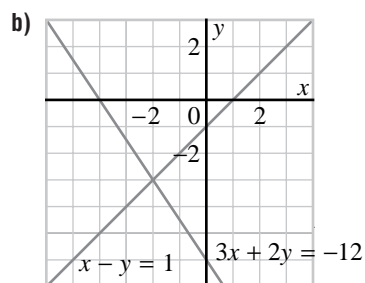
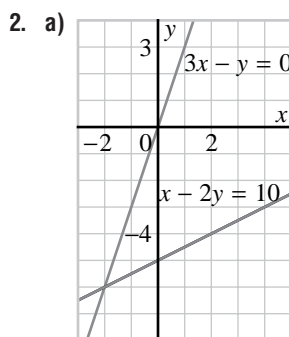
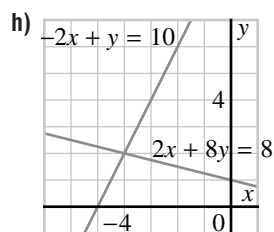
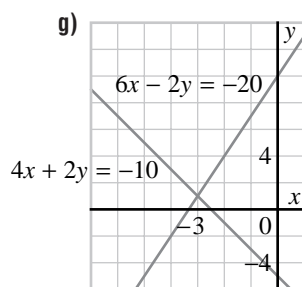
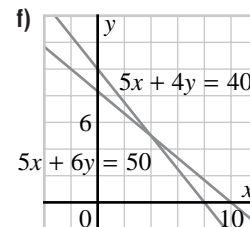
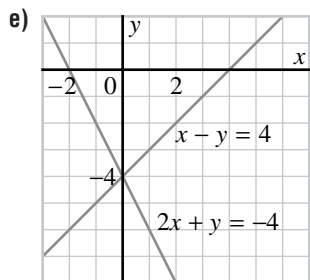
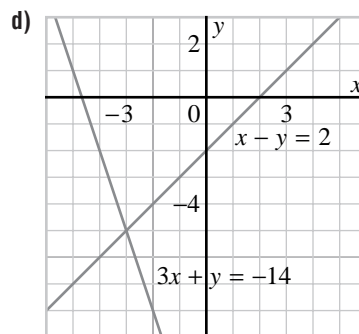
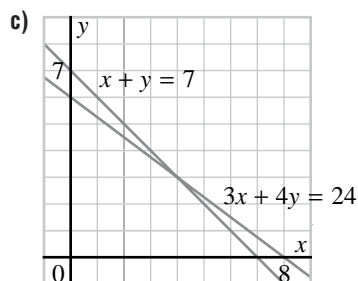
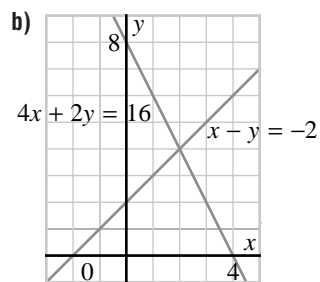
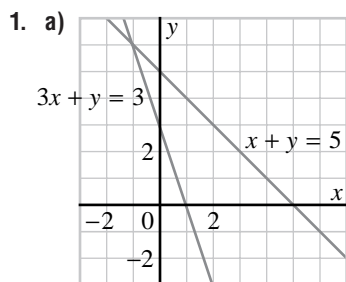
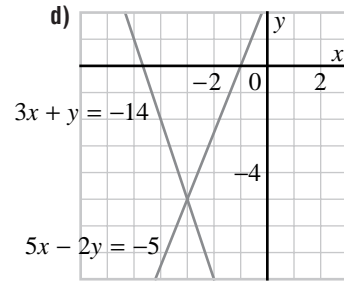
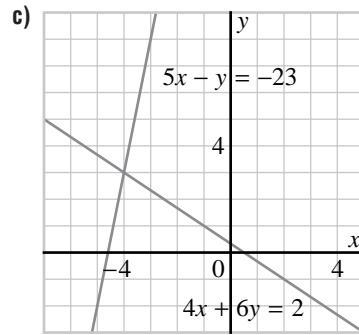


Selected Solutions — Chapter 5

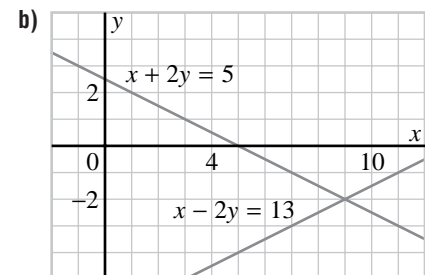
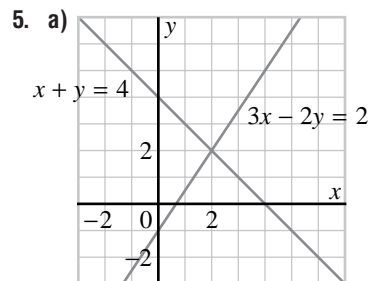
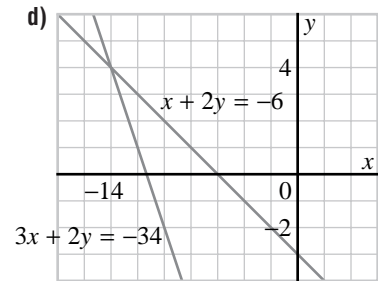
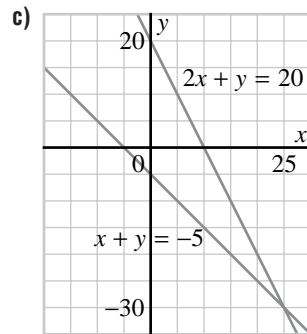
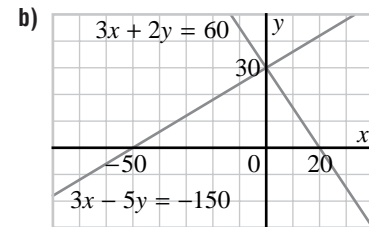
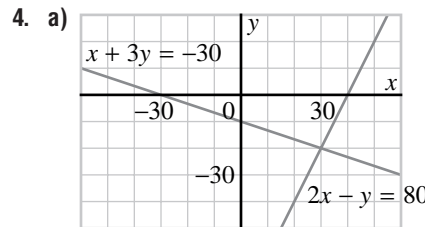
5.1 Exercises, page 305



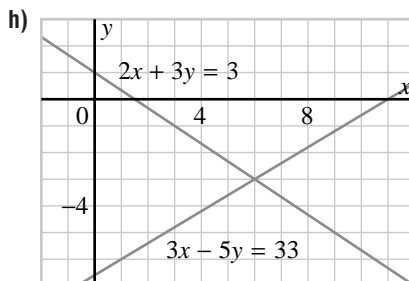
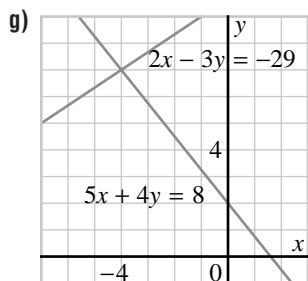
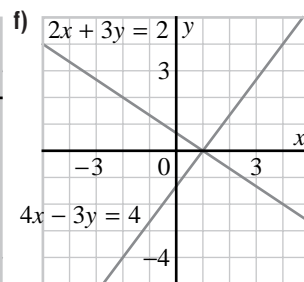
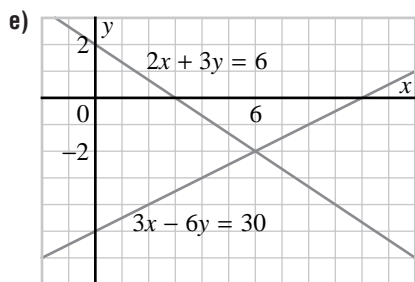
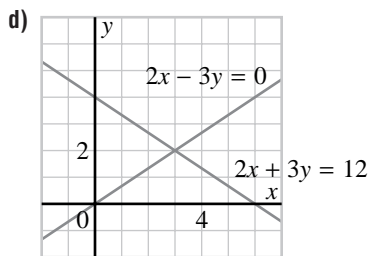
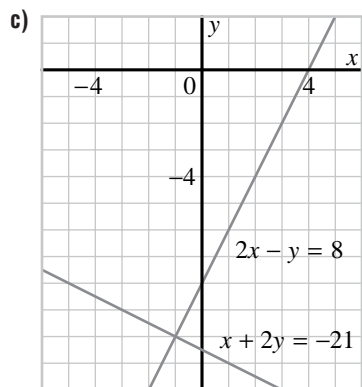
Selected Solutions — Chapter 5



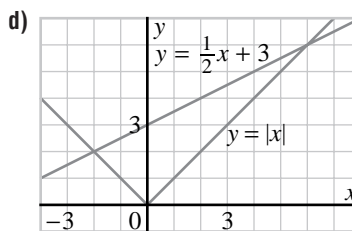
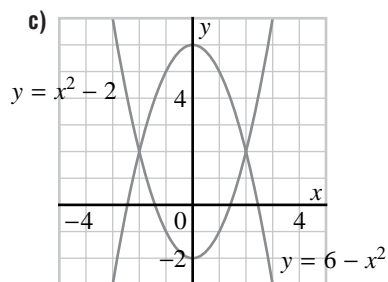
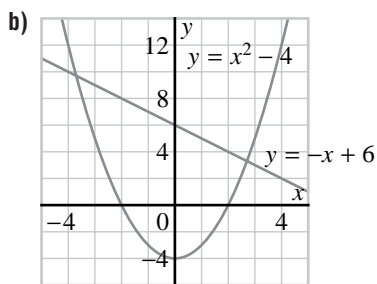
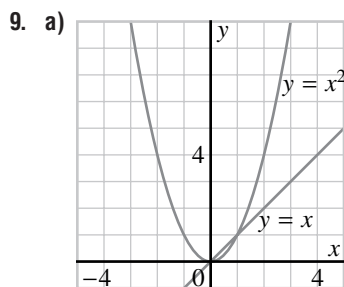
3. Explanations may vary. For part 2a: For equation $x - 2y = 10$, I rearranged the equation to $y = \frac{1}{2}x - 5$, then drew the graph by plotting the y -intercept at -5 , and using the slope $\frac{1}{2}$. For equation $3x - y = 0$, I rearranged the equation to $y = 3x$, then drew the graph by plotting the y -intercept 0 , and using the slope 3 . The point of intersection of the graphs, $(-2, -6)$, is the solution.



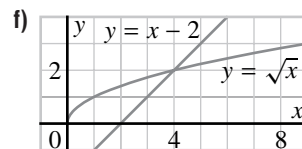
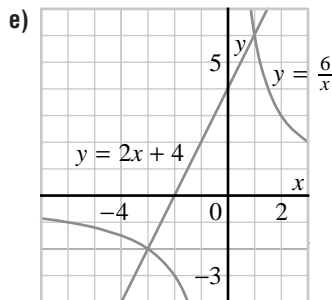
Selected Solutions — Chapter 5



8. The lines can be parallel, and not intersect at all, or they can be coincident, and intersect at every point.

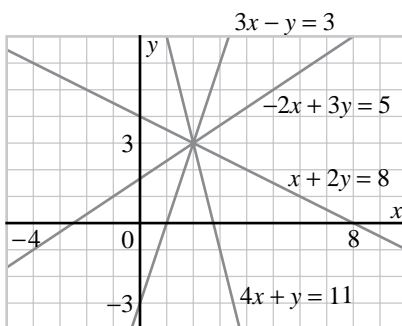


Selected Solutions — Chapter 5



10. Explanations may vary. In exercise 9a, I graphed $y = x^2$ as a parabola with vertex $(0, 0)$ passing through $(\pm 1, 1)$, $(\pm 2, 4)$. I graphed $y = x$ as a line with slope 1 and passing through $(0, 0)$. I determined the points of intersection, $(0, 0)$ and $(1, 1)$, from the graph.

