TABLE OF CONTENTS

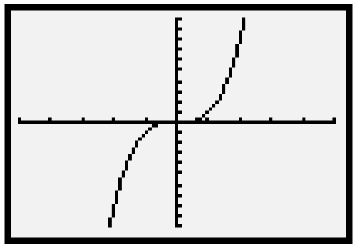
HELP

ADDISON-WESLEY
Mathematics 11

Selected Solutions — Chapter 3

Exploring with a Graphing Calculator:
Graphing Polynomial Functions, page 154

1. a)



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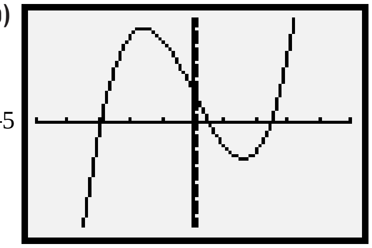

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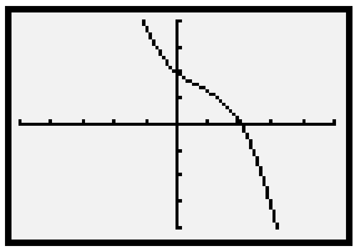

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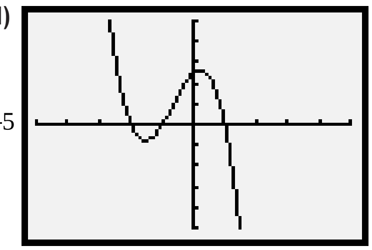

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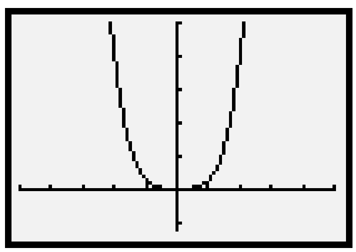

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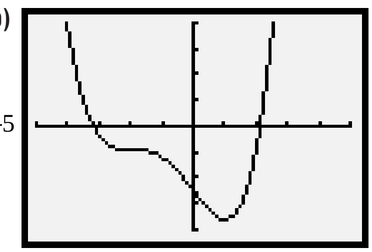

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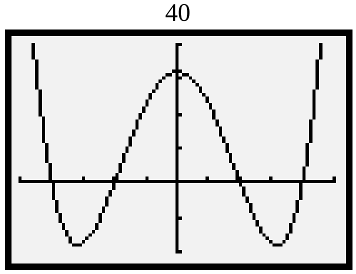

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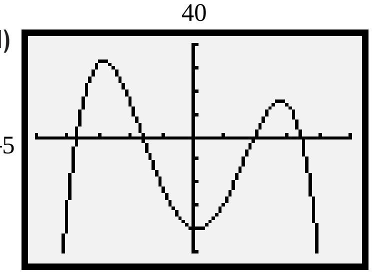

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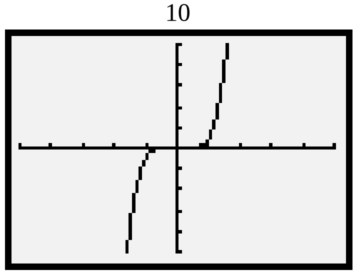

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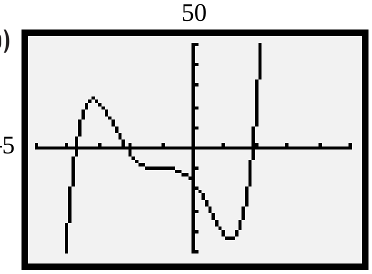

Selected Solutions — Chapter 3

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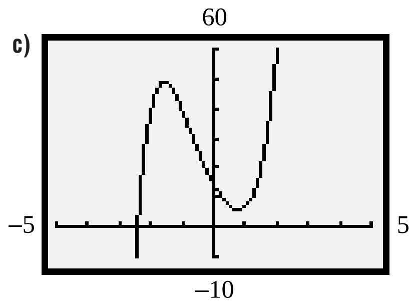

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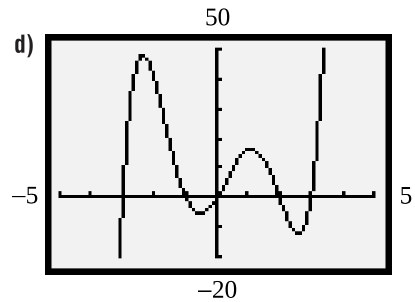

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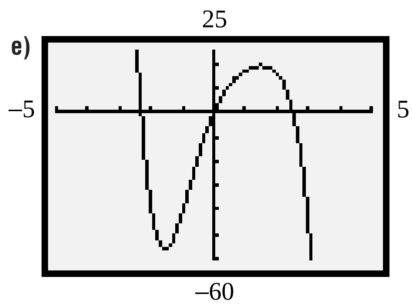


3.1 Exercises, page 160

1. **a)** Since $f(x) = 2x^3 + x^2 - 5$ is written with whole number exponents, and real number coefficients, $f(x)$ is a polynomial function.

b) Since $f(x) = x^2 + 3x - 2$ is written with whole number exponents and real number coefficients, $f(x)$ is a polynomial function.

c) Since $f(x) = 2x + 7$ is written with whole number exponents and real number coefficients, $f(x)$ is a polynomial function.

d) Since $f(x) = \sqrt{x + 1}$ has a variable under a square root, $f(x)$ is not a polynomial function.

e) Since $y = \frac{x^2 + x - 4}{x + 2}$ has a variable in the denominator, it is not a polynomial function.

f) Since $f(x) = x(x - 1)^2$ is written with whole number exponents and real number coefficients, $f(x)$ is a polynomial function.

g) Since $g(x) = 1.2x^2 + \frac{1}{2}x - \pi$ is written with whole number exponents and real number coefficients, $g(x)$ is a polynomial function.

h) Since $y = \sqrt{2x^3} + 2x - 0.5$ has a variable under a square root, it is not a polynomial function.

2. The domain of a polynomial function is all the real numbers. Thus graph b is not a polynomial. Also, polynomial graphs have ranges that extend infinitely in at least one, and possibly both, directions. Thus, graph e is probably not a polynomial, since its range seems to be the finite interval $2 < y \leq 5$. Graphs a, c, d, and f might be polynomials.

Selected Solutions — Chapter 3

3. a) Graphs d, e, and f have line symmetry.
In graph d, the line itself is an axis of symmetry. Also, the graph has an infinite number of other lines of symmetry: every line perpendicular to the graph is a line of symmetry of the graph. In each of graphs e and f there is a single line of symmetry, the line $x = 0$.

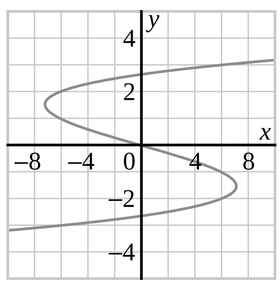
b) Graph c is symmetrical about the origin. Graph d is symmetrical about every one of its points.

4. a) All polynomial functions of degree 2 have axes of symmetry. Some polynomials of degree 4 have axes of symmetry; for example, the function at the lower left on page 157.

b) No, not all polynomials of even degree have axes of symmetry. For example, in the Example on page 159 graphs in parts b and c are both polynomials for degree 4 and yet they have no axis of symmetry.

5. Graph b passes through the origin, so it matches with function $k(x)$, since $k(x)$ is the only one of the three functions that has a value of 0 at $x = 0$. Graph a has three hills and valleys, so it must correspond to a polynomial of degree 4; the only remaining function is $g(x)$. By a process of elimination, graph c matches with function $f(x)$. As a check, when $x = 0$, $f(x) = 16$, which corresponds to the y-intercept.

6. a) i)



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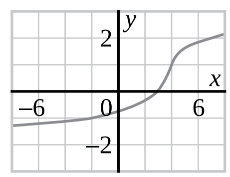


b) The inverse of $f(x) = x^3 - 2x^2 + 2x + 3$ is a function, since there is exactly one y-value for each x-value. The inverse of the function $f(x) = x^3 - 7x$ is not a function, since for some x-values there is more than one y-value.

7. a) Polynomial functions with no hills or valleys have inverses that are functions—otherwise the inverse will have more than one y-value for some x-values.

b) No, a polynomial function of the form $y = a$, where a is a real number, is a horizontal linear function. Its inverse is a vertical line, which is not a function.

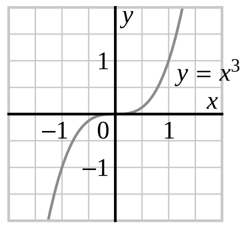
8. a) All the graphs open up (since the degrees are even) and all have axis of symmetry $x = 0$ (since the formulas contain only even powers of x). Only the graph of $y = x^2$ is a parabola, but they all have shapes like the cross-section of a bowl.

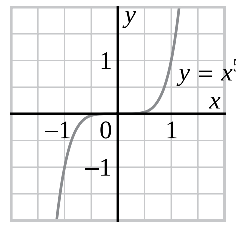
b) For very large even values of n, the graph would look similar, but have a nearly flat bottom and nearly vertical sides.

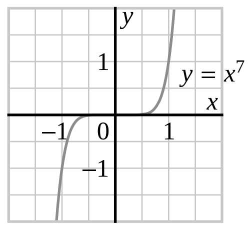
Selected Solutions — Chapter 3

c) The bottom of the graph gets flatter, and the sides get closer to the vertical. Think about the line segments joining the points (–1, 1) to (–1, 0), (–1, 0) to (1, 0), and (1, 0) to (1, 1). As n increases, the graph approaches these line segments.

9.a)







b) The graph of $y = x^n$ looks similar to the other graphs, but is flatter in the interval $-1 < x < 1$ and closer to the vertical outside that interval.

c) Consider the ray $x = -1$ from (–1, 0) down, the line segment joining the points (–1, 0) to (1, 0), and the ray $x = 1$ from (1, 0) upwards. As n increases, the curve gets closer and closer to the rays and line segment.

3.2 Exercises, page 166

1. **a)** False, a counter example is the cubic function $f(x) = x^2 - 2x^2 + 2x + 3$, on page 157. It has no relative maximum or minimum points.

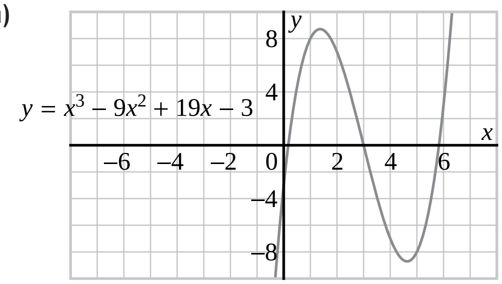
   b) True, the domain is all real numbers, since that is true of any polynomial function. Since the graphs of cubic functions extend from quadrant 2 to quadrant 4 or from quadrant 3 to quadrant 1, and the graphs are continuous, the range must be all the real numbers.

   c) True. The graph of a quartic function extends from quadrant 2 to quadrant 1 or from quadrant 3 to quadrant 2. Since the graph is continuous, it must change direction, since the quadrants are adjacent. Where the graph changes direction, it has an absolute maximum point or an absolute minimum point.

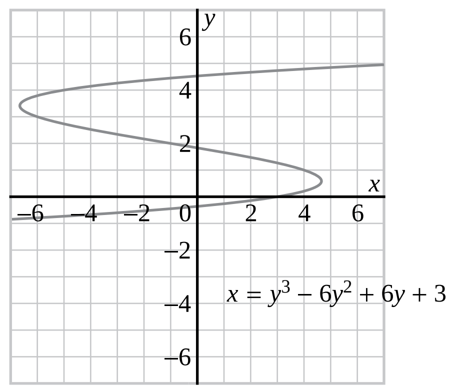
   d) False, as in part c, since a quartic function must have either an absolute minimum or an absolute maximum, it cannot have a range that is all the real numbers. A counterexample is $y = x^4$, whose range is the real numbers greater than or equal to 0.

