Selected Solutions — Chapter 2

2.1 Exercises, page 94

1. **b)** Explanations may vary.
For part i: I recognized that $y = 3x^2 + 7x - 2$ can be written in the form of a general quadratic function $y = ax^2 + bx + c$, where $a = 3$, $b = 7$, and $c = -2$; hence, the function is quadratic.
For part ii: I recognized that $f(x) = x^2 + \sqrt{x}$ cannot be written in the form, $f(x) = ax^2 + bx + c$, because the function contains the square root of a variable; hence, the function is not quadratic.

3. The vertex of a parabola lies on its axis of symmetry; in other words, the vertex is the point of intersection of the parabola with its axis of symmetry.

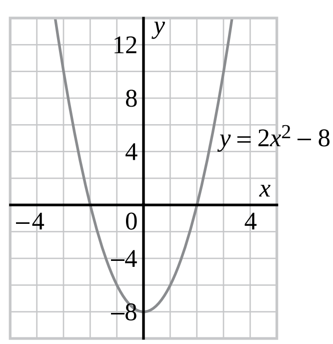
4. **a)** The graph has zero x-intercepts if it is entirely above or below the x-axis. It has one x-intercept if its vertex is on the x-axis. It has two x-intercepts otherwise.

b) It has exactly one y-intercept, which occurs when $x = 0$.

c) Yes, if the graph passes through the origin, then both of its intercepts are 0.

5. When the parabola opens up and the vertex is (a, b), the range is $y \geq b$.
When the parabola opens down and the vertex is (a, b), the range is $y \leq b$.

6. **a)**



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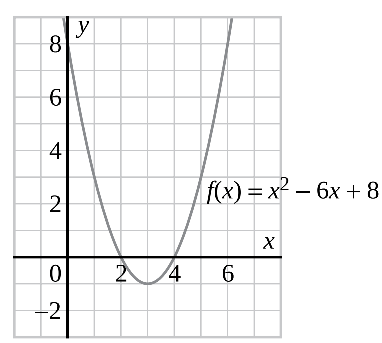

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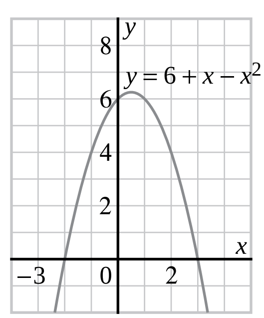

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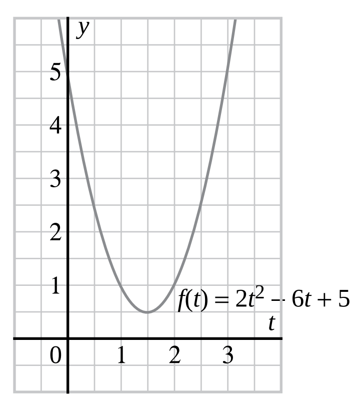
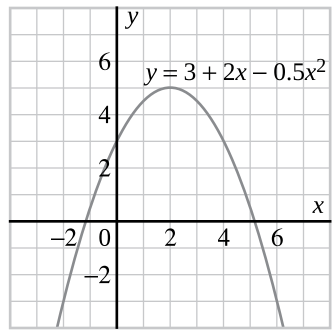


Selected Solutions — Chapter 2

e)