11. a) The solution of the system is $(2, 3)$.



- b)
$$\begin{array}{r} x + 2y = 8 \quad \textcircled{1} \\ 3x - y = 3 \quad \textcircled{2} \\ \hline \text{Add. } 4x + y = 11 \\ \text{Rearrange. } y = 11 - 4x \end{array}$$

The line passes through $(2, 3)$.

c)
$$\begin{array}{r} x + 2y = 8 \quad \textcircled{1} \\ 3x - y = 3 \quad \textcircled{2} \\ \hline \text{Subtract. } -2x + 3y = 5 \\ \text{Rearrange. } y = \frac{2x + 5}{3} \end{array}$$

The line passes through $(2, 3)$.

12. a) Since one equation is $x + y = 6$, try substituting in the second equation two numbers whose sum is 6.
 Try $(1, 5)$, $(2, 4)$.
 $(2, 4)$ is the solution.
- b) Since one equation is $x - y = 1$, try substituting in the second equation two numbers whose difference is 1.
 Try $(1, 0)$.
 $(1, 0)$ is the solution.
- c) Since one equation is $x + y = -8$, try substituting two numbers whose sum is -8 .
 Try $(-1, -7)$, $(-2, -6)$, $(-3, -5)$.
 $(-3, -5)$ is the solution.

Selected Solutions — Chapter 5

- d) Since one equation is $2x + y = 13$, try substituting numbers x and y that satisfy this equation in the second equation.

For example, $x = 6, y = 1$

$$\begin{aligned} \text{L.S.} &= 6 + 2 \\ &= 8 \neq \text{R.S.} \end{aligned}$$

For example, $x = 5, y = 3$

$$\begin{aligned} \text{L.S.} &= 5 + 6 \\ &= 11 \neq \text{R.S.} \end{aligned}$$

The sum of $x + 2y$ is more than 7; so, if we continue with the value of x decreasing and y increasing, we will continue to get the right side too big.

Try fractional value of x and y .

For example, $x = 5.5, y = 2$

$$\begin{aligned} \text{L.S.} &= 5.5 + 4 \\ &= 9.5 \neq \text{R.S.} \end{aligned}$$

Values of x that involve 0.5 will always make the right side involve 0.5, too.

Try $x = 5\frac{1}{3}, y = 2\frac{2}{3}$

$$\begin{aligned} \text{L.S.} &= 5\frac{1}{3} + 5\frac{1}{3} \\ &= 10\frac{2}{3} \neq \text{R.S.} \end{aligned}$$

Try $x = 6\frac{1}{3}, y = \frac{1}{3}$

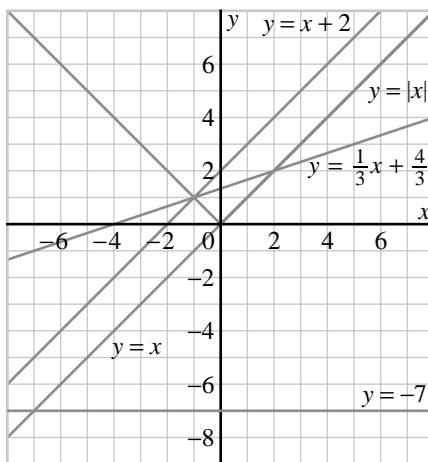
$$\begin{aligned} \text{L.S.} &= 6\frac{1}{3} + \frac{2}{3} \\ &= 7 = \text{R.S.} \end{aligned}$$

The solution is $\left(\frac{19}{3}, \frac{1}{3}\right)$

13. Graphs may vary.

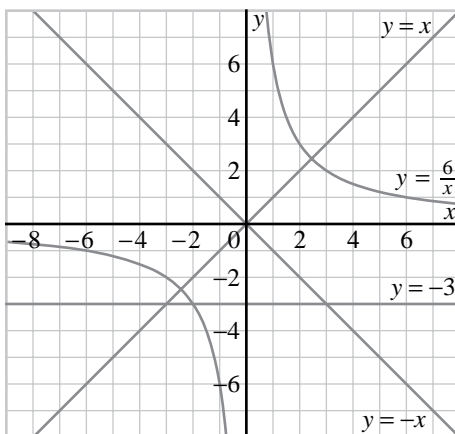
- a) The graphs can have 0 points of intersection

($y = -7$), 1 point of intersection ($y = x + 2$), 2 points of intersection ($y = \frac{1}{3}x + \frac{4}{3}$), or infinitely many points of intersection ($y = x$).

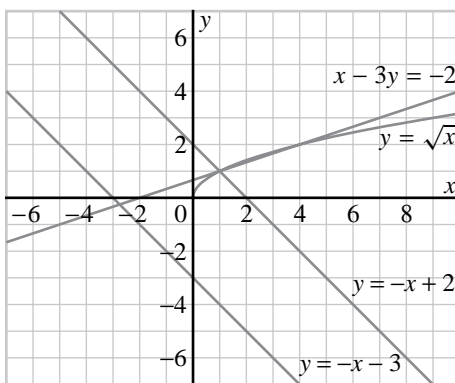


Selected Solutions — Chapter 5

- b) The graphs can have 0 points of intersection ($y = -x$), 1 point of intersection ($y = -3$), or 2 points of intersection ($y = x$).

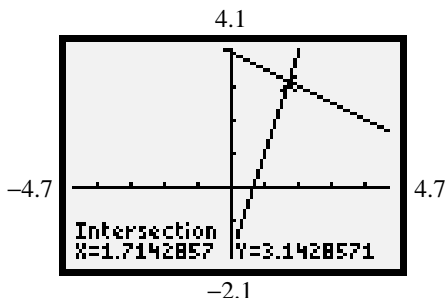


- c) The graphs can have 0 points of intersection ($y = -x - 3$), 1 point of intersection ($y = -x + 2$), or 2 points of intersection ($x - 3y = -2$).

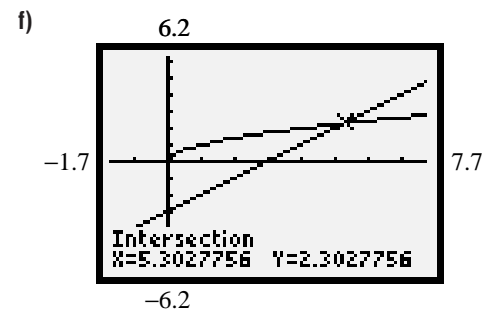
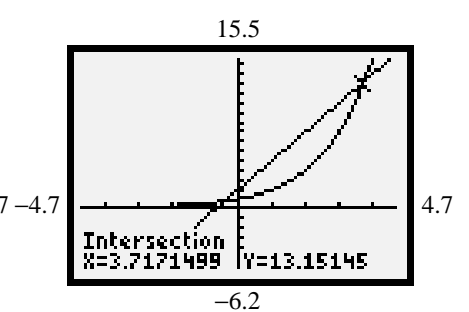
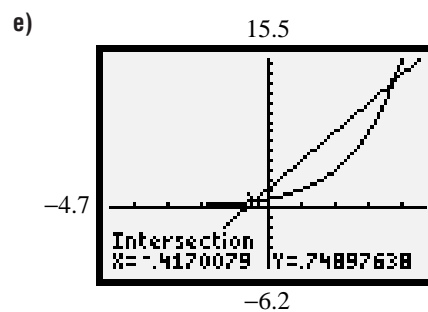
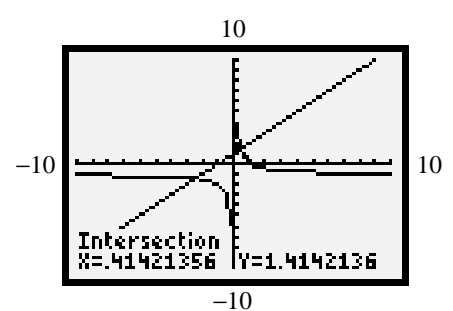
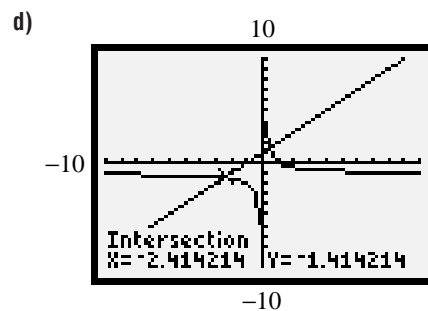
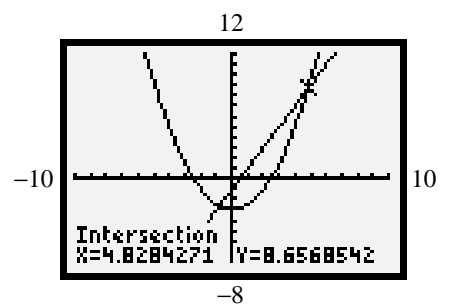
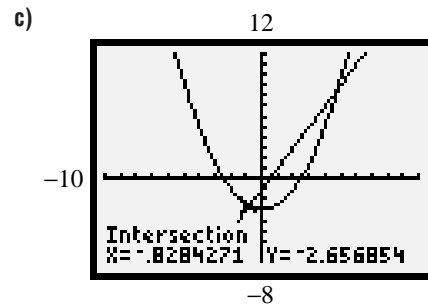
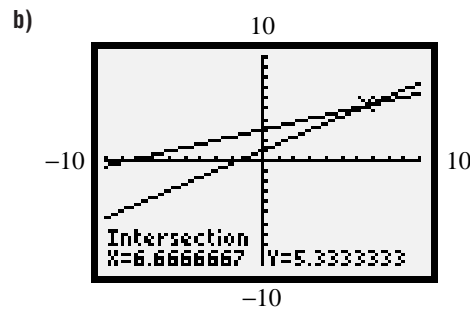


Exploring With a Graphing Calculator:
Solving Systems of Equations, page 309

2. a)



Selected Solutions — Chapter 5



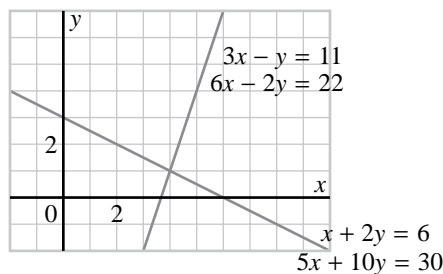
4. Answers may vary. Graph $y = x^2 + 1$ and $y = 0.99$ on the calculator. These appear to intersect at $x = 0$. Attempt to find the intersection point using the calculator.

Selected Solutions — Chapter 5

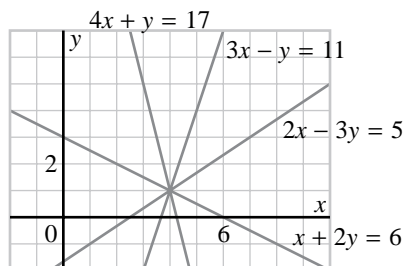
The closest points are $(0, 1)$ on the parabola and $(0, 0.99)$ on the line. Use the zoom feature on the calculator. Zoom in several times on the apparent intersection point, to show that the line and curve do not intersect.

Investigate, page 310

1. a), b)



3. b), d)



6. Explanations may vary. In exercise 5a, multiply equation ② by 2.

This does not change the solution.

Copy ① underneath the new equation.

Add the equations in the new system.

$$\begin{array}{r} 2x - 2y = 2 \\ 3x + 2y = 18 \\ \hline 5x = 20 \\ x = 4 \end{array}$$

Substitute $x = 4$ into $3x + 2y = 18$.

$$\begin{array}{r} 3(4) + 2y = 18 \\ 2y = 6 \\ y = 3 \end{array}$$

The solution is $(4, 3)$.**Modelling the Temperature of Earth's Crust, page 314**

0.021 represents the rise in temperature for each metre below the surface.
17.95 represents the temperature at the surface of Earth.