Selected Solutions — Chapter 3

2. a)

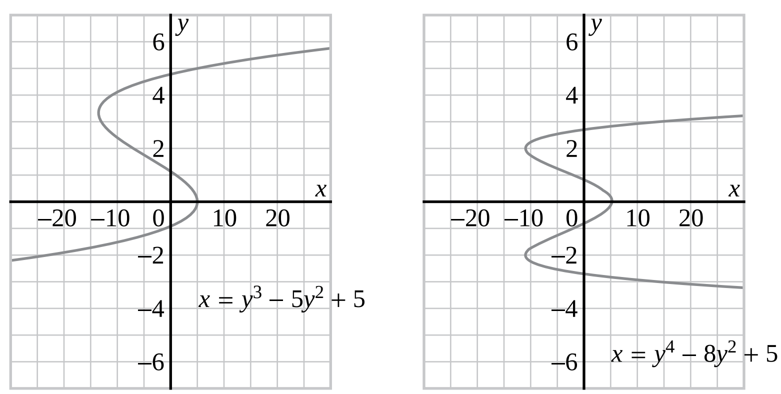


5. a)



c) From the graph of the inverse, we can see that a vertical line could intersect the graph in more than one point; therefore, the inverse is not a function.

6. a)



7. No, explanations may vary. Visualize the graph. For the inverse of a polynomial function to have a maximum or minimum point, the original function would have opened to the left, or to the right. In each case, it would have not passed the vertical line test and hence would not be a function.

3.3 Exercises, page 173

4. Answers may vary. For exercise 2a, since the zeros are 1, 2, and 3, the factors are $(x - 1)$, $(x - 2)$, and $(x - 3)$ in the function. Since the function is cubic, these are the only allowable factors, since there can be no more than 3 factors. Thus the function has the form $y = a(x - 1)(x - 2)(x - 3)$. Choosing any non-zero value of a, such as $a = 1$, gives a specific function that satisfies the required properties: $y = (x - 1)(x - 2)(x - 3)$.

Selected Solutions — Chapter 3

5. **a)** True; for example, $y = x^2 + 1$ has no zeros.

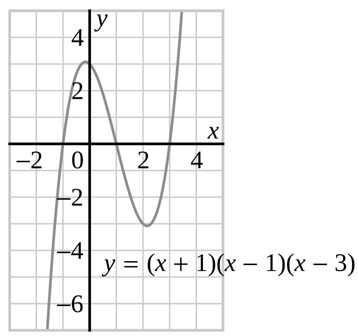
b) False, since the range of a cubic function is all the real numbers, every cubic function must pass through $y = 0$ for some value of x.

c) True, for the reason given in part b.

d) False; for example, $y = x^4 + 1$ has no zeros.

6. The functions are not unique. Any multiple of each function has the same zeros.

a)



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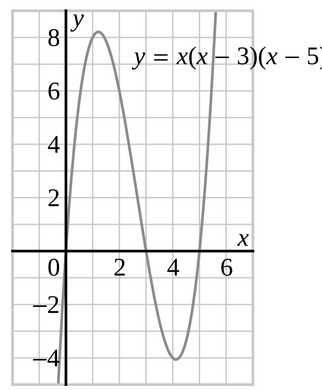

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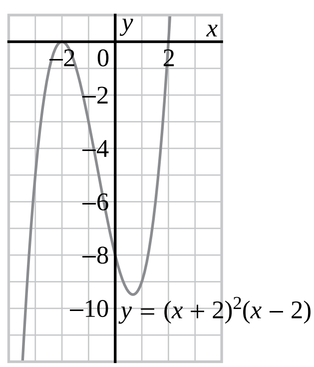

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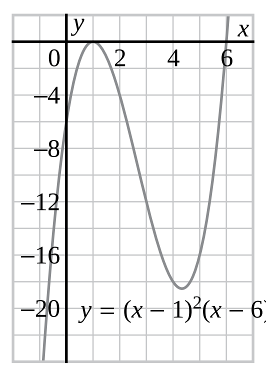
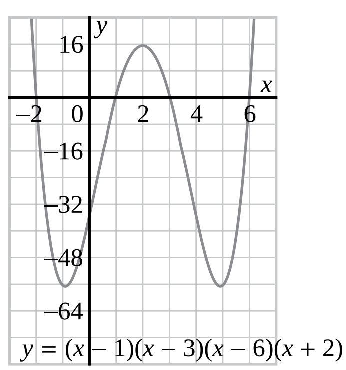


7. **a)**



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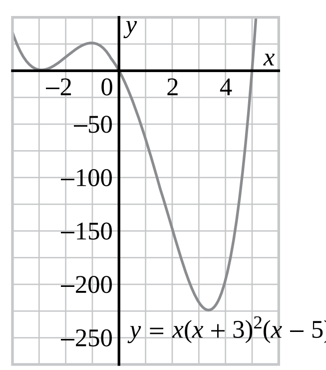

Selected Solutions — Chapter 3

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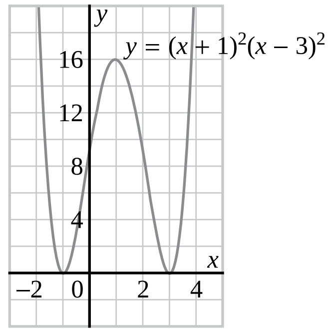

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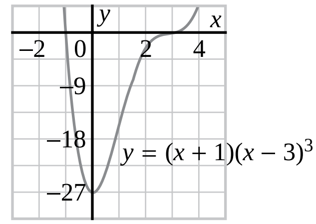

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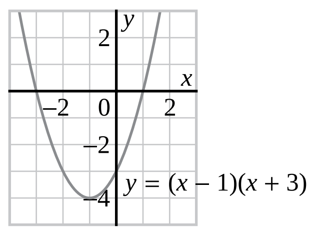

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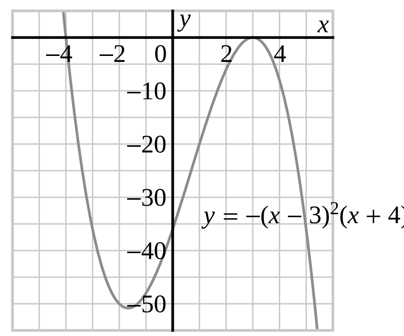

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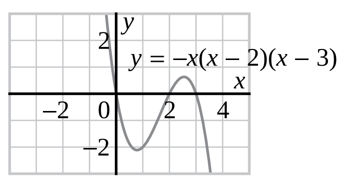

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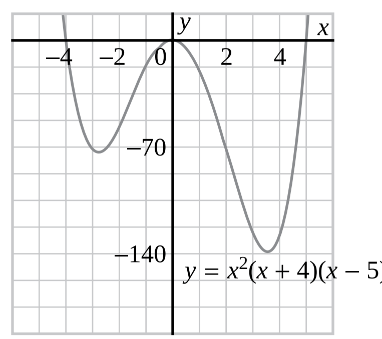

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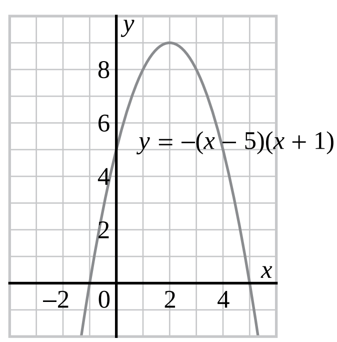

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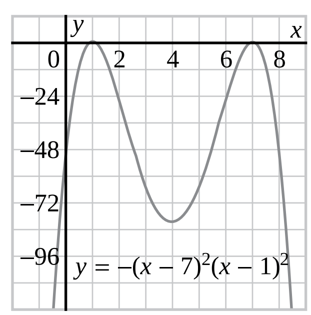
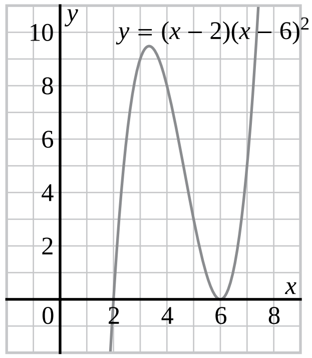
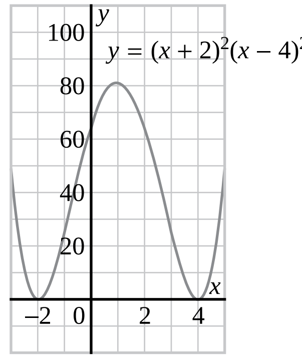
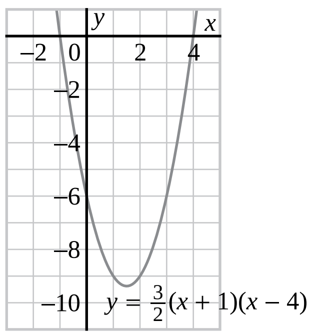
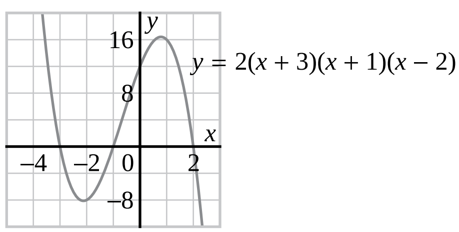


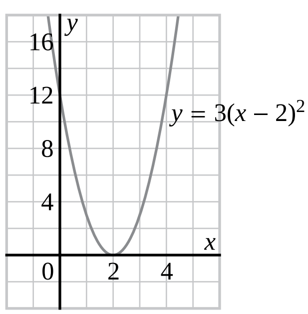
g) 

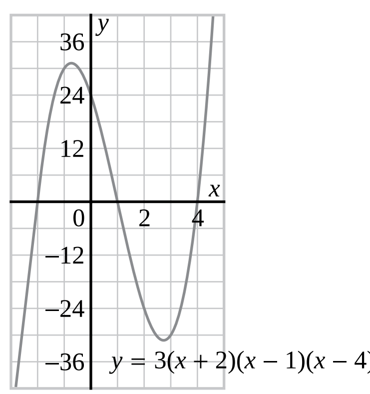
h) 

9. Explanations may vary. For exercise 8b: The function is a cubic, and since the coefficient of x^3 is negative, the graph extends from quadrant 2 to quadrant 4. The zeros are at $x = -4$ and $x = 3$. However, $x = 3$ is a double zero, so there is a hill or valley at that point. Since the graph goes down and to the right after that point, it must be a hill. From all of these considerations I sketched the graph. To get more detail, I determined the y-intercept, which is -36.

10. b) 

11. b) 

12. a) 

b) 

Selected Solutions — Chapter 3

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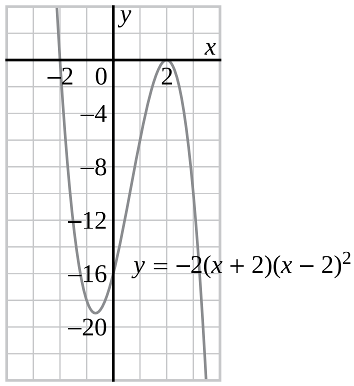

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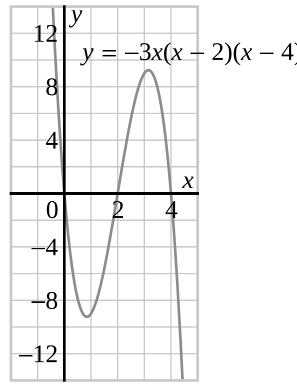
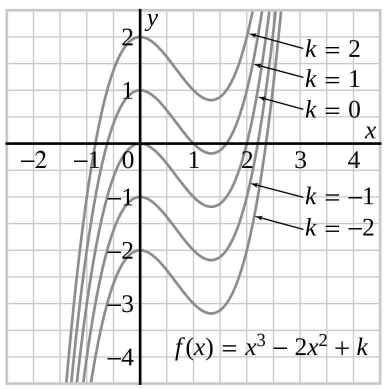


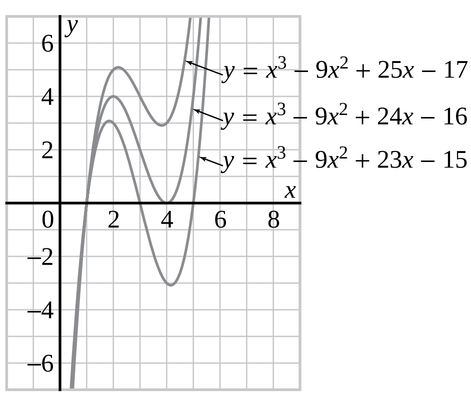
15. Explanations may vary. For exercise 4j:
I substituted $f(x) = 0$ to get $0 = x^3 - 64$. I rewrote this as $x^3 = 64$, then took the cube root of each side to get $x = 4$.