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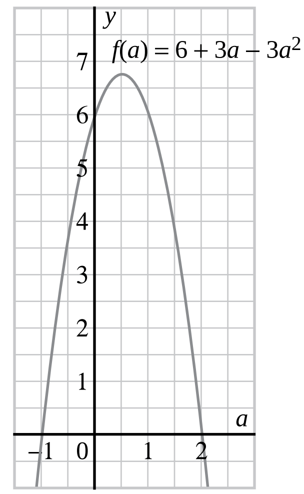
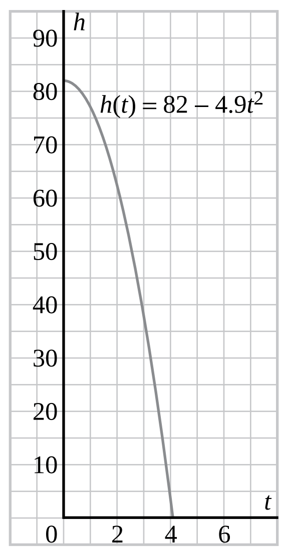
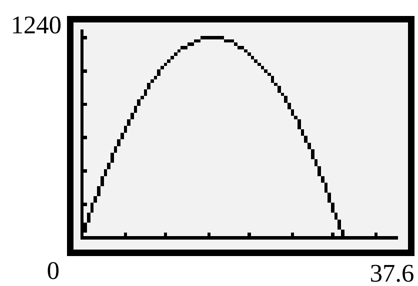


7. a)



8. a) Answers may vary.



The window setting used is $-10 \le X \le 40$ and $-1200 \le Y \le 1200$. This setting shows when the flare is at its maximum height and when $h = 0$.

d) The coordinates of the vertex represent the maximum height of the flare and the time at which the maximum height is reached.

e) The domain represents the times the flare is in the air. The range represents the heights the flare can reach.

Selected Solutions — Chapter 2

Modelling the Height of a Projectile

- Substitute $g \doteq 9.8$, $v = 153.2$, and $s = 0$ in the formula ①.
  $h(t) = \frac{1}{2}(9.8)t^2 + 153.2t + 0$
  $h(t) = -4.9t^2 + 153.2t$
- The heights in the table of values would be slightly greater. The vertex would now be at (10.2, 510.2), and the horizontal intercept would be at (20.4, 0).
- The model of formula ① assumes that there is no air resistance or other influences that might disturb the flight of the flare. Air resistance will be present, and there might be wind or rain as well.

9. b) The only way $5x^2 + kx + 4$ can be factored is in the form $(5x + a)(x + b)$. Now expand the factors to get $5x^2 + (5b + a)x + ab$. The two quadratic expressions are equal, so compare coefficients to get $5b + a = k$ and $ab = 4$. Now a and b are integers, so their only possible values are the possible factors of 4; that is, a and b must be chosen from the list ±1, ±2, ±4. Substituting all of these acceptable values of a and b into the equation in k will give us all of the values of k for which the original quadratic equation can be factored. The results are summarized in the table below:

a	$b = \frac{4}{a}$	$k = 5b + a$
1	4	21
–1	–4	–21
2	2	12
–2	–2	–12
4	1	9
–4	–1	–9

Thus, the only values of k for which the trinomial $5x^2 + kx + 4$ can be factored are ± 9, ± 12, and ± 21.

10. In part b, for example the factors of $2x^2 + mx + 5$ are $(2x + a)(x + b)$. Expand the factors. $2x^2 + (a + 2b)x + ab$
Since the two quadratic expressions are equal, compare the coefficients. $a + 2b = m$ and $ab = 5$
Since a and b are integers, their only possible values are factors of 5; that is, ±1 and ±5. Substitute these values of a and b into the equation in m to give all the values of m for which the original trinomial can be factored. The results are summarized in the table below.

a	$b > \frac{5}{a}$	$m = a + 2b$
–5	–1	–7
–1	–5	–11
1	5	11
5	1	7

Selected Solutions — Chapter 2

Thus, the only values of m for which the trinomial $2x^2 + mx + 5$ can be factored are –11, –7, 7, or 11.

16. **a)** The y-coordinate will get closer and closer to 0, but will never reach 0.

b) Answers may vary. The calculator only "samples" the x-axis at certain points, so if the zero of the function does not happen to be at one of these sampling points, we will only get approximations no matter how much we zoom in.