The graph is a straight line with slope 0.021 and y-intercept 17.95.

The temperature at the bottom of the mine is about 93°C . The temperature at the bottom of the geological drilling is about 333°C .

Assuming Earth's crust has the same composition at all depths, the function is reasonable for a domain from $d = 0$ to $d =$ width of Earth's crust.

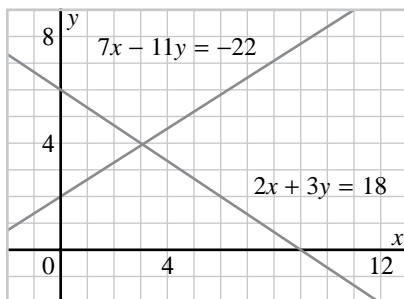
Selected Solutions — Chapter 5

5.2 Exercises, page 315

4. Explanations may vary. In exercise 2a, I subtracted equation ② from equation ① to get $6y = 24$. Therefore, $y = 4$. I substituted this into ① to get $2x + 3(4) = 18$. Solve this for x to get $x = 3$. Therefore, the solution is $(3, 4)$.

Explanations may vary. In exercise 3a, I multiplied equation ① by 3 to get $9x + 3y = 9$. Then, I subtracted equation ② from this new equation to get $7x = 14$. Therefore, $x = 2$. I substituted this into equation ① to get $3(2) + y = 3$. I solved this for y to get $y = -3$. Therefore, the solution is $(2, -3)$.

7. c) m represents the rate at which the temperature decreases. b is the temperature at Earth's surface.
 d) The domain is from 0 to the highest point on Earth's surface.
8. d) Answers may vary. The domain could be 0 to 100 000 km.
18. b) Explanations may vary. In exercise 18a, I, substituted 5 for x and -3 for y in $mx - y = 23$ to get $5m - (-3) = 23$. I solved this for m to get $m = 4$. To find n , I substituted 5 for x and -3 for y in $nx + y = 12$ to get $5n + (-3) = 12$. I solved this for n to get $n = 3$.
19. a) The solution appears to be $(3, 4)$.



b) $7x - 11y = -22$	①
$2x + 3y = 18$	②
Multiply ① by 2.	$14x - 22y = -44$
Multiply ② by 7.	$14x + 21y = 126$
Subtract ① from ②.	$43y = 170$
	$y = \frac{170}{43}$
Multiply ① by 3.	$21x - 33y = -66$
Multiply ② by 11.	$22x + 33y = 198$
Add.	$43x = 132$
	$x = \frac{132}{43}$

The solution is $\left(\frac{132}{43}, \frac{170}{43}\right)$ or to 2 decimal places is $(3.07, 3.95)$.

- c) The results are different. The point of intersection is close to, but not exactly, $(3, 4)$.

Selected Solutions — Chapter 5

20. a) Substitute in $2x - 3y = 10$.

$$\begin{aligned}(2, -2) : \quad \text{L.S.} &= 2(2) - 3(-2) \\ &= 10 \\ &= \text{R.S.}\end{aligned}$$

Substitute in $-4x + 6y = -20$.

$$\begin{aligned}(2, -2) : \quad \text{L.S.} &= -4(2) + 6(-2) \\ &= -20 \\ &= \text{R.S.}\end{aligned}$$

Substitute in $2x - 3y = 10$.

$$\begin{aligned}(5, 0) : \quad \text{L.S.} &= 2(5) - 3(0) \\ &= 10 \\ &= \text{R.S.}\end{aligned}$$

Substitute in $-4x + 6y = -20$.

$$\begin{aligned}(5, 0) : \quad \text{L.S.} &= -4(5) + 6(0) \\ &= -20 \\ &= \text{R.S.}\end{aligned}$$

Substitute in $2x - 3y = 10$.

$$\begin{aligned}(8, 2) : \quad \text{L.S.} &= 2(8) - 3(2) \\ &= 10 \\ &= \text{R.S.}\end{aligned}$$

Substitute in $-4x + 6y = -20$.

$$\begin{aligned}(8, 2) : \quad \text{L.S.} &= -4(8) + 6(2) \\ &= -20 \\ &= \text{R.S.}\end{aligned}$$

b) There are patterns in the x - and y -coordinates.

The x -coordinates increase by 3.

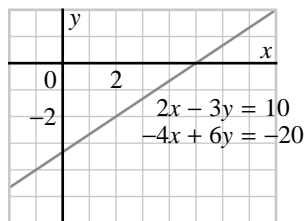
The y -coordinates increase by 2.

Two possible solutions are $(8 + 3, 2 + 2)$, or $(11, 4)$, and

$(11 + 3, 4 + 2)$, or $(14, 6)$.

Substitute to check.

c) The two lines coincide, providing an infinite number of solutions.



d) $2x - 3y = 10$ ①

$-4x + 6y = -30$ ②

Copy ①: $2x - 3y = 10$

Divide ② by -2 : $2x - 3y = 10$

Subtract: $0 = 0$

Any ordered pair that satisfies the first equation satisfies the second equation.

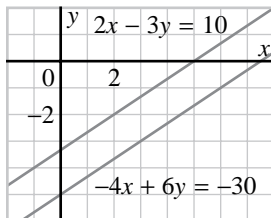
Selected Solutions — Chapter 5

$$21. \text{ a) } \begin{aligned} 2x - 3y &= 10 & \textcircled{1} \\ -4x + 6y &= -30 & \textcircled{2} \end{aligned}$$

$$\begin{array}{l} \text{Copy } \textcircled{1}. \qquad 2x - 3y = 10 \\ \text{Divide } \textcircled{2} \text{ by } 2. \quad \underline{-2x + 3y = -15} \\ \text{Add.} \qquad \qquad \qquad 0 = -5 \end{array}$$

This is impossible. There are no ordered pairs that satisfy this system.

b) The lines are parallel. They do not intersect.



c) The lines do not intersect; therefore, there is no solution.

5.3 Exercises, page 321

4. b) Explanations may vary. In exercise 4a, part i, the lines are not parallel because the coefficients of x and y are not in the same ratio. Thus, they intersect, and the system is consistent.

8. a) A system of lines that are coincident has an infinite number of solutions. For example,

$$\begin{aligned} 3x + 3y &= 6 & \textcircled{1} \\ x + y &= 2 & \textcircled{2} \end{aligned}$$

Test Property 1

$$\text{Multiply } \textcircled{2} \text{ by } 2. \qquad 2x + 2y = 4$$

This line is coincident with $\textcircled{1}$ and $\textcircled{2}$; therefore, the new system has an infinite number of solutions.

Test Property 2

$$\text{i) Add } \textcircled{1} \text{ and } \textcircled{2}. \qquad 4x + 4y = 8$$

This line is coincident with $\textcircled{1}$ and $\textcircled{2}$; therefore, the new system has an infinite number of solutions.

$$\text{ii) Subtract } \textcircled{2} \text{ from } \textcircled{1}. \qquad 2x + 2y = 4$$

This line is also coincident with $\textcircled{1}$ and $\textcircled{2}$.

Combining the two tests, the Properties of Linear Systems apply to a system with infinitely many solutions.

b) A system of lines that are parallel has no solution. For example,

$$\begin{aligned} 3x + 3y &= 6 & \textcircled{1} \\ x + y &= 1 & \textcircled{2} \end{aligned}$$

Test Property 1

$$\text{Multiply } \textcircled{2} \text{ by } 2. \qquad 2x + 2y = 2$$

This line is parallel to $\textcircled{1}$; therefore, the new system has no solution.

Selected Solutions — Chapter 5

Test Property 2

i) Add ① and ②. $4x + 4y = 7$

This line is parallel to both ① and ②; therefore, the new system has no solution.

ii) Subtract ② from ①. $2x + 2y = 5$

This line is parallel to both ① and ②.

Combining the two tests, the Properties of Linear Systems apply to a system with no solution.

9. a) $3x + y = 9$ ①

$6x + 2y = 18$ ③

(1, 6): Equation ① L.S. = $3(1) + 6$
 $= 9$
 $= \text{R.S.}$

Equation ③ L.S. = $6(1) + 2(6)$
 $= 18$
 $= \text{R.S.}$

(2, 3): Equation ① L.S. = $3(2) + 3$
 $= 9$
 $= \text{R.S.}$

Equation ③ L.S. = $6(2) + 2(3)$
 $= 18$
 $= \text{R.S.}$

(3, 0): Equation ① L.S. = $3(3) + 0$
 $= 9$
 $= \text{R.S.}$

Equation ③ L.S. = $6(3) + 2(0)$
 $= 18$
 $= \text{R.S.}$

(4, -3): Equation ① L.S. = $3(4) - 3$
 $= 9$
 $= \text{R.S.}$

Equation ③ L.S. = $6(4) + 2(-3)$
 $= 18$
 $= \text{R.S.}$

(5, -6): Equation ① L.S. = $3(5) - 6$
 $= 9$
 $= \text{R.S.}$

Equation ③ L.S. = $6(5) + 2(-6)$
 $= 18$
 $= \text{R.S.}$

- b) For the n th ordered pair, $x = n$.
 Substitute $x = n$ in equation ①.

$$3n + y = 9$$

$$y = 9 - 3n$$

The ordered pair is $(n, 9 - 3n)$.

Substitute this ordered pair in each equation to check.

Selected Solutions — Chapter 5

$$\begin{aligned}\text{Equation ① L.S.} &= 3n + 9 - 3n \\ &= 9 \\ &= \text{R.S.}\end{aligned}$$

$$\begin{aligned}\text{Equation ③ L.S.} &= 6n + 2(9 - 3n) \\ &= 6n + 18 - 6n \\ &= 18 \\ &= \text{R.S.}\end{aligned}$$

c) All values of n are possible, since the domain of both functions is the set of real numbers.

d) The system in exercise 6 with infinitely many solutions is:

$$\begin{aligned}4x - y &= 11 && \text{①} \\ -12x + 3y &= -33 && \text{②}\end{aligned}$$

$$\begin{aligned}\text{Substitute } x = n \text{ in equation ①.} \quad 4n - y &= 11 \\ y &= 4n - 11\end{aligned}$$

The ordered pair is $(n, 4n - 11)$.

Substitute this ordered pair in each equation to check.

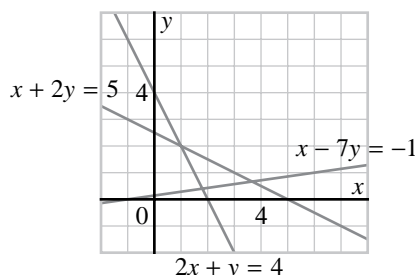
$$\begin{aligned}\text{Equation ① L.S.} &= 4n - (4n - 11) \\ &= 11 \\ &= \text{R.S.}\end{aligned}$$

$$\begin{aligned}\text{Equation ② L.S.} &= -12n + 3(4n - 11) \\ &= -12n + 12n - 33 \\ &= -33 \\ &= \text{R.S.}\end{aligned}$$

10. a) • Three lines can intersect at one point. This may occur in two ways: i) 3 lines have different slopes, or ii) 2 lines are coincident and the third line has a different slope.
- Three lines might not intersect at all. This may occur in 3 ways: i) 3 parallel but not coincident lines, ii) 2 parallel but not coincident lines, and the third line a different slope, or iii) 3 lines with different slopes intersecting in pairs.
- Three lines may have an infinite number of point of intersection. This occurs when all three lines are coincident.

b) i) $x + 2y = 5$
 $x - 7y = -1$
 $2x + y = 4$

Since no two equations represent parallel lines or coincident lines, the lines may intersect in pairs to form a triangle or intersect at one point. Graph to check.