19.



20. **a)** The pattern is that the constant decreases by 1, while the coefficient of x increases by 1. The next three functions are:
$y = x^3 - 9x^2 + 26x - 18$
$y = x^3 - 9x^2 + 27x - 19$
$y = x^3 - 9x^2 + 28x - 20$

b)



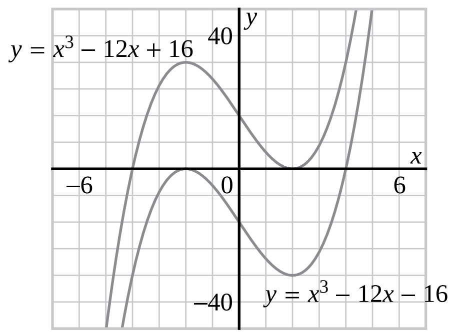
c) As k increases through positive values, both the hill and valley get higher. The x-values of the hill and valley get closer together, and the y-values of the hill and valley also get closer together. When $k = 4$, the hill and valley "merge" so that there is no more hill or valley. For $k > 4$, there is no hill or valley. All the curves pass through $(1, 0)$.

Selected Solutions — Chapter 3

d) In the preceding direction, the pattern is the constant increases by 1, while the coefficient of x decreases by 1. The preceding three functions are:
$y = x^3 - 9x^2 + 22x - 14$
$y = x^3 - 9x^2 + 21x - 13$
$y = x^3 - 9x^2 + 20x - 12$

e) As k decreases through negative values, both the hill and valley get lower. The x-values of the hill and valley get farther apart, and the y-values of the hill and valley also get farther apart. For negative values of k, the curves have the same features of a hill and a valley. All the curves pass through (1, 0).

21. Sketch the graphs of the functions $y = x^3 - 12x + 16$ and $y = x^3 - 12x - 16$.



It's easier to complete part b first, then apply the conclusions to part a.

b) i) For 3 different roots, $-16 \leq k < 16$

ii) For 2 different roots, $k = \pm 16$

iii) For only 1 root, $k > 16$ or $k < -16$

a) Use the table from part b.

i) The equation $x^3 - 12x + 10 = 0$ has 3 different roots.

ii) The equations $x^3 - 12x + 20 = 0$ and $x^3 - 12x - 20 = 0$ have only 1 root each.

Problem Solving: Visual Proofs, page 176

1. The large square is divided into two smaller squares and two equal rectangles. The area of the large square is equal to the sum of the areas of the 4 pieces. The large square has sides of length $a + b$, so its area is $(a + b)^2$. Similarly, the smaller squares have areas a^2 and b^2, and each rectangle has area ab. Thus,
$(a + b)^2 = a^2 + b^2 + ab + ab$, which means
$(a + b)^2 = a^2 + 2ab + b^2$.

2. Cutting out the small square leaves an area of $a^2 - b^2$. When the resulting trapezoids are arranged to form a rectangle, the length is $a + b$ and the width is $a - b$. Thus, the area of the rectangle is $(a + b)(a - b)$. Since the area has not been changed by rearranging the trapezoids, we have $a^2 - b^2 = (a + b)(a - b)$. This is the result of factoring the difference of two squares.

Selected Solutions — Chapter 3

3. a) Since the dots form a square with sides of length 5, then

$$1 + 3 + 5 + 7 + 9 = 5^2 = 25$$

b) The side length of the square is equal to the number of numbers on the list, which in this case is $\frac{2n - 1 + 1}{2} = n$.

Thus, $1 + 3 + 5 + 7 + ... + (2n - 1) = n^2$

4. a) The parallelogram has 5 rows of 6 dots, so it has $5 \times 6 = 30$ dots in total. The triangle has half as many dots, so

$$1 + 2 + 3 + 4 + 5 = \frac{30}{2} = 15$$

b) The sum $1 + 2 + 3 + 4... + n$ can be visualized as a triangle. Put two such triangles together to get a parallelogram. The parallelogram has n rows of $(n + 1)$ dots, for a total of $n(n + 1)$ dots. The original triangle has half as many, so

$$1 + 2 + 3 + 4... + n = \frac{n(n + 1)}{2}$$

c) The first 6 triangular number can be obtained in a number of ways: by forming the triangles, adding up numbers, or using the formula in part b. The triangular numbers are 1, 3, 6, 10, 15, 21. The 100th triangular number is obtained by using the formula in part b; substituting $n = 100$ into the formula to get 5050.

5. a) The pyramid with red balls has $16 + 9 + 4 + 1 = 30$ and there are 30 red dots in the squares on the left of the rectangle.
The pyramid with blue balls is the same as the red pyramid. The blue dots are arranged in squares on the right of the rectangle.
The multicoloured pyramid has 16 yellow balls, 9 pink balls, 4 green balls and 1 red ball that are arranged in the middle of the rectangle.

b) The rectangle has dimensions 10 by 9, so there are 90 balls in the rectangle.

c) Since the rectangle contains the balls that would fit in 3 pyramids, the number of balls in one pyramid is $\frac{90}{3} = 30$. Thus,

$$1^2 + 2^2 + 3^2 + 4^2 = 30$$

d) One dimension of the rectangle is $2n + 1$ (twice the side length of the base of a pyramid + 1). The other dimension is $1 + 2 + 3 + 4 + ... + n$ (the sum of the side length of each layer in the pyramid). However, the result of exercise 4 is that $1 + 2 + 3 + 4 + ... + n = \frac{n(n + 1)}{2}$. Thus, the dimensions of the rectangle are $2n + 1$ and $\frac{n(n + 1)}{2}$. The number of balls in the rectangle is $(2n + 1) \times \frac{n(n + 1)}{2} = \frac{n(n + 1)(2n + 1)}{2}$.

Selected Solutions — Chapter 3

e) Since the rectangle contains all the balls from 3 pyramids, the number of balls in one pyramid is
$\frac{1}{3} \times \frac{n(n+1)(2n+1}{2} = \frac{n(n+1)(2n+1)}{6}$. Thus,
$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$.

f) The first 4 pyramidal numbers can be calculated from the pyramid in the text. The next 2 numbers and the 100th number can be calculated by the formula.
The first 6 pyramidal numbers are 1, 5, 14, 30, 55, 91. The 100th pyramidal number is 338 350.

6. a) i) $(a + b)^2$

ii) $c^2 + 4\left(\frac{1}{2}ab\right) = c^2 + 2ab$

b) From part a, the two expressions for area are equal.
$(a + b)^2 = c^2 + 2ab$
Expand.
$a^2 + 2ab + b^2 = c^2 + 2ab$
$a^2 + b^2 = c^2$

3.4 Exercises, page 180

6. a) By using a graphing calculator and zooming in if necessary, it seems clear that there are no negative roots.

b) In other cases, there could be two unequal negative roots, very close together, or there could be no negative roots.

c) In exercise 1b, it looks as if the two equal negative zeros are –2.5. Determine the value of the function for values of *x* close to –2.5, for example, –2.4, –2.45, –2.49, –2.51, –2.55, –2.6. If the function is always positive, there are no negative roots. If the function has negative values, there are no unequal negative roots.

7. c) i) No. The two methods give the same results.

ii) The second method may be more reliable, since only one graph must be drawn.

8. c) i) No. The two give exactly the same results.

ii) The second method may be more reliable, since only one graph must be drawn.

iii) Answers may vary.

9. a) The leading coefficient is positive, so the graph stretches from quadrant 3 to quadrant 1. Thus, the graph must “turn around” at some point farther to the right along the *x*-axis and head back up into quadrant 1. Thus, the graph must cross the *x*-axis again at some positive value of *x*.

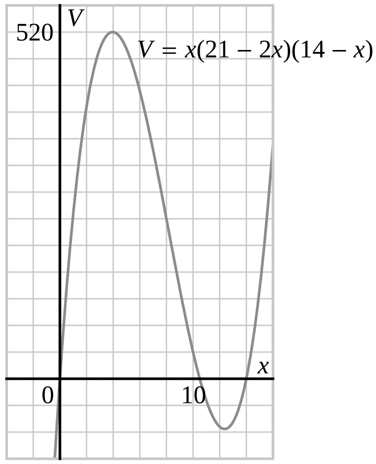
b) The third zero is positive as discussed in part a.

c) No, if it were, there would have to be an equal number of zeros on each side of the *y*-axis. There is not. There are two positive zeros and one negative zero.

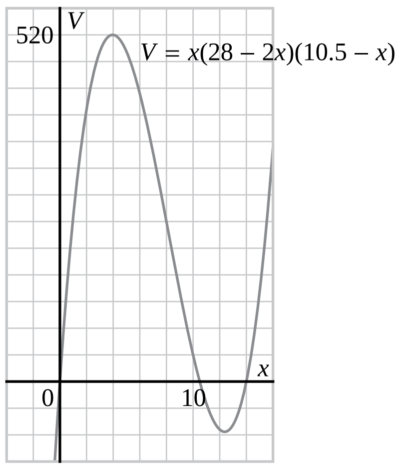
Selected Solutions — Chapter 3

3.5 Exercises, page 185

3. **b)**



4. **b)**



Modelling Domed Stadium Design

- Answers may vary. Probably the most important quantity is the area that the stadium covers on the ground. Then the shape of the dome is decided by structural and design considerations. Also, the seating capacity of the stadium will be dependent on its volume.
- The height may have to be greater because of structural considerations; a higher arch may be stronger than a hemispherical one.

8. **a)** **i)** From the example, the maximum volume of the box with no top is approximately 1040 cm^3.
From exercise 3d, the maximum volume of the box with a top is $(13.08 \times 10.04 \times 3.96)$ cm^3, or approximately 520 cm^3.
From exercise 4d, the maximum volume of the box with a top is $(20.08 \times 6.54 \times 3.96)$ cm^3, or approximately 520 cm^3.
The box with no top has a maximum volume that is twice that of each box with a top.

ii) The sizes of the squares removed are the same for all three boxes.

iii) For the boxes with tops, the heights are the same, and the width of each box is half the length of the other box. For the box with no top, its length is equal to the length of one box with no top, and its width is double the width of one box with no top. For the box with no top, its width is equal to the length of the other box with no top, and its length is double the width of the other box with no top.

b) The box with no top has volume function $V = x(28 - 2x)(21 - 2x)$. Both the boxes with tops have volume functions $V = x(14 - x)(21 - 2x)$. The first volume function is twice the second volume function, but both functions have their maximums at the same value of x.

