanations may vary. For part a:
The formula for the accumulated amount is $A = P(1 + i)^n$. Since interest is 4% compounded semi-annually, $i = 0.02$. The principal is $P = 500$. It is invested for 5 years; therefore, $n = 10$. Using the calculator, $A = \$500(1 + 0.02)^{10}$ is evaluated to be \$609.50.

4. Compare the two funds at the end of 1 year.

a) Use $A = P(1 + i)^n$ for each deposit.
The January 1st deposit accumulates to

$$A = 300\left(1 + \frac{0.075}{4}\right)^4$$
$$= 323.14$$

The April 1st deposit accumulates to

$$A = 300\left(1 + \frac{0.075}{4}\right)^3$$
$$= 317.19$$

The July 1st deposit accumulates to

$$A = 300\left(1 + \frac{0.075}{4}\right)^2$$
$$= 311.36$$

The October 1st deposit accumulates to

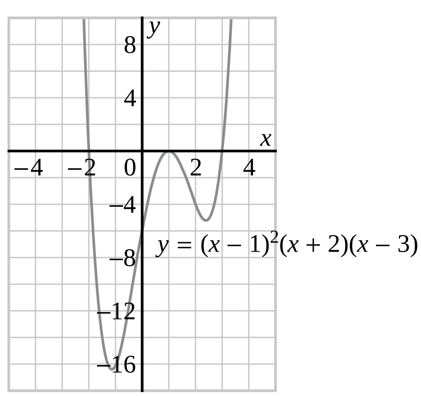
$$A = 300\left(1 + \frac{0.075}{4}\right)^1$$
$$= 305.63$$

The total in the fund at the end of 1 year :
$323.14 + 317.19 + 311.36 + 305.63 = 1257.32$

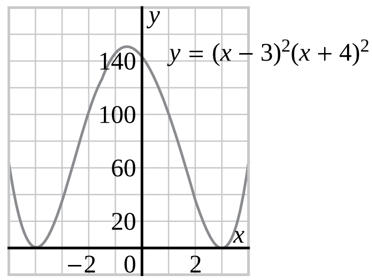
b) The same procedure as above may be used, or the TVM solver on the T1-83 (page 28) may be used to obtain the total in the fund at the end of 1 year. The total is \$1248.18.

Therefore, plan a is better because more money is accumulated in any given year. In the end, her son will have more money for his education.

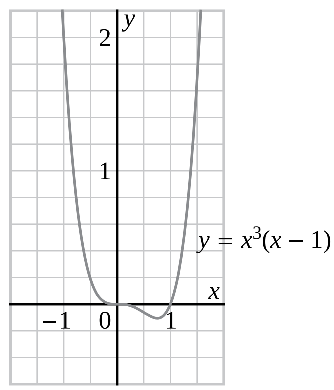
8. a)



b)



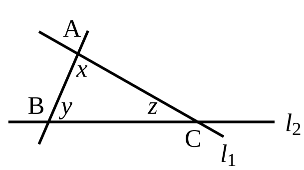
c)



11. Explanations may vary. For part b:
To determine $f(g(x))$, substitute the function $g(x) = 2x + 3$ for every x that occurs in $f(x) = x^2$. Specifically, $f(g(x)) = (2x + 3)^2$.
Now expand and simplify to get $4x^2 + 12x + 9$.
To determine $g(f(x))$, substitute the function $f(x) = x^2$ for every x in $g(x) = 2x + 3$. Specifically, $g(f(x)) = 2x^2 + 3$.

15. Explanations may vary. For part a: the trinomial cannot be factored; therefore, the quadratic formula is used to find the solution.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 1.5$, $b = 2.0$, $c = -3.2$.
The values are substituted to get $x = \frac{-2 \pm \sqrt{23.2}}{3}$.
The solutions are $x = 0.94$ and -2.27.

17. Use indirect proof.
Assume that l_1 and l_2 are not parallel.
Since l_1 and l_2 are not parallel, they will meet at some point C, as illustrated below.



In △ABC, $x + y + z = 180^\circ$, where $z \neq 0^\circ$.
This contradicts the given information that $x + y = 180^\circ$.
The assumption that l_1 and l_2 are not parallel is incorrect.
Therefore, l_1 and l_2 are parallel.