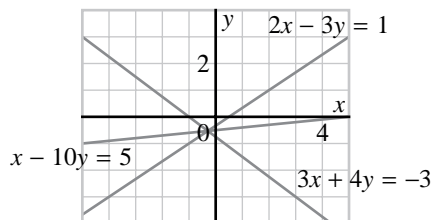


The system is inconsistent because the lines intersect in pairs.

Selected Solutions — Chapter 5

$$\begin{aligned} \text{ii) } 2x - 3y &= 1 & \textcircled{1} \\ 3x + 4y &= -3 & \textcircled{2} \\ x - 10y &= 5 & \textcircled{3} \end{aligned}$$

Same discussion applies as in part i: Graph to check.



The lines appear to intersect at one point. Check algebraically.

$$\begin{array}{rcl} \text{Multiply } \textcircled{1} \text{ by } 3. & 6x - 9y = 3 & \textcircled{4} \\ \text{Multiply } \textcircled{2} \text{ by } 2. & 6x + 8y = -6 & \textcircled{5} \\ \text{Subtract } \textcircled{5} \text{ from } \textcircled{4}. & -17y = 9 & \\ & y = \frac{7}{-17} & \end{array}$$

$$\begin{array}{rcl} \text{Multiply } \textcircled{1} \text{ by } 4. & 8x - 12y = 4 & \textcircled{6} \\ \text{Multiply } \textcircled{2} \text{ by } 3. & 9x + 12y = -9 & \textcircled{7} \\ \text{Add.} & 17x = -5 & \\ & x = \frac{-5}{17} & \end{array}$$

Lines $\textcircled{1}$ and $\textcircled{2}$ intersect at $\left(-\frac{5}{17}, -\frac{9}{17}\right)$.

$$\begin{array}{rcl} \text{Multiply } \textcircled{3} \text{ by } 3. & 3x - 30y = 15 & \textcircled{8} \\ \text{Subtract } \textcircled{2} \text{ from } \textcircled{8}. & 3x + 4y = -3 & \textcircled{2} \\ & -34y = 18 & \\ & y = \frac{-9}{17} & \end{array}$$

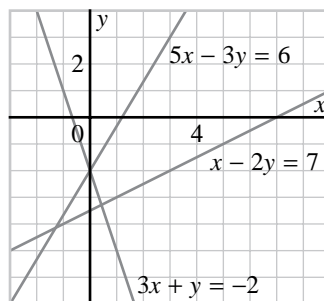
$$\begin{array}{rcl} \text{Multiply } \textcircled{2} \text{ by } \textcircled{10}. & 30x + 40y = -30 & \\ \text{Multiply } \textcircled{3} \text{ by } \textcircled{4}. & 4x - 40y = 20 & \\ \text{Add.} & 34x = -10 & \\ & x = \frac{-5}{17} & \end{array}$$

Lines $\textcircled{2}$ and $\textcircled{3}$ intersect at $\left(\frac{-5}{17}, \frac{-9}{17}\right)$.

The lines intersect at one point, so the system is consistent.

$$\begin{aligned} \text{iii) } 3x + y &= -2 & \textcircled{1} \\ x - 2y &= 7 & \textcircled{2} \\ 5x - 3y &= 6 & \textcircled{3} \end{aligned}$$

Same discussion applies as in part i. Graph to check.



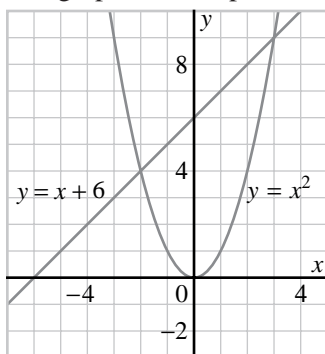
Selected Solutions — Chapter 5

The system is inconsistent because the lines intersect in pairs.

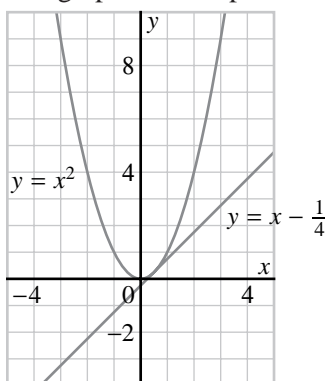
5.4 Exercises, page 325

3. Explanations may vary. For exercise 2a: I labelled $x + y = 9$ as equation ①, then solved for y to get $y = 9 - x$, which I labelled as equation ③. I substituted for y from ③ into $2x + y = 11$ (which I labelled equation ②) to get $2x + (9 - x) = 11$. I solved this equation for x to get $x = 2$. I substituted $x = 2$ in equation ③ to get $y = 7$. The solution is $(2, 7)$.

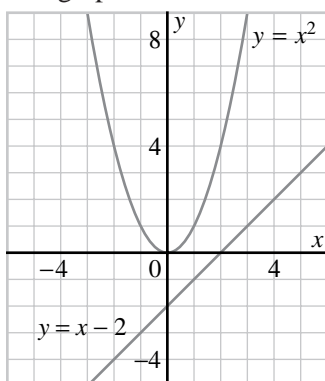
6. a) The graphs have 2 points of intersection.



- b) The graphs have 1 point of intersection.



- c) The graphs do not intersect.



Selected Solutions — Chapter 5

7. c) The equations have the form $y = 2x + 2k + 1$, where k is an integer.

The system is

$$y = x^2 \quad \textcircled{1}$$

$$y = 2x + 2k + 1 \quad \textcircled{2}$$

Substitute x^2 for y in equation $\textcircled{2}$.

$$x^2 = 2x + 2k + 1$$

$$x^2 - 2x - 2k - 1 = 0$$

Use the quadratic formula.

$$x = \frac{2 \pm \sqrt{(-2)^2 + 4(1)(-2k - 1)}}{2}$$

$$= \frac{2 \pm \sqrt{8 + 8k}}{2}$$

$$= \frac{2 \pm 2\sqrt{2k + 2}}{2}$$

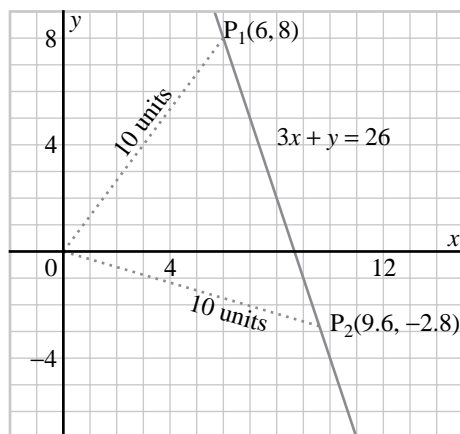
$$= 1 \pm \sqrt{2k + 2}$$

which is equal to $1 \pm \sqrt{0}$, $1 \pm \sqrt{2}$, $1 \pm \sqrt{4}$, ... for

$k = -1, 0, 1, \dots$

12. Explanations may vary. For exercise 8a, I substituted x^2 for y in the second equation to get $x^2 = 8 - x^2$. I simplified and solved for x to get $x = \pm 2$. I substituted 2 for x in the first equation to get $y = 4$. I substituted -2 for x in the first equation to get $y = 4$. The solutions are $(2, 4)$ and $(-2, 4)$.

13.



Modelling a Skydiver's Height, page 327

Before opening the parachute, the height is a parabolic function, as the skydiver's speed is affected by gravity only. After opening the parachute, the speed is affected by gravity and the drag on the parachute.

Before she releases the parachute, she is accelerating, which implies a quadratic function. After she releases the parachute, she is traveling at a constant speed, which implies a linear function.

Before she releases her parachute, the graph is a parabola opening down. After she releases her parachute, the graph is a straight line with a negative slope.

Selected Solutions — Chapter 5

Many factors may affect the height, such as air friction, the time the parachute takes to release, and the reaction time of the skydiver; that is, she may not open her parachute when she has fallen 1000 m.

$$17. \text{ a) i) } y = 3x^2 + 4x + 2 \quad \textcircled{1}$$

$$y = 0.5x^2 + 4x + 2 \quad \textcircled{2}$$

Substitute $3x^2 + 4x + 2$ for y in equation $\textcircled{2}$.

$$3x^2 + 4x + 2 = 0.5x^2 + 4x + 2$$

$$2.5x^2 = 0$$

$$x = 0$$

Substitute 0 for x in equation $\textcircled{1}$.

$$y = 3(0)^2 + 4(0) + 2$$

$$= 2$$

The solution is $(0, 2)$.

$$\text{ii) } y = 5x^2 - 2x + 3 \quad \textcircled{1}$$

$$y = -2x^2 - 2x + 3 \quad \textcircled{2}$$

Substitute $5x^2 - 2x + 3$ for y in equation $\textcircled{2}$.

$$5x^2 - 2x + 3 = -2x^2 - 2x + 3$$

$$7x^2 = 0$$

$$x = 0$$

Substitute 0 for x in equation $\textcircled{1}$.

$$y = 5(0)^2 - 2(0) + 3$$

$$= 3$$

The solution is $(0, 3)$.

b) The two parabolas defined by

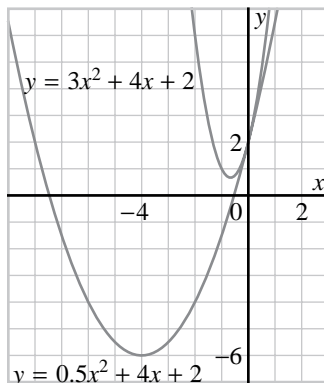
$$y = a_1x^2 + bx + c \quad \textcircled{1} \text{ and}$$

$$y = a_2x^2 + bx + c \quad \textcircled{2}$$

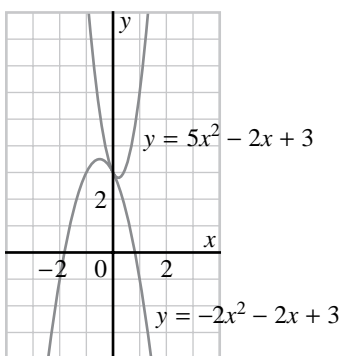
have the same y -intercept $(0, c)$. $(0, c)$ is also the only point of intersection.

Using algebra, substitute $a_2x^2 + bx + c$ for y in $\textcircled{1}$, to obtain $a_2x^2 + bx + c = a_1x^2 + bx + c$. This simplifies to $(a_2 - a_1)x^2 = 0$.

The only solutions to this are $a_1 = a_2$ (coincident parabolas) or $x = 0$. Therefore, the two parabolas, as defined, will always have only one point of intersection $(0, c)$.



Selected Solutions — Chapter 5



18. Solve the system of equations.

$$y = 2x + b \quad \text{①}$$

$$y = (x - 3)^2 + 2 \quad \text{②}$$

Substitute $2x + b$ for y in equation ②.

$$2x + b = (x - 3)^2 + 2$$

$$2x + b = x^2 - 6x + 11$$

$$x^2 - 8x + 11 - b = 0$$

For there to be only one solution, $x^2 - 8x + 11 - b$ must be a perfect square. The term $11 - b$ must equal $\left(\frac{1}{2} \text{ coefficient of } x\right)^2$. Thus,

$$11 - b = 16$$

$$b = -5$$

19. a) i) $y = \frac{6}{x} \quad \text{①}$

$$y = 1 - x^2 \quad \text{②}$$

Substitute $\frac{6}{x}$ for y in equation ②.

$$\frac{6}{x} = 1 - x^2$$

$$6 = x - x^3$$

$$x^3 - x + 6 = 0$$

Use the Factor Theorem.

$$\text{Let } f(x) = x^3 - x + 6$$

$$\begin{aligned} \text{Then } f(-2) &= (-2)^3 - (-2) + 6 \\ &= 0 \end{aligned}$$

Hence, $x + 2$ is a factor.