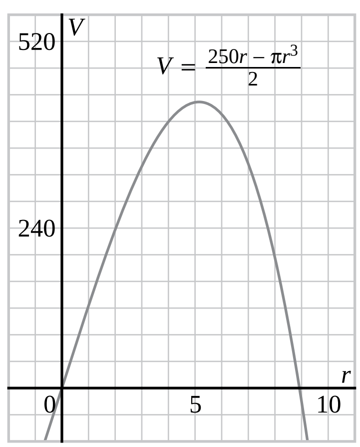
9. **a)** The volume of the can is $V = \pi r^2 h$. To write this as a cubic function in r, we need to eliminate h. We need some other information. The only other information we have is that the can is made from 250 cm^2 of metal. Thus, the surface area A is 250. Write a formula for A, solve it for h, and then substitute the result into the formula for V. The surface area of a cylinder with one base is

$$A = \pi r^2 + 2\pi rh$$
$$250 = \pi r^2 + 2\pi rh$$
$$250 - \pi r^2 = 2\pi rh$$
$$h = \frac{250 - \pi r^2}{2\pi r}$$

Substitute for h into the formula for V.

$$V = \pi r^2 h$$
$$= \pi r^2 \frac{(250 - \pi r^2)}{2\pi r}$$
$$V = \frac{250r - \pi r^3}{2}$$

b)



c) **i)** Use the trace feature of a graphing calculator to determine the value of r for $V = 200$. Or graph the line $V = 200$ and determine the r-coordinates of the points of intersection. There are two solutions, $r \doteq 1.66$ and $r \doteq 7.98$. Substitute each value of r into the equation for h.

Selected Solutions — Chapter 3

$h = \frac{250 - \pi r^2}{2\pi r}$

When $r \doteq 1.66$, $h \doteq \frac{250 - \pi(1.66)^2}{2\pi(1.66)} \doteq 23.14$

When $r \doteq 7.98$, $h \doteq \frac{250 - \pi(7.98)^2}{2\pi(7.98)} \doteq 1.00$

Thus, when the can has volume 200 mL, its dimensions are $r = 1.66$ cm and $h = 23.14$ cm, or $r = 7.98$ cm and $h = 1.00$ cm.

ii) Use the trace feature to determine the value of r when the volume is a maximum. Or, determine the value of the graph. The result is approximately $r = 5.15$ cm. Then substitute this value of r into the equation for h.

$$h = \frac{250 - \pi(5.15)^2}{2\pi(5.15)}$$
$$\doteq 5.15$$

The height is equal to the radius, 5.15 cm.

10. Answers may vary. Suppose that the triangle has n rows and the rectangle has h rows. The number of dots in each row of the triangle follow the pattern $1, 2, 3, ..., n$. Thus, the nth row of the triangle has n dots in it. The total number of dots in the triangular pattern is $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$. (See page 176, exercise 4.) The total number of dots in the rectangular pattern is nh. Thus, the total number of dots on the page is $\frac{n(n+1)}{2} + nh = 5000$.

Solve for h.

$$nh = 5000 - \frac{n(n+1)}{2}$$
$$h = \frac{5000}{n} - \frac{n+1}{2}$$

There are many solutions to this equation, but we need whole number values for n and h. Use guess and check. From the diagram, we can see that n is much greater than h, so start with $n = 50$.

n	50	60	70	80	90	100	110
h	74.5	52.83	35.9	22	10.1	–0.5	–9.5

From the table, one possible solution is $n = 80$, $h = 22$.

The base of the rectangle has 80 dots, the height of the rectangle has 22 dots, and each leg of the isosceles triangle is 80.

Mathematics File: Function Operations, page 188

5. In each case the functions are not unique. There is an infinite number of ways to make each function.

For example, in part e, $f(x)$ could be 2 and $g(x)$ could be $\frac{1}{2}(x^2 - 3x + 4)$ or $f(x)$ could be 3 and $g(x)$ could be $\frac{1}{3}(x^2 - 3x + 4)$ and so on.

6. There is only one solution in parts a and b. There are two solutions in part c, but the two solutions are:

$f(x) = x(x+1), g(x) = x + 1$ and

$f(x) = x + 1, g(x) = x(x+1)$.

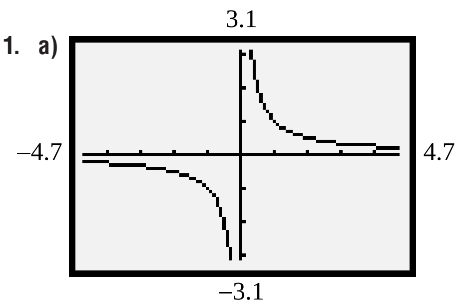
Selected Solutions — Chapter 3

Thus the solution to part c is “essentially unique”; that is, there is only one pair of functions that works, although we can choose which to label $f(x)$ and which to label $g(x)$.

7. If $f(x) = 0$, then $g(x) \div f(x)$ doesn’t make sense. Otherwise, all functions can be combined algebraically.

Exploring with a Graphing Calculator: Graphing Rational Functions, page 189

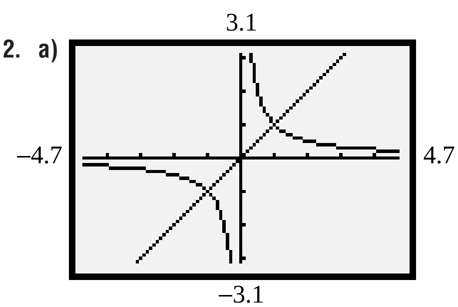
1. a)



c) There is no y-coordinate when $x = 0$, $\frac{1}{0}$ is not defined.

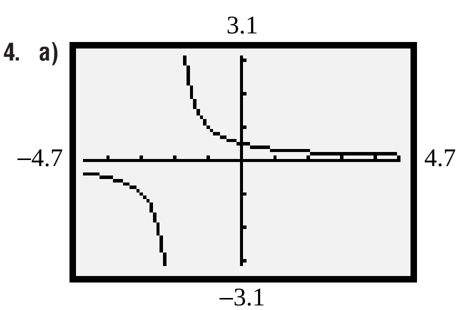
d) No, the two branches are not connected.

2. a)

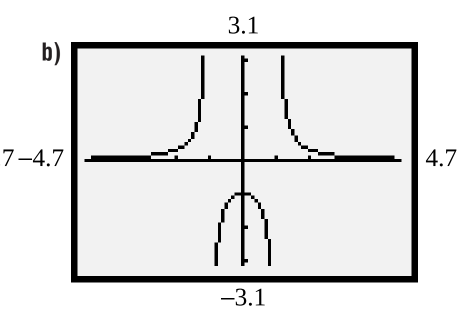


b) Corresponding y-coordinates are reciprocals.

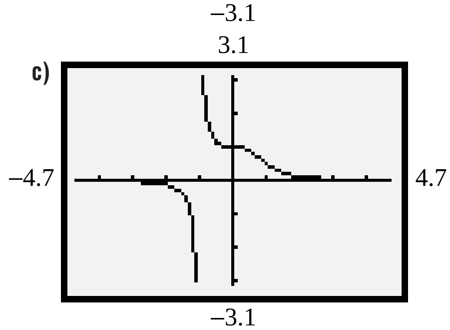
4. a)



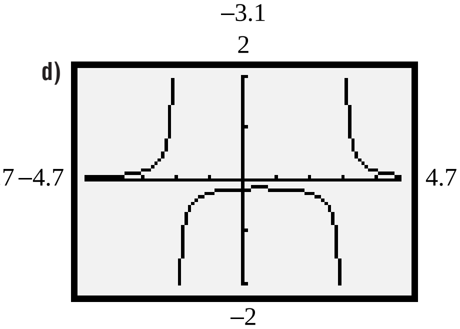
b)



c)

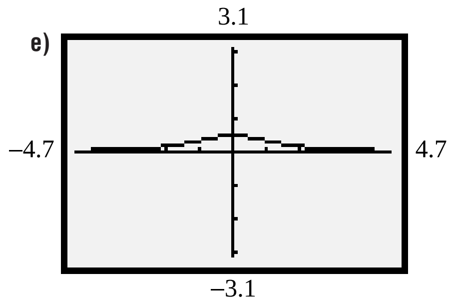


d)

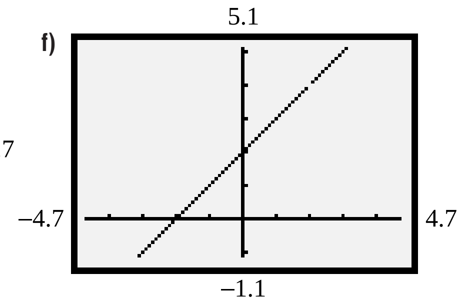


Selected Solutions — Chapter 3

e)

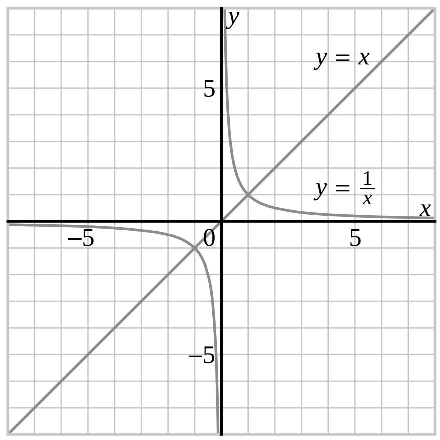


f)



Investigate, page 191

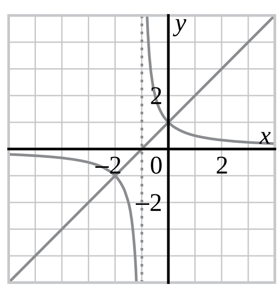
1. a), b)



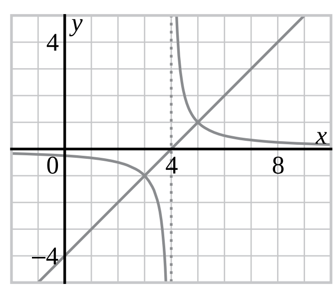
2. a) The two values are reciprocals, so when x has a large absolute value, $\frac{1}{x}$ has a small absolute value.

3.6 Exercises, page 194

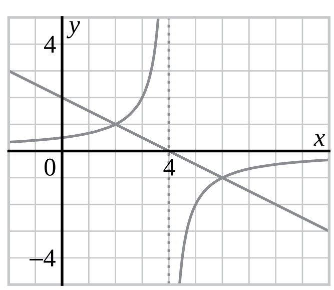
2. a)



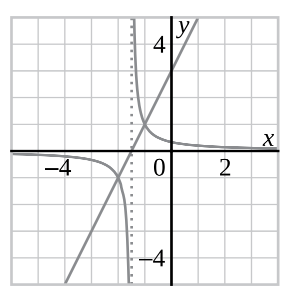
b)



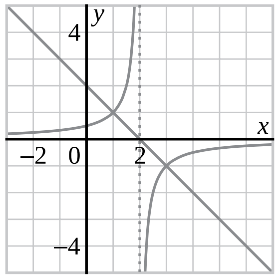
c)



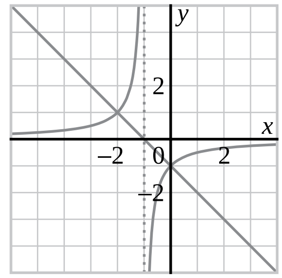
d)



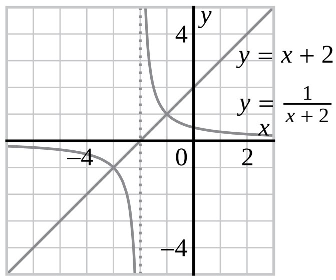
e)



f)



3. a)



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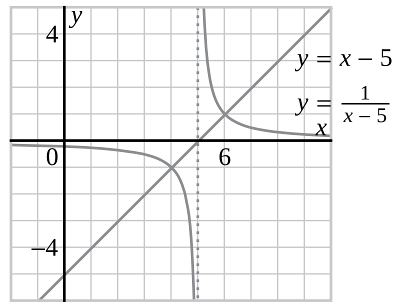
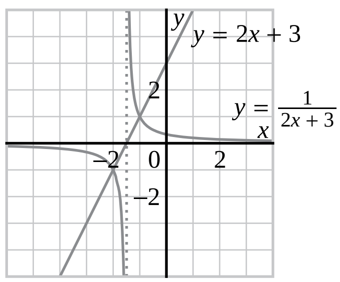


c)



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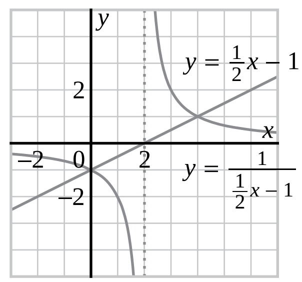
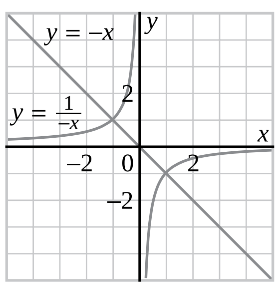


e)



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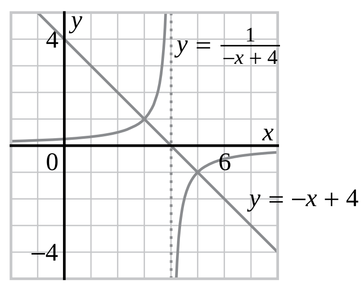
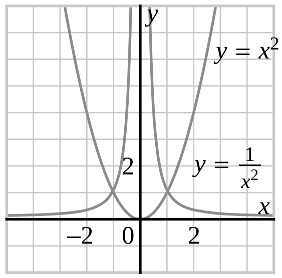


4. Answers may vary. For part a:
The graph of the function $y = x + 2$ is a straight line, so I determined the intercepts and then drew a straight line through the points. To graph the function $y = \frac{1}{x+2}$, I noticed that the function is not defined when $x = -2$. Thus, the graph has two branches, on either side of the vertical line $x = -2$. I constructed two tables of values, one for each branch, then joined the points of each branch with a smooth curve.

Selected Solutions — Chapter 3

5. a)



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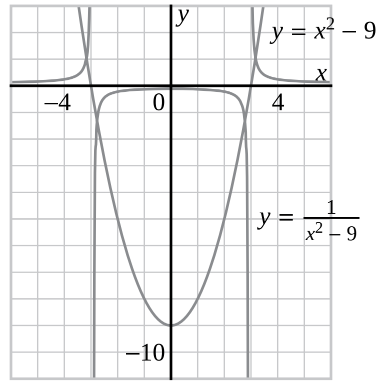
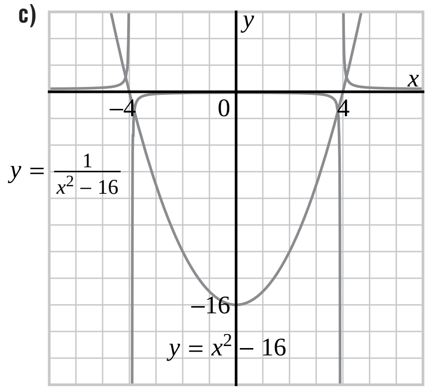


c)



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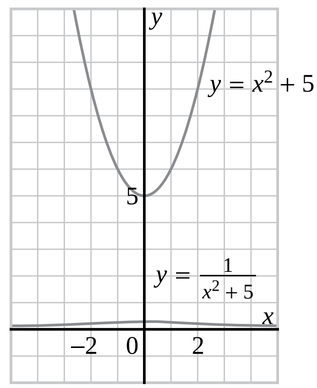
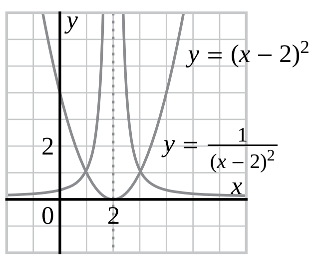
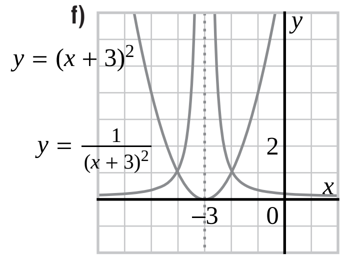


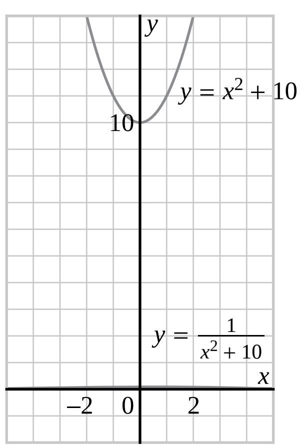
e)



f)



6. a)



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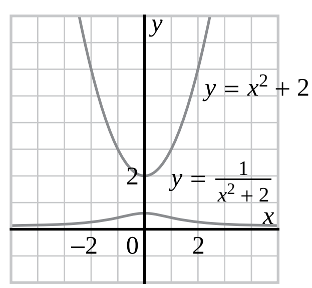
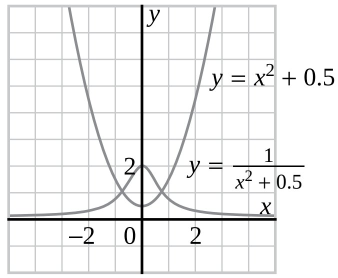


Selected Solutions — Chapter 3

c)



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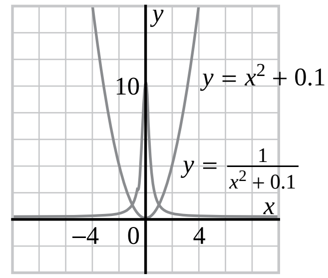
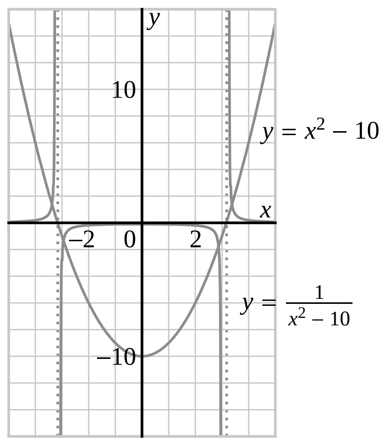


7. a)



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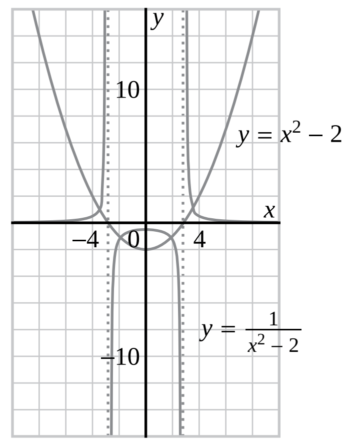
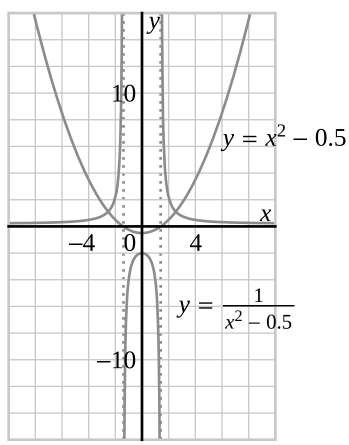


c)



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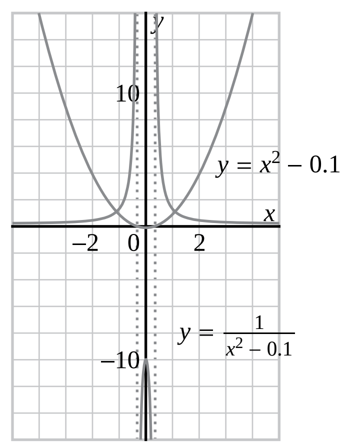
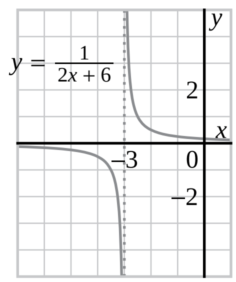


8. The graph of $y = x^2 + c$ has the same shape for all values of c. As c increases, the graph is translated upwards, and as c decreases the graph is translated downwards.

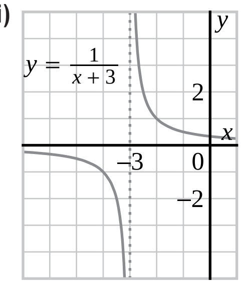
The graph of the function $y = \frac{1}{x^2 + c}$ changes its shape as c changes.

When $c = 0$, the graph consists of two branches, both entirely above the x-axis, and both branches shooting upwards to infinitely large values of y as x approaches 0. When c increases to a small positive value, the two branches join together so that there is only one continuous curve. There is a very high hill with its peak at $(0, \frac{1}{c})$, and as before, the curve approaches the x-axis for infinitely large and small values of x. As c increases, the curve has the same general shape, but the height of the hill decreases, always with a height of $\frac{1}{c}$. When c is negative, the shape of the graph is quite different. There are two values for which the function is undefined, so the graph consists of three disconnected branches. To obtain these two values, solve the equation $x^2 - c = 0$ to obtain $x = \pm\sqrt{c}$. The vertical lines $x = \pm\sqrt{c}$ are called vertical asymptotes. To the right of $x = \sqrt{c}$ and to the left of $x = -\sqrt{c}$, the graph looks similar to the graph of $y = \frac{1}{x^2}$: far off to the right and left of the graph, the curves gradually approaches the x-axis, and the curves get infinitely high as they approach the vertical asymptote. Between the vertical asymptote, the middle branch of the graph looks like an upside-down U, reaching a peak at $(0, \frac{1}{c})$, and plunging down infinitely as it nears the two vertical asymptotes. As c decreases through negative values, the vertical asymptotes get pushed outwards, and the U-shaped middle branch gets higher (closer and closer to the x-axis, but never crossing it) and stretches outwards, following the vertical asymptotes.

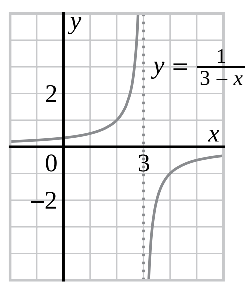
9. a) i)



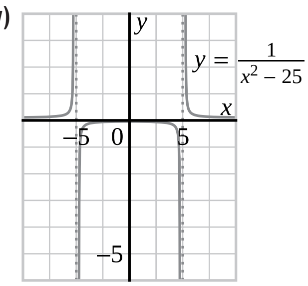
ii)



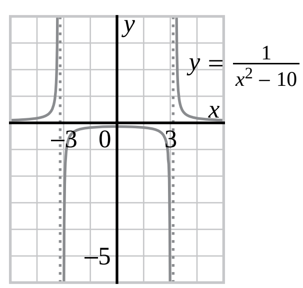
iii)



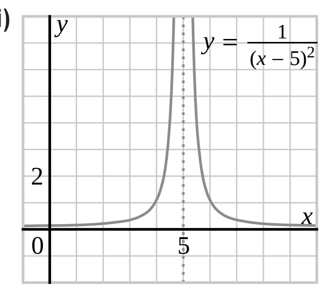
iv)



v)



vi)



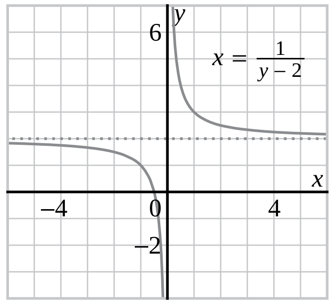
10. The answer to each question is no, and a counterexample for each part is any constant function. For example, consider the constant function $f(x) = \frac{1}{3}$. The reciprocal function is $\frac{1}{f(x)} = 3$, and its graph is a horizontal line, has no asymptote, and is defined for all values of x.

11. a) To get the domain of $\frac{1}{f(x)}$, take the domain of $f(x)$ and remove any points at which $f(x) = 0$.

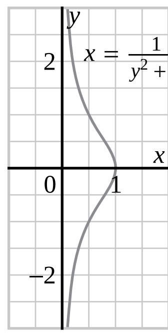
b) The range of $\frac{1}{f(x)}$ is the set of all reciprocals of non-zero elements of the range of $f(x)$.