Use long division to find another factor.

$$\begin{array}{r} x^2 - 2x + 3 \\ x + 2 \overline{) x^3 + 0x^2 - x + 6} \\ \underline{x^3 + 2x^2} \\ -2x^2 - x \\ \underline{-2x^2 - 4x} \\ 3x + 6 \\ \underline{3x + 6} \\ 0 \end{array}$$

The factors are $(x + 2)(x^2 - 2x + 3)$.

Use the quadratic formula to solve $x^2 - 2x + 3 = 0$.

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(3)}}{2}$$

Selected Solutions — Chapter 5

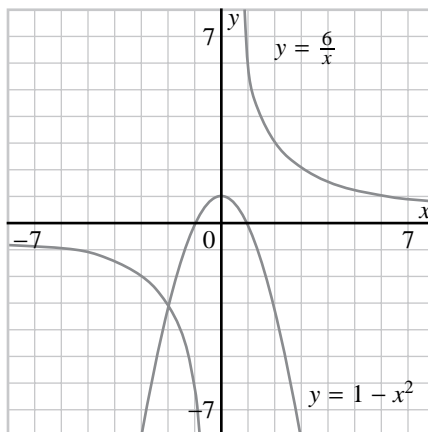
Since the discriminant is negative, there are no real roots.

The only root is $x = -2$.

Substitute -2 for x in equation ①.

$$\begin{aligned} y &= \frac{6}{-2} \\ &= -3 \end{aligned}$$

The solution is $(-2, -3)$.



ii) $y = \frac{6}{x}$ ①

$y = 10 - x^2$ ②

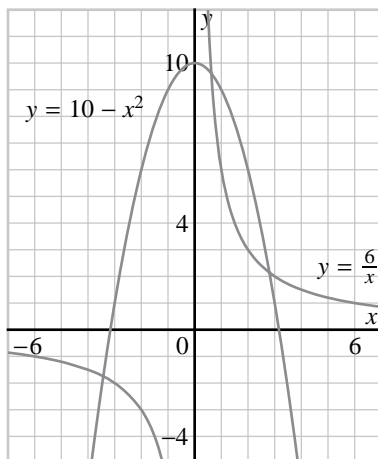
Substitute $\frac{6}{x}$ for y in equation ②.

$$\begin{aligned} \frac{6}{x} &= 10 - x^2 \\ 6 &= 10x - x^3 \end{aligned}$$

$$x^3 - 10x + 6 = 0$$

Use a graphing calculator to find approximate values for x and y .

$(0.6244, 9.6092)$, $(2.8035, 2.1402)$, $(-3.4279, -1.7503)$



The parabola in part ii is congruent to the parabola in part i, and has been translated 9 units up. In the parabola's new position, the parabola intersects both branches of the hyperbola, $y = \frac{6}{x}$, providing two additional points of intersection.

Selected Solutions — Chapter 5

b) Solve the system.

$$y = k - x^2 \quad \text{①}$$

$$y = \frac{6}{x} \quad \text{②}$$

Substitute $k - x^2$ for y in equation ②.

$$k - x^2 = \frac{6}{x}$$

$$kx - x^3 = 6$$

$$x^3 - kx + 6 = 0$$

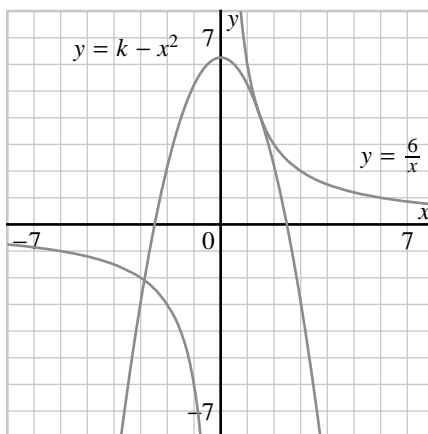
A cubic equation always has at least one real solution, so the system must have at least one solution.

Graphically, it is not possible to shift a parabola opening down with vertex on the y -axis without it intersecting $y = \frac{6}{x}$.

c) It is possible for the cubic equation in part b,

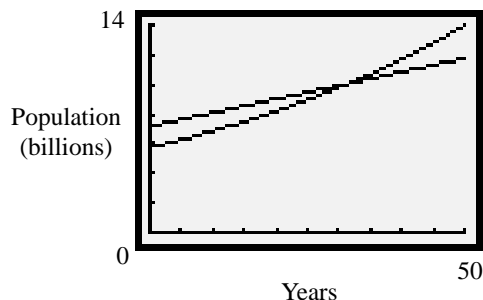
$x^3 - kx + 6 = 0$, to have two roots. One of them will be a double root. The cubic would have the factored form $(x - a)(x - b)^2 = 0$.

Graphically, the parabola will just touch one branch of the hyperbola and intersect the second branch as illustrated below.



Mathematical Modelling: Will There Be Enough Food? (Part I), page 328

1. Population will exceed grain production after several years.
2. Population exceeds grain production after about 31 years.



3. a) (31.58, 10.07)
- b) i) $P_1 = P_2$ is true when the two lines intersect; that is, when $n = 31.58$.
 ii) $P_1 < P_2$ occurs when $0 \leq n < 31.58$.
 iii) $P_1 > P_2$ occurs when $n > 31.58$.

Selected Solutions — Chapter 5

4. After about 31.58 years, there will not be enough grain to support the world population.
5. It is assumed that population growth and grain production will follow these growth equations. These are reasonable assumptions until P_1 gets close to P_2 . At this point the growth of the world's population will slow down due to limited food supplies, or more growth in grain production might occur to feed people.
6. The population growth is exponential, with the form $y = 2^x$. The food supply is linear, with the form $y = x$.
7. a) A thousand million or 1 billion.
b) Two hundred years have elapsed since 1798. The population doubles every 25 years. The population should be $1 \times 2^{\frac{200}{25}}$ billion or 2^8 billion, or 256 billion according to this model.
8. The time for the population to double should be longer. Diseases, famine, and birth control reduce the population.
9. As the population grows, there is a greater chance of more children being born. That is, as the population grows, the chances of increased population also grows. Population may therefore increase exponentially.

On the other hand, food supply depends on land space and resources, which are limited. Food supply may still increase because of more efficient methods of production or more people in the food industry; however, with limited resources a linear relationship is more probable.

Linking Ideas: Mathematics & Technology**Solving a Two-Variable Linear System, page 330**

9. b) The first system has 2 coincident lines. The second system has two parallel, but not coincident lines. The third system has a coefficient of 0, leading to the error.
10. b) See exercise 9b.

5.5 Exercises, page 334

19. Let x represent the number of adults on both nights. Let y represent the number of students on both nights.

$$x + y = 1390 \quad \textcircled{1}$$

$$4x + 2.5y = 4285 \quad \textcircled{2}$$

$$\text{From } \textcircled{1}, x = 1390 - y \quad \textcircled{3}$$

Substitute $1390 - y$ for x in equation $\textcircled{2}$.

$$4(1390 - y) + 2.5y = 4285$$

$$5560 - 4y + 2.5y = 4285$$

$$1.5y = 1275$$

$$y = 850$$

Selected Solutions — Chapter 5

Substitute $y = 850$ in equation ③.

$$\begin{aligned}x &= 1390 - 850 \\ &= 540\end{aligned}$$

Therefore, 540 adults and 850 students attended the play on both nights.

Let a represent the number of adults on the first night. Then $(540 - a)$ represents the number of adults on the second night.

Let b represent the number of students on the first night. Then $(850 - b)$ represents the number of students on the second night.

On the first night,

$$\frac{a}{b} = \frac{3}{5} \quad \text{④}$$

$$5a - 3b = 0$$

On the second night,

$$\frac{540 - a}{850 - b} = \frac{2}{3} \quad \text{⑤}$$

$$1620 - 3a = 1700 - 2b$$

$$3a - 2b = -80$$

Multiply ④ by 3. $15a - 9b = 0$

Multiply ⑤ by 5. $15a - 10b = -400$

Subtract. $b = 400$

Substitute 400 for b in $850 - b$ to get 450.

Therefore, 400 students attended on the first night, and 450 on the second night.

20. The plane's outbound speed is 270 km/h, and its return speed is 330 km/h. Let x hours represent the time of the original trip.

The distance travelled is $270x$ kilometres.

Let y hours represent the time of the return trip.

The distance travelled is $330y$ kilometres.

$$x + y = 8 \quad \text{①}$$

$$270x = 330y \quad \text{②}$$

Solve ② for y . $y = \frac{27}{33}x$

$$y = \frac{9}{11}x \quad \text{③}$$

Substitute for y from ③ into ①.

$$x + \frac{9}{11}x = 8$$

$$\frac{20}{11}x = 8$$

$$x = 4.4$$

$$\begin{aligned}\text{Distance travelled} &= (4.4)(270) \\ &= 1188\end{aligned}$$

The patrol plane can travel 1188 km against the headwind and return safely.

21. Let x kilometres per hour represent the wind speed.

Let y kilometres per hour represent the speed of the aircraft in still air.

The speed from Montreal to Paris is $(y + x)$ kilometres per hour.

The speed from Paris to Montreal is $(y - x)$ kilometres per hour.

Selected Solutions — Chapter 5

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\frac{\text{Distance}}{\text{Speed}} = \text{Time}$$

The time to travel from Paris to Montreal is $\frac{5432}{y-x} = 8$,
which simplifies to $\frac{679}{y-x} = 1$ ①

The time to travel from Montreal to Paris is $\frac{5432}{y+x} = 7$,
which simplifies to $\frac{776}{y+x} = 1$ ②

From ①, $y - x = 679$ ③

From ②, $y + x = 776$ ④

Add. $2y = 1455$

$$y = 727.5$$

Substitute 727.5 for y in ④.

$$x = 48.5$$

The wind speed is 48.5 km/h. The speed of the aircraft in still air is 727.5 km/h.

22. Let t dollars represent the cost of a kilogram of tea and c dollars represent the cost of a kilogram of coffee.

$$5t + 8c = 58 \quad \text{①}$$

$$5(1.15t) + 8(1.10c) = 65.30$$

$$5.75t + 8.8c = 65.30 \quad \text{②}$$

Divide ② by 0.05. $115t + 176c = 1306$

Multiply ① by 23. $115t + 184c = 1334$

Subtract. $8c = 28$

$$c = 3.5$$

Substitute 3.5 for c in ①. $t = 6$

The new prices are $1.15(\$6/\text{kg}) = \$6.90/\text{kg}$ for tea, and $1.10(\$3.5/\text{kg}) = \$3.85/\text{kg}$ for coffee.

23. Let x represent the hourly rate in dollars and y represent the overtime hourly rate in dollars.

$$20x + 4y = 166.40 \quad \text{①}$$

$$20x + 7.5y = 200.00 \quad \text{②}$$

Subtract. $3.5y = 33.6$

$$y = 9.60$$

Substitute for y in ①. $x = 6.40$

Therefore, the lifeguard's hourly rate is \$6.40 and her overtime hourly rate is \$9.60.

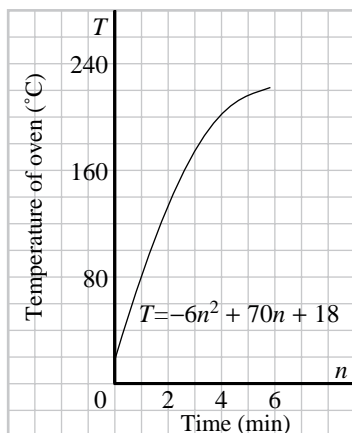
Modelling the Temperature of an Oven, page 340

The part of the graph that represents the time it takes the oven to reach the indicated temperature.