Selected Solutions — Chapter 3

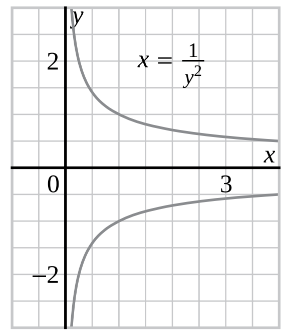
12. a)



13. a) i)



ii)



14. a) Predictions may vary. All the curves pass through the points (1, 1) and (–1, –1). All the curves have the same general shape and the same general features.

b) Consider the branch of the curve for $x > 0$. Imagine the L-shape formed by the rays $x = 1$(for $y > 0$), and $y = 0$ (for $x > 1$). Then as n gets larger, the upper part of the curve (that is, for $0 < x < 1$) approaches the vertical part of the L, and the lower part of the curve (that is, for $x > 0$) approaches the horizontal part of the L.

Rotate the whole picture (and the discussion!) by 180° to see what happens to the curve for $x < 0$. Consider the branch of the curve for $x < 0$. Imagine the L-shape formed by the rays $x = -1$ (for $y < 0$), and $y = 0$ (for $x < -1$). As n gets larger, the lower part of the curve approaches the vertical part of the L and the upper part of the curve approaches the horizontal part of the L.

15. a) All the curves pass through the points (1, 1) and (–1, 1). All of the curves have the same general shape and the same general features.

b) Consider the branch of the curve for $x > 0$. Imagine the L-shape formed by the rays $x = 1$(for $y > 0$), and $y = 0$ (for $x > 1$). Then as n gets larger, the upper part of the curve (that is, for $0 < x < 1$) approaches the vertical part of the L, and the lower part of the curve (that is, for $x > 0$) approaches the horizontal part of the L.

Thus, for $x > 0$, the behaviour here is much the same as the behaviour in exercise 14 b. Reflect the whole picture (and the discussion!) in the y-axis to see what happens to the curve for $x < 0$. Consider the branch of the curve for $x < 0$. Imagine the L-shape formed by the rays $x = -1$ (for $y > 0$), and $y = 0$ (for $x < -1$). As n gets larger, the upper part of the curve approaches the vertical part of the L, and the lower part of the curve approaches the horizontal part of the L.

Selected Solutions — Chapter 3

3.7 Exercises, page 200

1. a) The function is a polynomial function because it has the form $y = ax^2 + bx + c$.

b) The function is a rational function because it has a variable in the numerator.

c) The function is a polynomial function because it has the form $y = ax^3 + bx^2 + cx + d$.

d) The function is a rational function because it can be written $y = \frac{1}{x+2}$, and it has a variable in the denominator.

e) The function is a polynomial function because it can be written $y = \frac{1}{4}x^2 - \frac{1}{2}x - 2$.

f) The function is some other type because it has the variable as an exponent.

g) The function is a polynomial function because it has the form $y = ax^2 + bx + c$.

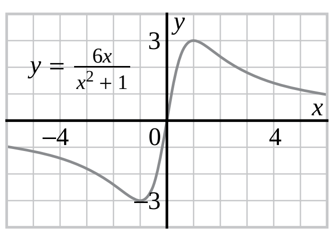
h) The function is some other type because it has a variable in a square root.

2. The graphs of polynomials do not have asymptotes, so it is possible that graphs b and d are graphs of polynomial functions. The other 4 graphs have asymptotes, so they might be the graphs of rational functions.

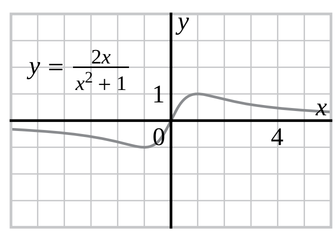
3. a) Yes. Let $p(x)$ represent the polynomial function. Then its reciprocal is $\frac{1}{p(x)}$, which is the ratio of two polynomial functions, since its numerator, 1, is also a rational function.

b) No. For example, consider the rational function $f(x) = \frac{x-1}{x}$. Its reciprocal is $g(x) = \frac{x}{x-1}$, which is also a rational function.

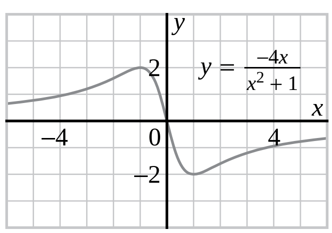
5. a)



b)

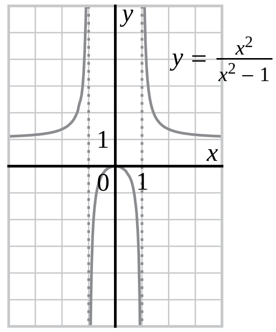


c)



Selected Solutions — Chapter 3

6. a)



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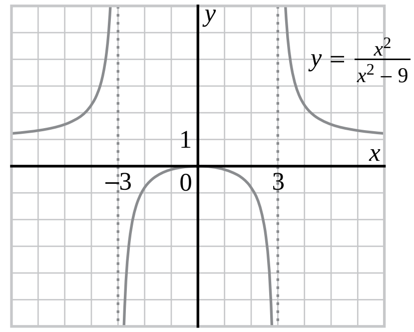

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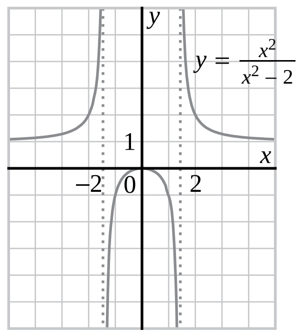
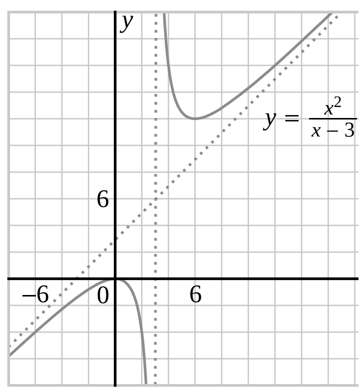


7. a)



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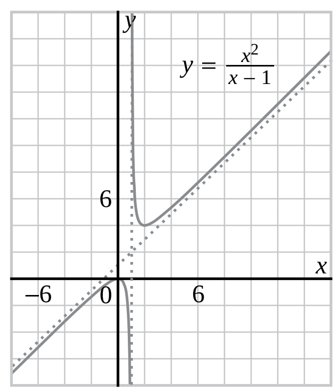

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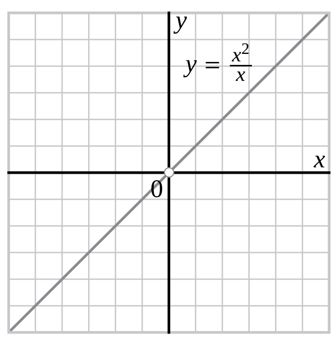
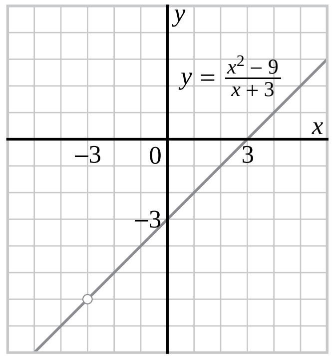


8. a)



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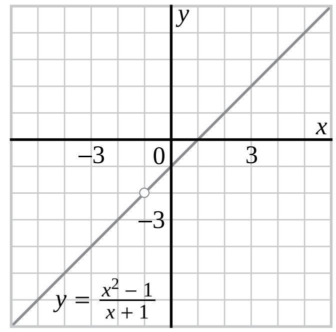

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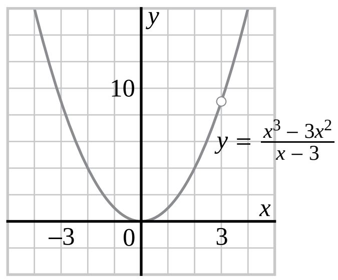
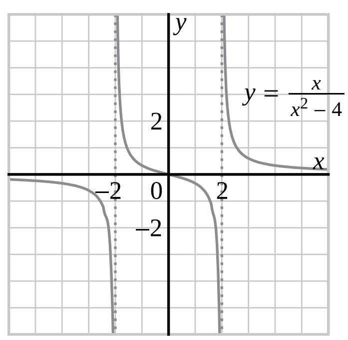
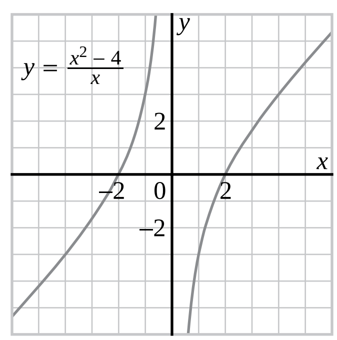


9. Explanations may vary. For exercise 5b:
The function $y = \frac{2x}{x^2 + 1}$ is a vertical compression of factor $\frac{1}{2}$ of the graph of $y = \frac{4x}{x^2 + 1}$ on page 198. So I used the points on the graph of $y = \frac{4x}{x^2 + 1}$ and plotted corresponding points with the same x-coordinate and half the y-coordinate, to get the graph of $y = \frac{2x}{x^2 + 1}$.

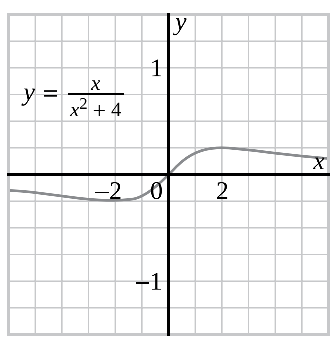
10. a) i)



ii)



iii)



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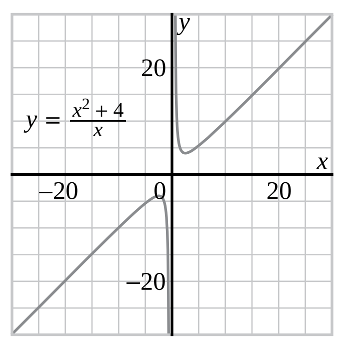
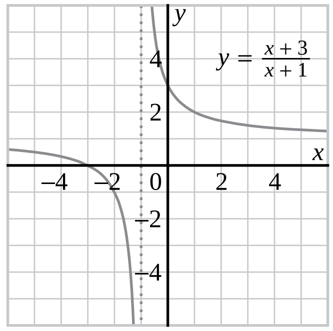
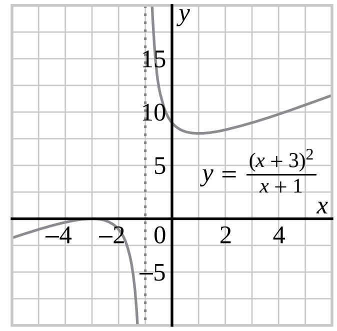


Selected Solutions — Chapter 3

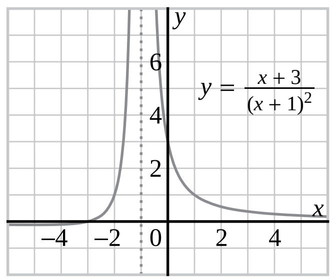
11. a) i)



ii)



iii)



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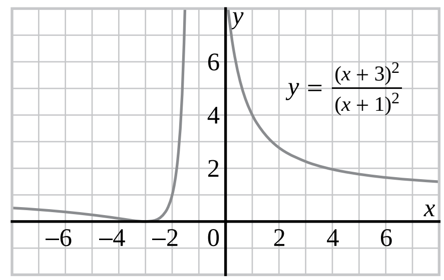
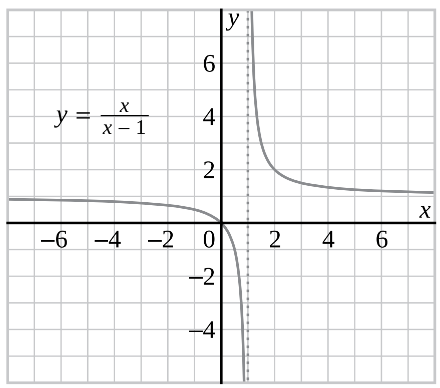
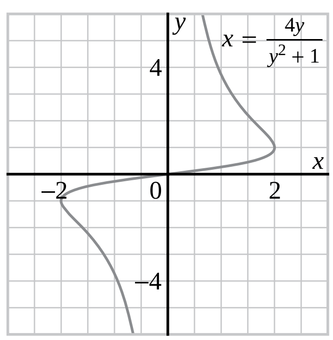


13. a)



14. a) i)



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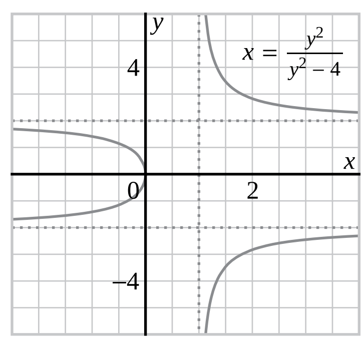
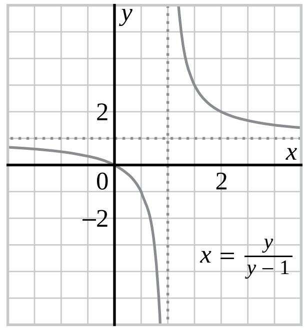


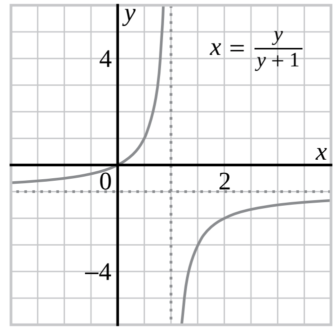
c) Neither inverse is a function. Each inverse has x-values that correspond to more than one y-value.

Selected Solutions — Chapter 3

15. a) i)



ii)



c) Both inverses are functions. There is only one y-value for each x-value.

16. Answers may vary. In each case, let the rational function be $f(x) = \frac{m(x)}{n(x)}$.

a) $f(x)$ is continuous for all values of x if $n(x)$ is never zero for any value of x.

b) The graph of $f(x)$ has holes when the only factors of $n(x)$ that are zero for some values of x are also factors of $m(x)$. There is one hole for each distinct factor.

c) The graph of $f(x)$ has vertical asymptotes when there are factors of $n(x)$ that have zeros and these factors are not also factors of $m(x)$. There is one vertical asymptote for each such factor.

d) The graph of $f(x)$ has horizontal asymptotes when the degree of $m(x)$ is less than or equal to the degree of $n(x)$.

e) The graph of $f(x)$ has an inclined asymptote when the degree of $m(x)$ is one more than the degree of $n(x)$.

17. a) No. For example, the graph of the function $f(x) = x$ has point symmetry about every point on the line, and it has line symmetry with respect to every line perpendicular to it. However, the reciprocal function $\frac{1}{f(x)} = \frac{1}{x}$ has point symmetry only about the origin, and line symmetry only with respect to the line $y = -x$.

b) Yes. To obtain the inverse of the function $f(x)$, the graph is reflected in the line $y = x$. However, all lines of symmetry and points of symmetry are also reflected, so the inverse has exactly the same symmetry as the original function.

Modelling How Travelling Time Changes when Speed Changes, page 205

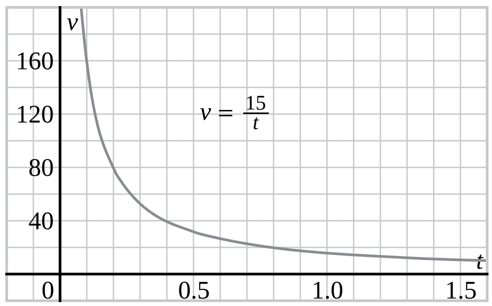
- A faster speed means that less time is needed for the journey.
- When the speed is slower, more time is needed for the journey.
- The graph of t against s has a negative slope.
- In this way increases in speed and increases in time are both considered to be positive changes, whereas decreases in speed and decreases in time are both considered to be negative changes. If one sign were negative, then the corresponding increase would be interpreted as a negative change, and that might be confusing. The

Selected Solutions — Chapter 3

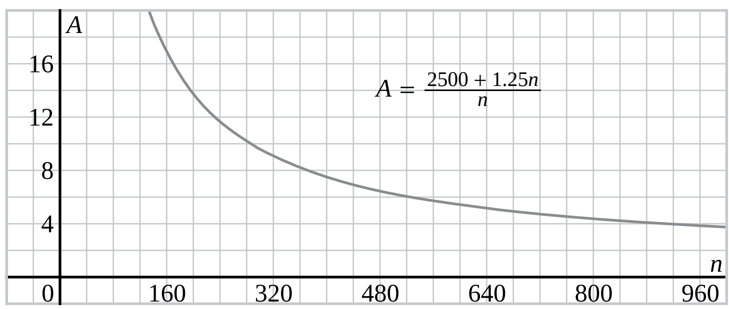
conclusions of the solution would not change, but the graph would have a positive slope, and some positive values would now be negative, and vice versa.

3.8 Exercises, page 206

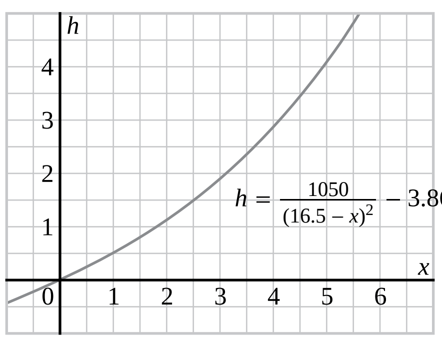
1. b)



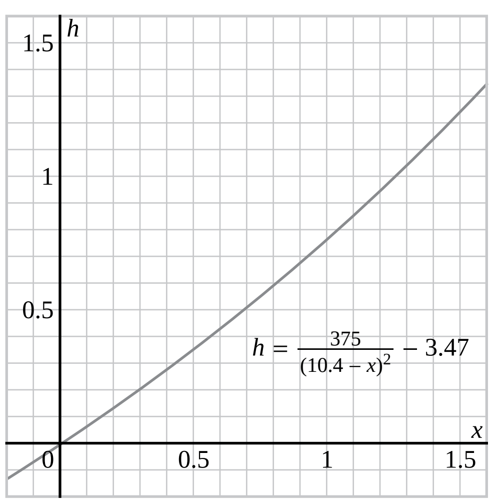
3. a)



4. c)

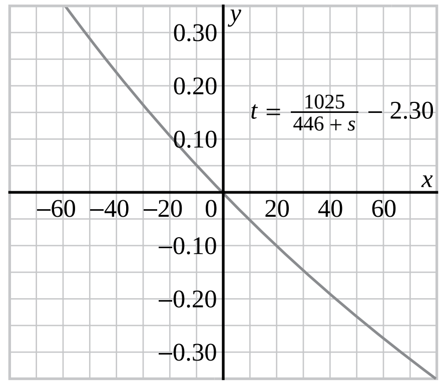


5. b)



Selected Solutions — Chapter 3

6. c)



Mathematical Modelling: The Water and the Wine, page 208

1. a) Yes. In each pile, ten cards were taken away and ten cards were added. Thus, each pile has the same number of cards it started out with, 26.

   b) Call the pile that used to have all red cards the red pile and call the pile that used to have all black cards the black pile. Let x represent the number of black cards that are in the red pile after the exchange of cards. Since there are 26 cards in the red pile, the other $(26 - x)$ cards in the red pile must be red. This leaves $(26 - (26 - x))$ or x red cards that are in the black pile. Thus there are x black cards in the red pile, and x red cards in the black pile.

2. Let the black cards represent water and the red cards represent wine. Then the same reasoning that was used in exercise 1b shows that there is as much water in the wine jug as wine in the water jug after the exchanges are made.

3. a) No. In the solution to exercise 1b no mention was made of the number of cards in the black pile. Thus, the reasoning is valid no matter how many cards are in each pile to start with, provided that each pile ends up with the same number of cards that it started with.

   b) It doesn't matter how much fluid each jug starts with, provided that each jug ends up with the same volume of fluid that it started with.

4. a) No. Shuffling may affect the value of x in the solution to exercise 1b, but it doesn't affect the validity of the reasoning. Thus, shuffling may affect how many red cards end up in the black pile, and vice versa, but it won't change the fact that they are equal.

   b) Stirring may affect how much water ends up in the wine jug and vice versa, but it won't change the fact that they are equal.

5. a) Yes. If the number of cards transferred is not the same then the reasoning in the solution to exercise 1b is not valid. For example, you could transfer 10 red cards to the black pile and then transfer only 2 cards from the black pile back to the red pile. There is no

way that the number of red cards in the black pile can be equal to the number of black cards in the red pile.

b) Again, the volumes of liquid transferred must be equal or the volume of wine in the water jug will not be equal to the volume of water in the wine jug.

6. a)

Jug	Volume at start (L)	Volume after first step (L)
1st (water jug)	t	$t - x$
2nd (wine jug)	n	$n - x$

i) Using the table, the volume of water in the first jug is $(t - x)$ litres.

ii) The volume in the second jug is $(n + x)$. (The volume of wine is n litres and the volume of water is x litres.)

iii) Fraction of water in second jug $= \frac{x}{n+x}$;
fraction of wine in second jug $= \frac{n}{n+x}$.

b) i) Assume that the fluid in the wine jug is now evenly mixed. The volume of wine transferred back to the water jug is the volume of fluid transferred × the fraction of the fluid that is wine:

$$\text{Volume of wine transferred} = x \times \frac{n}{n+x} = \frac{nx}{n+x}$$

ii) The volume of water transferred back to the water jug is the volume of fluid transferred × the fraction of the fluid that is water:

$$\text{Volume of water transferred} = x \times \frac{x}{n+x} = \frac{x^2}{n+x}$$

But there were x litres of water in the wine jug, so the volume of water that now remains is

$$x - \frac{x^2}{n+x} = \frac{x(n+x) - x^2}{n+x} = \frac{nx}{n+x}$$

Compare this result with the result of part bi. The volume of wine in the water jug is equal to the volume of water in the wine jug.

7. a) The four functions are:

$$y = \frac{x}{x+1}$$

$$y = \frac{2x}{x+2}$$

$$y = \frac{3x}{x+3}$$

$$y = \frac{4x}{x+4}$$

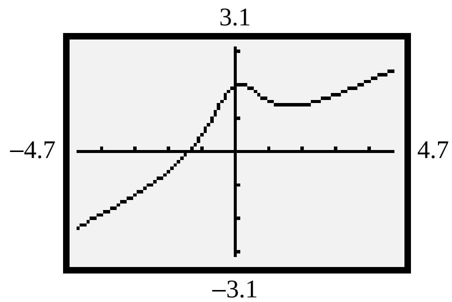
b) The mixing takes place in the jug that holds n litres of fluid.

c) The domain of each function is the intersection of the two intervals $0 < x \le n$ and $0 < x \le t$. If we assume that $t \ge n$, then the domains are $0 < x \le 1$, $0 < x \le 2$, $0 < x \le 3$, and $0 < x \le 4$ respectively.