The coordinates of the vertex are $\left(\frac{35}{6}, \frac{7998}{36}\right)$

A reasonable domain is approximately $0 \leq n \leq 6$ min. After this time, the temperature in the oven remains more or less constant. A new function is required for $n > 6$ min.

Selected Solutions — Chapter 5

**5.6 Exercises, page 341**

1. Substitute each possible solution into the system. Part c is the only system where the solution satisfies all three questions; therefore, it is the only solution.
2. Substitute the solution into each system. Only in part a does the solution satisfy the three equations.
4. Explanations may vary. In exercise 3a, I subtracted equation ② from equation ① to obtain $y + 3z = 7$. I labelled this equation ④. I subtracted equation ③ from equation ② to obtain $y = 4$. Then I substituted 4 for y in equation ④ to obtain $4 + 3z = 7$. I solved for z to obtain $z = 1$. I substituted 4 for y and 1 for z in equation ③ to obtain $x + 4 + 1 = 8$ and $x = 3$. The solution is $(3, 4, 1)$.
11. **c)** Answers may vary. A reasonable domain for the price of a book is $0 \leq p \leq \$70$.
To determine the domain, factor the revenue function to $r = -5p(p - 70)$.
The revenue is zero when $p = 0$ or $p = 70$. These values determine the domain.
12. **b)** \$35 125
c) Answers may vary. To determine the domain, set the revenue function to zero; that is, $0 = -500x^2 + 500x + 35\,000$. Using the quadratic formula, the solution is $x = -7.88$ or $x = 8.88$. Since x represents an increase in ticket prices, a reasonable value for the domain is $0 \leq x \leq \$8.50$.
13. **b)** There is an infinite number of systems with this solution. An infinite number of planes can pass through a given point.
14. **a)** $2x - y - z = -3$
 $x + 2y - 2z = -2$
 $x + ky + z = 5$
Substitute $(a, b, 2)$ in the three equations.
 $2a - b - 2 = -3$
 $a + 2b - 4 = -2$

Selected Solutions — Chapter 5

$$a + kb + 2 = 5$$

These equations simplify to

$$2a - b = -1 \quad \textcircled{1}$$

$$a + 2b = 2 \quad \textcircled{2}$$

$$a + kb = 3 \quad \textcircled{3}$$

Copy $\textcircled{1}$.

$$2a - b = -1$$

Multiply $\textcircled{2}$ by 2.

$$\underline{2a + 4b = 4}$$

Subtract.

$$5b = 5$$

$$b = 1$$

Substitute 1 for b in equation $\textcircled{2}$.

$$a + 2 = 2$$

$$a = 0$$

Substitute 0 for a and 1 for b in equation $\textcircled{3}$.

$$0 + k = 3$$

$$k = 3$$

Thus, $a = 0$, $b = 1$, and $k = 3$

- b) To ensure that the linear system has exactly one point of intersection, k can only have one value. The values of a and b are determined by the first two equations, and the value of k is determined by these values.

15. a) Three planes can coincide, they can be parallel, two can be parallel and one can intersect both, they can all intersect in a line, and so on.

Linking Ideas: Mathematics & Technology

Solving a Three-Variable Linear System, page 345

9. b) The first system has three coincident planes; therefore, there is an infinite number of solutions.
The second system has three parallel planes; therefore, there are no solutions.
The third system has a coefficient of zero, leading to the error.
10. b) See exercise 9b.

Modelling a Farmer's Planting Arrangement, page 349

The model includes points in the shaded region because the farmer may decide to plant less than 80 ha; that is, $c + w \leq 80$.

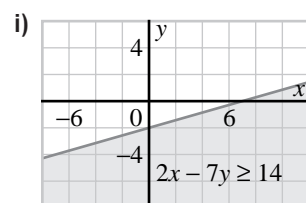
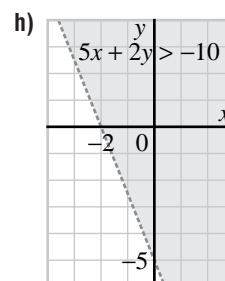
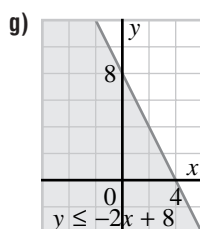
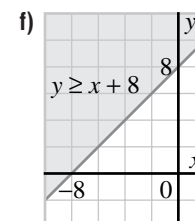
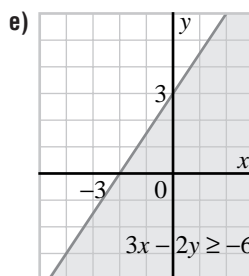
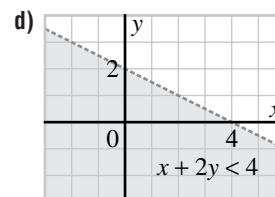
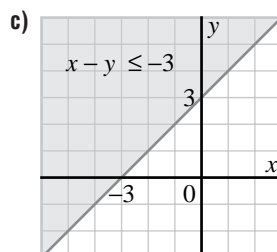
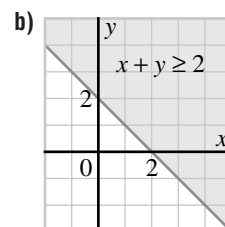
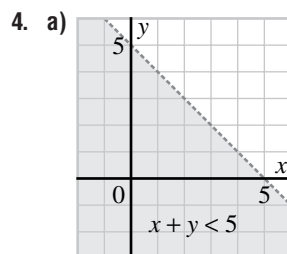
As a farmer, profit is the concern. If corn is a more lucrative crop, the farmer might insist on planting more corn. That is, $c > w$.

In addition, many soils need variation in crops to keep the soil healthy. To ensure fertile soil, the farmer might rule out points $(0, 80)$, $(80, 0)$ and points close to these.

The x -intercept and y -intercept of $c + w = 80$ are equal, so the graph is symmetrical about $w = c$.

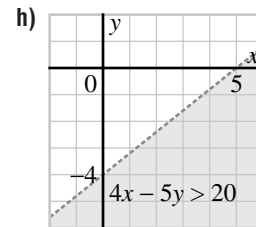
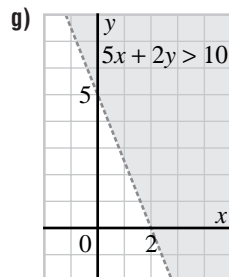
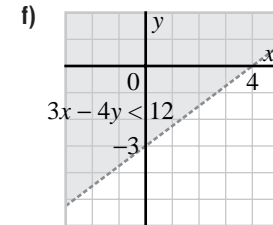
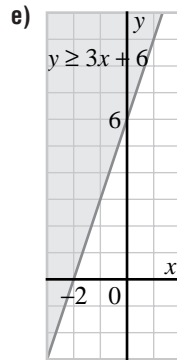
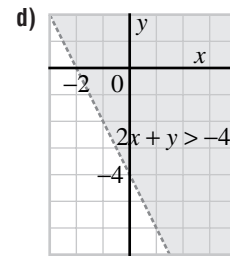
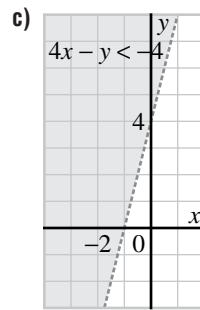
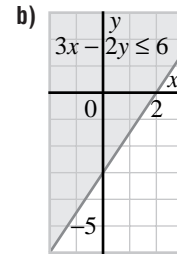
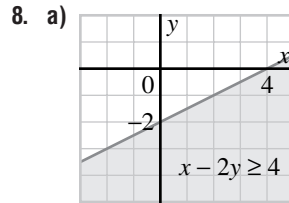
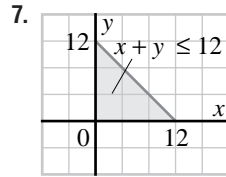
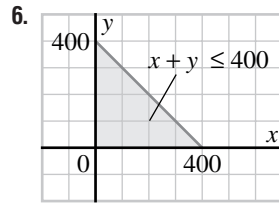
Selected Solutions — Chapter 5

5.7 Exercises, page 350

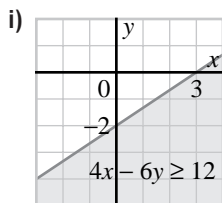


5. Explanations may vary. In exercise 4a, I graphed the line $x + y = 5$, using x - and y -intercepts of 5. I drew a broken line because the line is not part of the solution. I found a point that satisfies the inequality. For example, I checked $(0, 0)$; since $0 + 0 = 0$, which is less than 5, I shaded the region that has $(0, 0)$; that is, I shaded the region below the line.

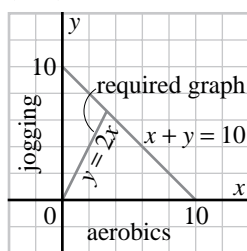
Selected Solutions — Chapter 5



Selected Solutions — Chapter 5



9. Let x represent the number of hours of aerobics.
 Let y represent the number of hours jogging.
 Therefore, $x + y \leq 10$
 and $y = 2x$
 The solution is that part of the graph of $y = 2x$ that lies between $(0, 0)$ and the line $x + y = 10$.



Modelling the Manufacture of Footballs and Soccer Balls, page 354

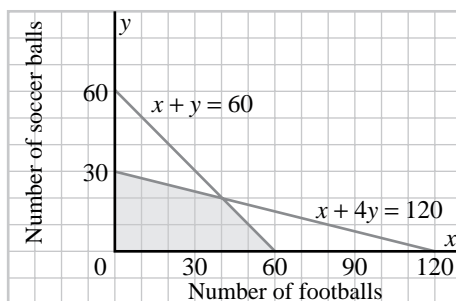
The manufacturer might leave the machines idle for awhile; thus, creating inequalities, which implies all points in the shaded region may be included in the model.

The x -intercept of $x + y = 30$ is not equal to the y -intercept of $x + 4y = 60$, nor is the y -intercept of $x + y = 30$ equal to the x -intercept of $x + 4y = 60$. Therefore, the graph is not symmetrical about $y = x$.

With two cutting machines and two stitching machines, the restrictions become $2x + 2y \leq 120$ which simplifies to

$$x + y \leq 60 \quad \text{and} \\ x + 4y \leq 120$$

The new restrictions increase (quadruple) the shaded region as illustrated below.