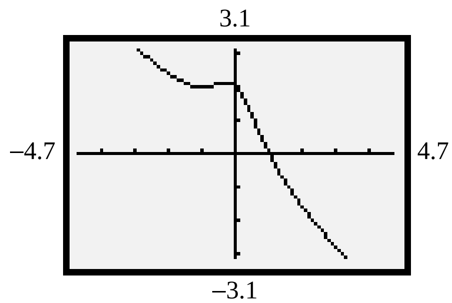
Selected Solutions — Chapter 3

Exploring with a Graphing Calculator: Graphing f(x) + g(x), page 212

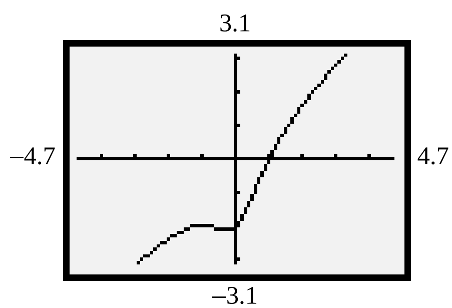
1. a) The graph has an asymptote $y = 0.5x$ for x-values that are far to the left and far to the right of the graph. Near the origin there is a relative maximum just past $x = 0$, and then a relative minimum a little after that.



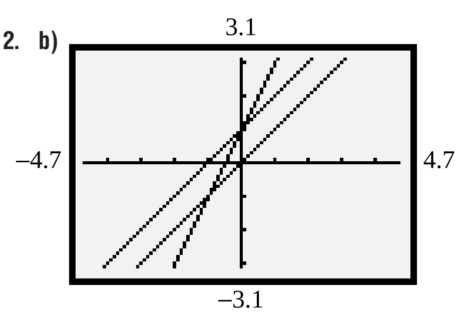
b) The graph is a reflection in the y-axis of $y = \frac{2}{x^2 + 1} + x$.



c) The graph is a reflection in the y-axis and then a reflection in the x-axis of $y = \frac{2}{x^2 + 1} + x$.



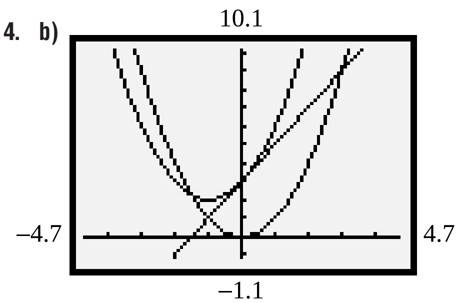
2. b)



Selected Solutions — Chapter 3

3. b) The coordinates are related in this way because the formulas are added to obtain the formula for the third function.

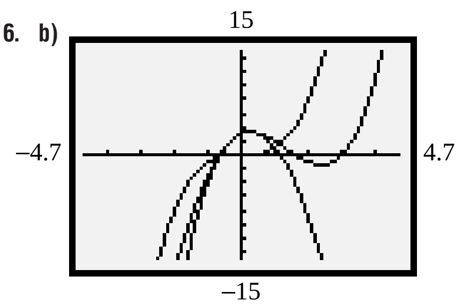
4. b)



c) For every x-value, the height difference between the two functions is x^2. Thus, when $x = 0$, the two graphs have the same height; in other words, the two graphs have the same y-intercept.

5. b) For every x-value, the height difference between the two functions is x^2. Thus, when $x = 0$, the two graphs have the same height; in other words, the two graphs have the same y-intercept.

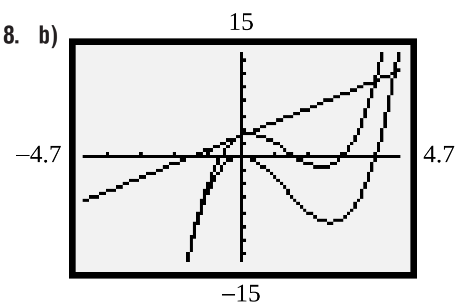
6. b)



c) For every x-value, the height difference between the two functions is x^3. Thus, when $x = 0$, the two graphs have the same height; in other words, the two graphs have the same y-intercept.

7. b) For every x-value, the height difference between the two functions is x^3. Thus, when $x = 0$, the two graphs have the same height; in other words, the two graphs have the same y-intercept.

8. b)

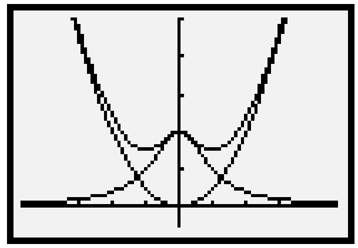


c) For every x-value, the height difference between the two functions is $x^3 - 4x^2$. Thus, when $x = 0$, the two graphs have the same height; in other words, the two graphs have the same y-intercept.

9. b) For every x-value, the height difference between the two functions is $x^3 - 4x^2$. Thus, when $x = 0$, the two graphs have the same height; in other words, the two graphs have the same y-intercept.

Selected Solutions — Chapter 3

10. b)



c) Express the function using a common denominator.

$$y = \frac{4}{x^2 + 1} + x^2$$
$$= \frac{4 + x^4 + x^2}{x^2 + 1}$$

Thus, the function is the quotient of two polynomial functions, so it is a rational function.

d) Its graph looks like the graph of a polynomial function. This is because for values of x that have large absolute values, the term $\frac{4}{x^2 + 1}$ is very small; thus the function $y = \frac{4}{x^2 + 1} + x^2$ approximates $y = x^2$ for values of x that have large absolute values.

3.9 Exercises, page 217

16. b) No. It is not true in general that the composition of two functions is the reciprocal of one of them, although it may be true in some special case. One special case is part a, but part b is not a special case.

Linking Ideas: Mathematics and Technology
Patterns in Graphs of Functions, page 220

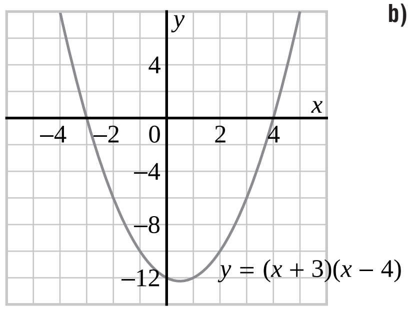
2. The straight line function is $y = 0$; the only function of the form $y = \frac{a}{x^2 + a}$ that is identical to $y = 0$ has $a = 0$.

3. b) The function $y = 0$ does not pass through the point $(0, 1)$, since all the y-values for this function are 0.

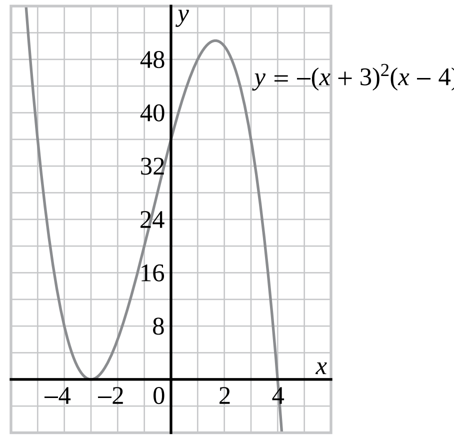
Selected Solutions — Chapter 3

3 Review, page 221

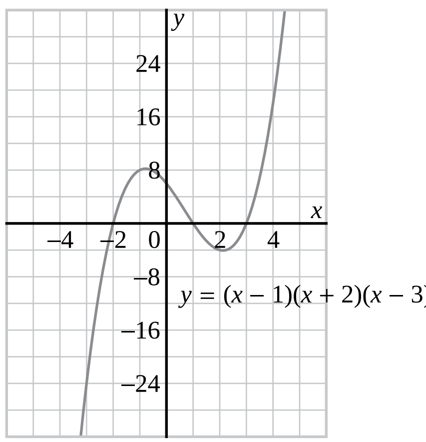
3. a)



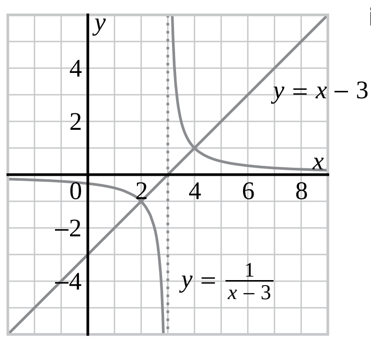
b)



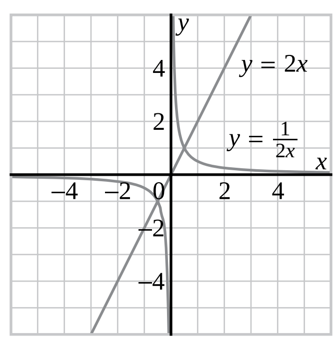
c)



5. a) i)



ii)



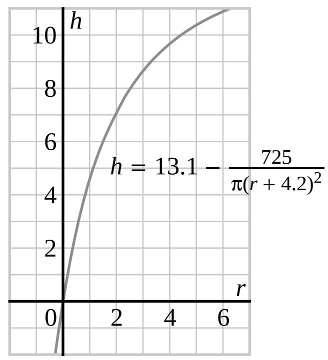
c) Explanations may vary. For part aii:
I graphed $y = 2x$ after finding the coordinates of the two points on the line, (0, 0) and (2, 4). Then, for each whole number value of x from -2 to 2, I took the reciprocal of the y-value and plotted a point. For example, when $x = 2$, $y = \frac{1}{4}$, when $x = -1$, $y = -1$. Since $x \neq 0$ for $y = \frac{1}{2x}$, I know that the y-axis is an asymptote. Also, $y \neq 0$, since 1 is the numerator of the function, so the x-axis is also an asymptote. I joined the points to form two smooth curves.

Selected Solutions — Chapter 3

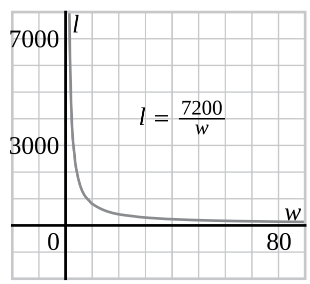
6. b) Functions i and ii have the form $y = ax^3 + bx^2 + cx + d$, so they are polynomial functions. Functions ii and iv have polynomials in the numerator and denominator, so they are rational functions. Functions v and vi involve radicals of x, so they are neither polynomials nor rational functions.

7. c) The function is not defined for $x = 3$; thus, no matter how much we zoom in there will always be a hole at $x = 3$.

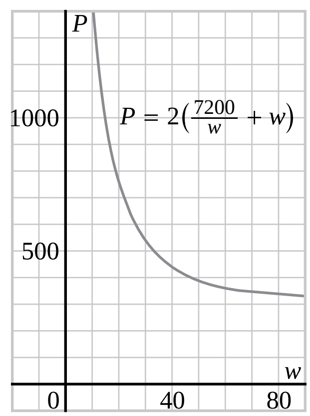
8. b)



9. b)



d)



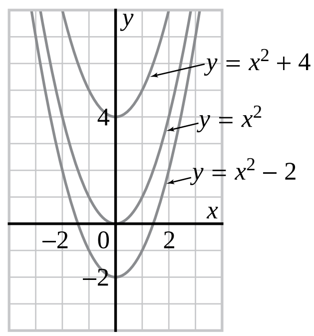
ADDISON-WESLEY Mathematics 11

Selected Solutions — Chapter 3

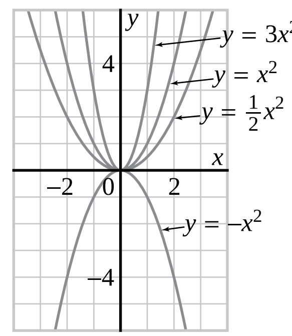
3 Cumulative Review, page 223

2. **b)** Explanations may vary. For part i:
I used the formula $A = P(1 + i)^n$. I need to find P, and everything else in the formula is known. So, I substituted the known quantities into the formula then solved for P.
$4250 = P(1.07)^4$ to get $P = \frac{4250}{(1.07)^4} = 3242.30$. You would have to invest about \$3242.30.
Then I checked the result.

5. **a)** **i)**



ii)



b) Answers may vary. For part i:
I drew $y = x^2$ using the method of differences, after plotting the vertex at (0, 0). For $y = x^4 + 4$, I plotted the vertex (0, 4), then used the method of differences. For $y = x^2 - 2$, I plotted the vertex (0, –2), then used the method of differences. All three graphs are congruent.