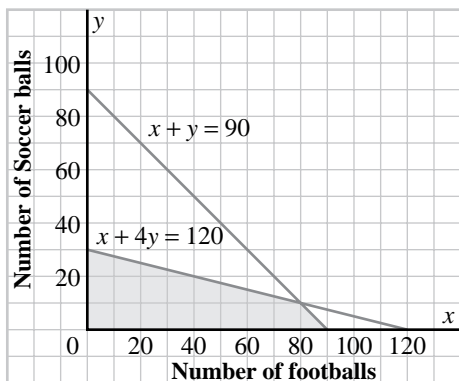


With three cutting machines and two stitching machines, the restrictions become $2x + 2y \leq 180$ which simplifies to

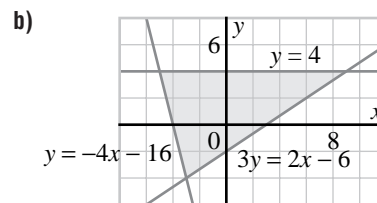
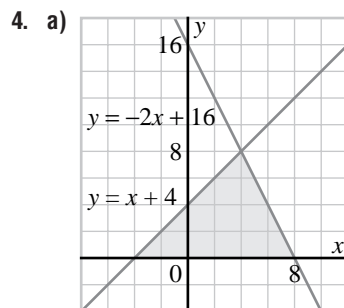
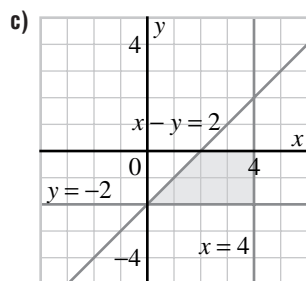
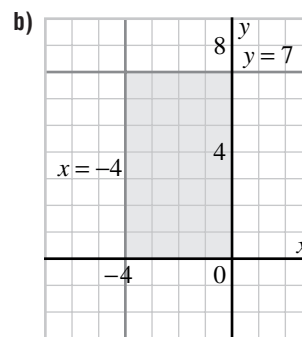
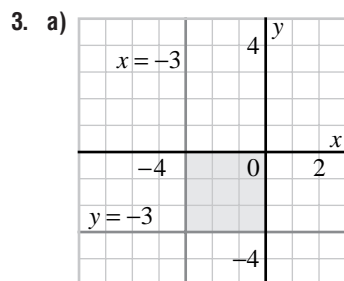
$$x + y \leq 90 \quad \text{and} \\ x + 4y \leq 120$$

Selected Solutions — Chapter 5

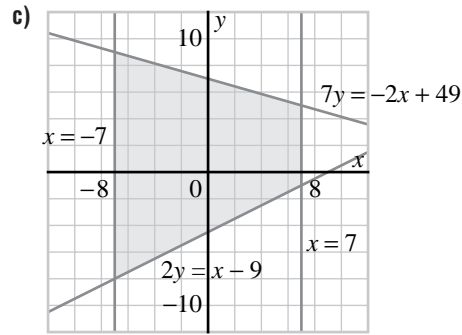
The new restrictions increase the shaded region as illustrated below.



5.8 Exercises, page 354



Selected Solutions — Chapter 5

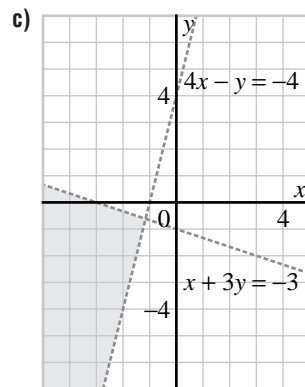
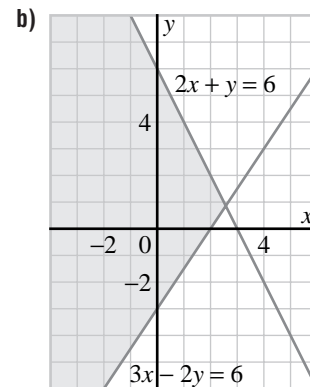
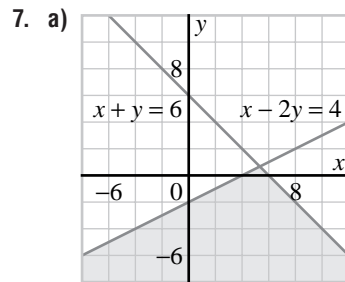


5. Explanations may vary. For exercise 4a: I graphed $y = -2x + 16$ using y-intercept 16 and x-intercept 8. The required region is below this line. I graphed $y = x + 4$ using x-intercept -4 and y-intercept 4. The required region is below this line. The line $y = 0$ is the x-axis. The required region is above this line. The region is a triangle.

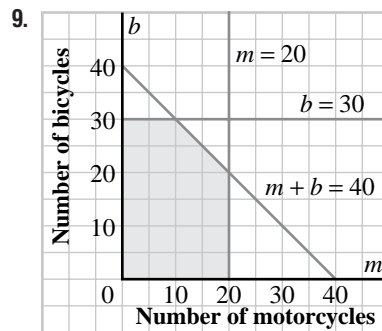
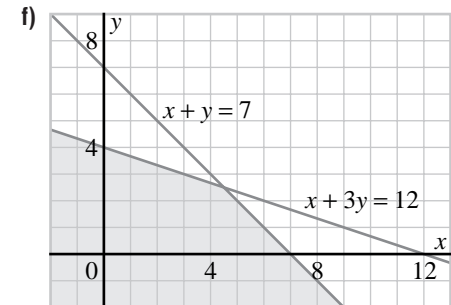
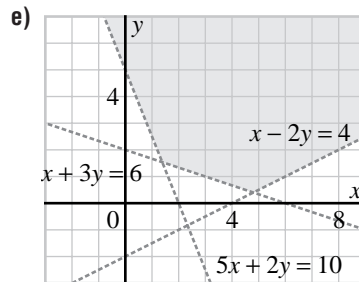
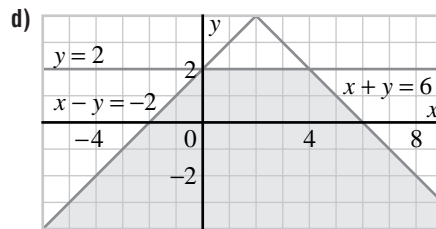
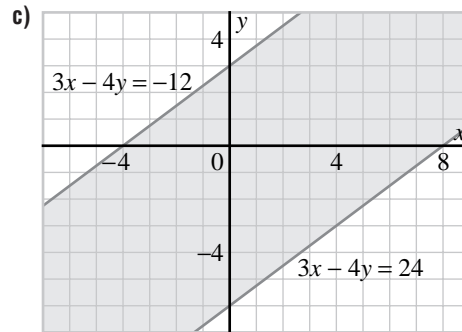
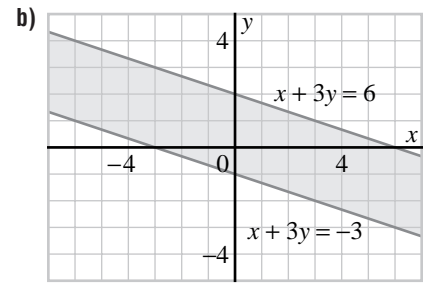
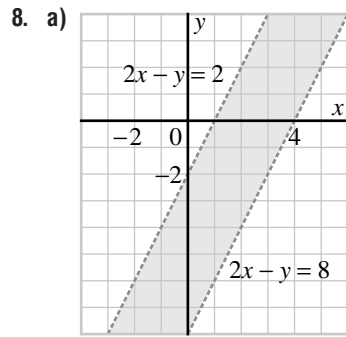
The area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$.

The base is 12 units and the height is 8 units.

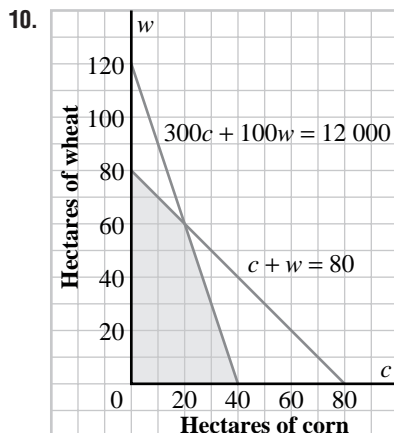
Therefore, the area = $\frac{1}{2} \times 12 \times 8$
 = 48 square units



Selected Solutions — Chapter 5

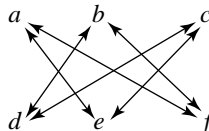


Selected Solutions — Chapter 5



Linking Ideas: Mathematics & Technology
Solving Linear Systems, page 357

2. b) The expressions for the solutions for both x and y are fractions. The patterns are best seen visually as the multiplication and subtraction of “diagonals”.



The denominators for both x and y are $ae - bd$.

The numerator for x is $-(bf - ce)$.

The numerator for y is $af - cd$.

The patterns arise because the steps eliminate y from equation ① and x from equation ② in a systematic way.

- c) A formula is efficient in a two-variable system since the formula has a pattern that permits memorization. With a formula, substitution of the coefficients and evaluation is straightforward.

3. Formulas are practical when they are simple to remember and straightforward to use. In some cases, as with a 3-variable system, the formulas are too complicated. The standard methods are safer to use.

Problem Solving: Solving Maximum-Minimum Problems, page 358

1. Let P dollars represent the profit.
 The coordinates of the vertices are $(0, 15)$, $(20, 10)$, and $(30, 0)$.
 Substitute the coordinates of each point into
 $P = 10x + 15y$.
 $(0, 15)$: $10(0) + 15(15) = 225$
 $(20, 10)$: $10(20) + 15(10) = 350$
 $(30, 0)$: $10(30) + 15(0) = 300$
 The vertex $(20, 10)$ yields the greatest profit of \$350.
 Twenty footballs and 10 soccer balls should be made.

Selected Solutions — Chapter 5

2. Let P dollars represent profit.

a) $P = 20x + 4y$

(0, 15): $20(0) + 4(15) = 60$

(20, 10): $20(20) + 4(10) = 440$

(30, 0): $20(30) + 4(0) = 600$

The vertex (30, 0) yields the greatest profit of \$600.

Thirty footballs and 0 soccer balls should be made.

b) $P = 4x + 20y$

(0, 15): $4(0) + 20(15) = 300$

(20, 10): $4(20) + 20(10) = 280$

(30, 0): $4(30) + 20(0) = 120$

The vertex (0, 15) yields the greatest profit of \$300.

No footballs and 15 soccer balls should be made.

3. The coordinates of the vertices are (0, 30), (10, 30), (20, 20), and (20, 0).

a) $P = 50m + 25b$

(0, 30): $50(0) + 25(30) = 750$

(10, 30): $50(10) + 25(30) = 1250$

(20, 20): $50(20) + 25(20) = 1500$

(20, 0): $50(20) + 25(0) = 1000$

The vertex (20, 20) yields the greatest profit of \$1500.

Twenty motorcycles and 20 bicycles should be made.

b) $P = 25m + 50b$

(0, 30): $25(0) + 50(30) = 1500$

(10, 30): $25(10) + 50(30) = 1750$

(20, 20): $25(20) + 50(20) = 1500$

(20, 0): $25(20) + 50(0) = 500$

The vertex (10, 30) yields the greatest profit of \$1750.

Ten motorcycles and 30 bicycles should be made.

4. The coordinates of the vertices are (0, 80), (20, 60), and (40, 0).

a) $P = 200c + 300w$

(0, 80): $200(0) + 300(80) = 24\ 000$

(20, 60): $200(20) + 300(60) = 22\ 000$

(40, 0): $200(40) + 300(0) = 8000$

The vertex (0, 80) yields the greatest profit of \$24 000.

Zero hectares of corn and 80 ha of wheat should be sown.

b) $P = 300c + 200w$

(0, 80): $300(0) + 200(80) = 16\ 000$

(20, 60): $300(20) + 200(60) = 18\ 000$

(40, 0): $300(40) + 200(0) = 12\ 000$

The vertex (20, 60) yields the greatest profit of \$18 000.

Twenty hectares of corn and 60 ha of wheat should be sown.

c) $P = 400c + 100w$

(0, 80): $400(0) + 100(80) = 8000$

(20, 60): $400(20) + 100(60) = 14\ 000$

(40, 0): $400(40) + 100(0) = 16\ 000$

The vertex (40, 0) yields the greatest profit of \$16 000.

Forty hectares of corn and 0 ha of wheat should be sown.

Selected Solutions — Chapter 5

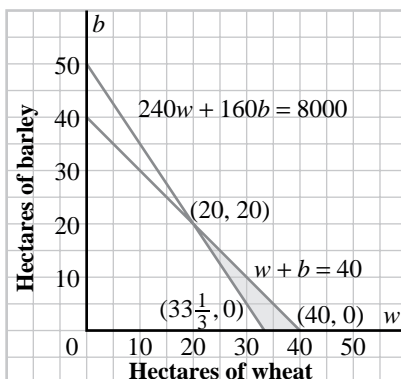
5. Let w represent the number of hectares of wheat, and b the number of hectares of barley. Thus, $w + b = 40$. Let F represent the amount of fertilizer in kilograms. We want to minimize this.

$$F = 400w + 240b$$

Let P represent the profit. $P \geq 8000$

a) $P = 240w + 160b \geq 8000$

Graph $w + b = 40$ and $240w + 160b \geq 8000$.



The minimum occurs at either $(20, 20)$, $(40, 0)$, or $(33\frac{1}{3}, 0)$

$$F = 400w + 240b$$

$$(20, 20): \quad 400(20) + 240(20) = 12\,800$$

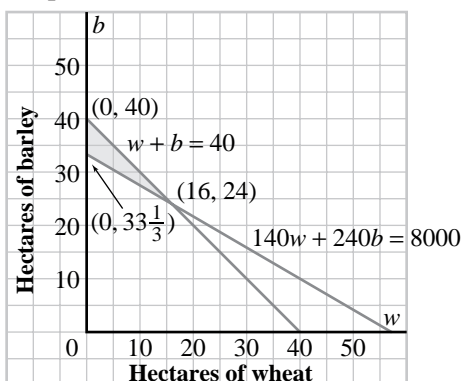
$$(40, 0): \quad 400(40) + 240(0) = 16\,000$$

$$(33\frac{1}{3}, 0): \quad 400(33\frac{1}{3}) + 240(0) = 13\,333.33$$

Fertilizer use is minimized with 20 ha of wheat and 20 ha of barley. The profit will be \$8000.

b) $P = 140w + 240b \geq 8000$

Graph $w + b = 40$ and $140w + 240b \geq 8000$.



The minimum occurs at either $(0, 40)$, $(16, 24)$ or $(0, 33\frac{1}{3})$

$$F = 400w + 240b$$

$$(0, 40): \quad 400(0) + 240(40) = 9600$$

$$(16, 24): \quad 400(16) + 240(24) = 12\,160$$

$$(0, 33\frac{1}{3}): \quad 400(0) + 240(33\frac{1}{3}) = 8000$$

Fertilizer use is minimized with 0 ha of wheat and $33\frac{1}{3}$ ha of barley. The profit will be \$8000

Selected Solutions — Chapter 5

Mathematical Modelling: Will There Be Enough Food? (Part II), page 361

1. Substitute each population for x in the formula

$$y = 0.001\ 26x(20 - x)$$

a) When $x = 8$ billion, $y = 0.001\ 26(8)(20 - 8)$
 $= 0.120\ 96$

The annual addition is $0.120\ 96 \times 10^9 = 120\ 960\ 000$

b) When $x = 12$ billion, $y = 0.001\ 26(12)(20 - 12)$
 $= 0.120\ 96$

The annual addition is $120\ 960\ 000$.

c) When $x = 16$ billion, $y = 0.001\ 26(16)(20 - 16)$
 $= 0.080\ 64$

The annual addition is $80\ 640\ 000$.

d) When $x = 20$ billion, $y = 0.001\ 26(20)(20 - 20)$
 $= 0$

The annual addition is 0 .

2. a) To find the maximum value of y , complete the square in

$$y = 0.001\ 26x(20 - x)$$

$$y = 0.0252x - 0.00126x^2$$

$$y = -0.001\ 26(x^2 - 20x + 100 - 100)$$

$$y = -0.001\ 26(x - 10)^2 + 0.126$$

When the annual addition, y , is a maximum, the population, x , is 10 billion.

- b) To find the population when the annual addition is 0, substitute $y = 0$.

$$0 = 0.001\ 26x(20 - x)$$

$$\text{Either } x = 0, \text{ or } x = 20$$

When the annual addition is 0, the population is 0 or 20 billion.

4. a) The graph approaches the maximum sustainable population and then levels off.

Therefore, if the maximum population is greater than 20 billion, the graph will have a steeper slope at the beginning. It will level off as it approaches the new maximum population.

For a value less than 20 billion, the graph will have a more gradual slope and level off at the new maximum population.

- b) a reflects the “speed” at which the maximum sustainable population is reached.

For $a > 0.001\ 26$, the graph approaches the maximum population quickly, then levels off.

For $a < 0.001\ 26$, the curve is flatter and the approach to 20 billion is more gradual.

- c) The initial population is represented by the y -intercept. Thus, for the initial population greater than 5.73 billion, the y -intercept is greater.

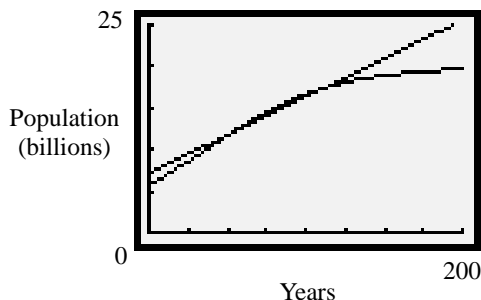
For the initial population less than 5.73 billion, the y -intercept is lower than on the original graph.

Selected Solutions — Chapter 5

5. Answers may vary. In the year 2000, the world population is estimated to reach 6 billion. Using $a = 0.00126$ and the maximum population as 12 billion, the EARTHPOP program predicts a population of 10.9 billion in 2150.

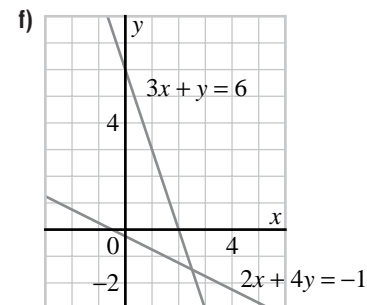
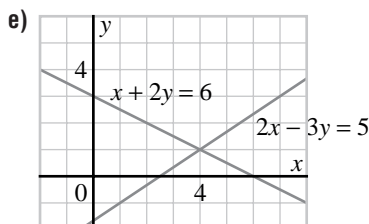
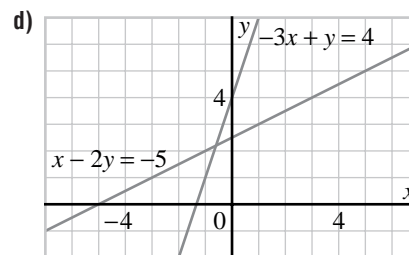
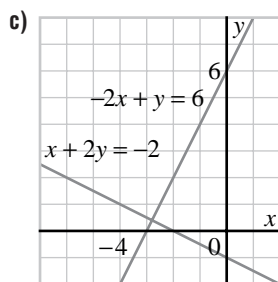
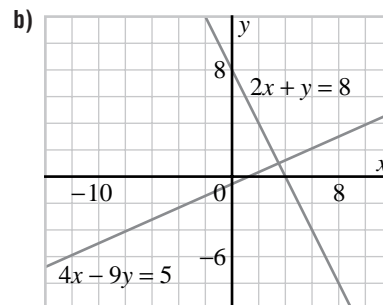
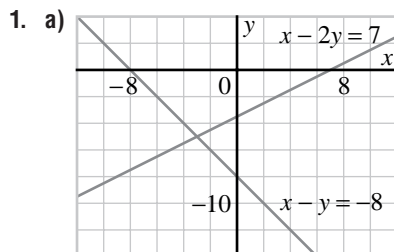
The program predicts a population less than United Nations' prediction. The difference is possible because the program uses mathematics and past population trends. United Nations uses mathematics, economics, and other factors in predicting future trends.

6. a, b)



7. For $51.95 < n < 110.48$, there will not be enough grain to feed the world population.

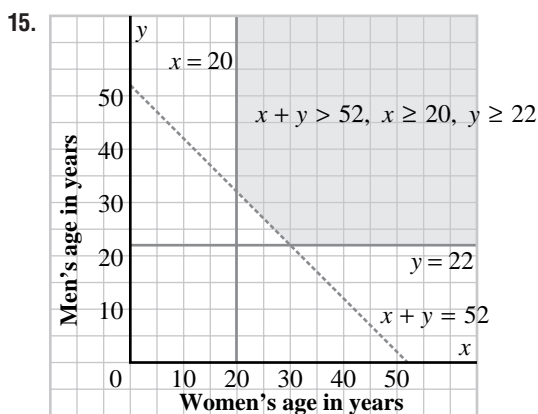
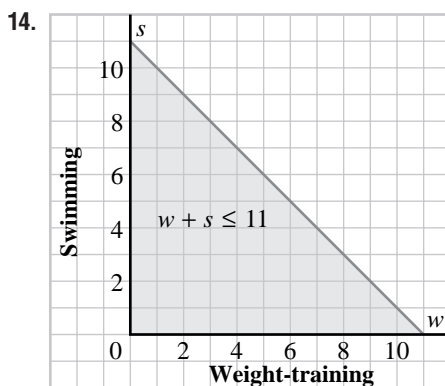
5 Review, page 364



Selected Solutions — Chapter 5

2. Explanations may vary. For exercise 1a: I drew the graph of $x + y = -8$, using the x - and y -intercepts, which are both -8 . I drew the graph of $x - 2y = 7$, using the x -intercept 7 and the slope $\frac{1}{2}$. The lines intersected at the point $(-3, -5)$, which is the solution of the system.
7. Explanations may vary. In exercise 5a, I labelled the equations as equation ① and equation ②. I subtracted equation ② from equation ① to obtain $-2y = 2$. This simplifies to $y = -1$. I substituted this into equation ② to obtain $3x - 4(-1) = 1$. I solved for x to obtain $x = -1$. The solution is $(-1, -1)$.

Explanations may vary. In exercise 6a, I labelled the equations as equation ① and equation ②. I solved equation ① for u to obtain $u = -17 - 5v$. I labelled this equation ③. I substituted equation ③ into equation ② to obtain $3(-17 - 5v) - 4v = 6$. I simplified and solved for v to obtain $v = -3$. I substituted -3 for v in equation ③ to obtain $u = -2$. The solution is $(-2, -3)$.

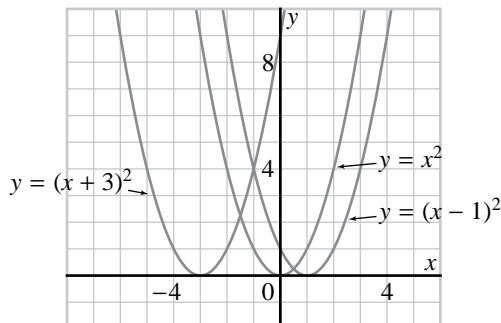


Selected Solutions — Chapter 5

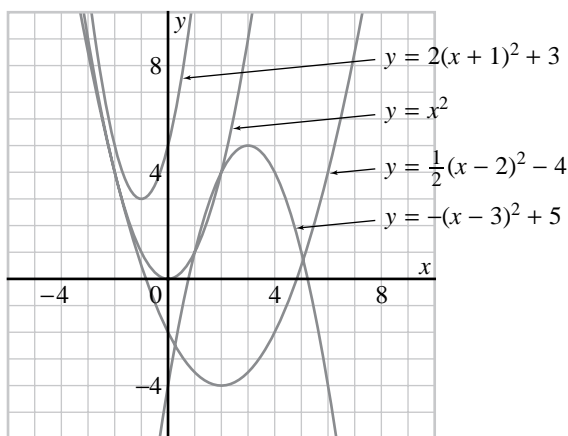
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3. a) Explanations may vary. In the summer, the LeBlancs use a lot of water on their lawn.

4. a)



b)



7. b)