17. **a)** Refer to the example on page 84. Adjust the viewing window so x runs from –8 to 8 with tick marks at every unit. Do not change the y range. Press GRAPH. The negative zero is also displayed. Each graphing calculator method below can be used to determine the negative zero.

- Press TRACE and ZOOM 2 repeatedly as on pages 85 to 86.
- Press CALC 2 to select the "zero" feature as on page 87.

b) Suppose that the cliff disappeared and the stone were projected upwards from the ground, at the same height as the beach, behind where the cliff used to be. Then if the stone were thrown at the time given by the negative solution to the equation, at just the right speed and in the right direction, it would retrace the path of the original stone, reaching all points along the path at the same times as the original stone.

18. For each part of this exercise, use this reasoning: Begin by factoring the given quadratic as $(x + a)(x + b)$. Expand the factors to get $x^2 + (b + a)x + ab$. Then compare this expanded version with the original quadratic to draw conclusions.

a) We have $ab = k$ and $a + b = -1$. Any k with factors whose sum is –1, such as $-6 = (-3)(2)$ or $-12 = (-4)(3)$, will work. All the possible values for k are 10, –2, –6, –12,… (To get a general formula for all k values, solve $a + b = -1$ for b, then substitute the result in $ab = k$ to get $k = -a^2 - a$). Finally, substitute all integer values for a to get all the k values.

b) We want $ab = k$ and $a + b = -2$. Thus, any number with factors whose sum is –2, such as $1 = (-1)(-1)$ or $-8 = (-4)(2)$ is possible. The complete list of possible k values is 1, 0, –3, –8, –15,…, and the general formula for the possible k values is $k = -a^2 - 2a$.

c) The possible values are any values of k with factors whose sum is –3, such as $-18 = (-6)(3)$ or $-28 = (-7)(4)$. The full list is 2, 0, –4, –10, –18, …, and the general formula for the k values is $k = -a^2 - 3a$.

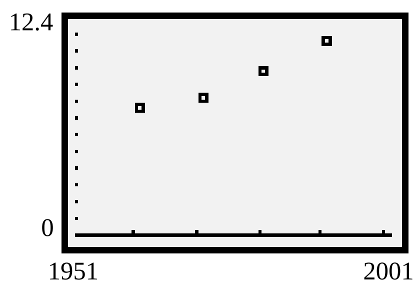
Selected Solutions — Chapter 2

Exploring with a Graphing Calculator: The Parabola of Best Fit, page 100

Modelling Ticket Price Increases

- Use the trace feature on the graphing calculator to determine that the maximum revenue results when ticket prices are $17.17.
- Use the trace feature to find: $14.88 < p < $19.47.
- The *R*-intercept is how much money the arena would lose if admission were free. The *p*-intercepts are the ticket prices for which the revenue is zero.
- Answers may vary. To generate revenue the ticket price must be between the *p*-intercept values. Therefore, a reasonable domain for the function is $2.13 \geq p \geq 32.22$.
- A quadratic function whose graph opens down is a reasonable model for this situation because if prices are too low, the revenue is low; as prices increase, so does revenue, but only to a point. Once prices pass a certain point, revenue begins to decrease.

1. a)



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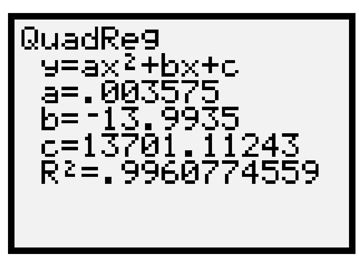
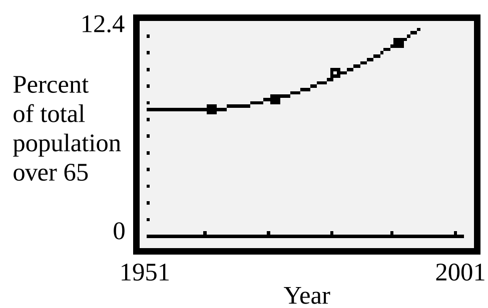
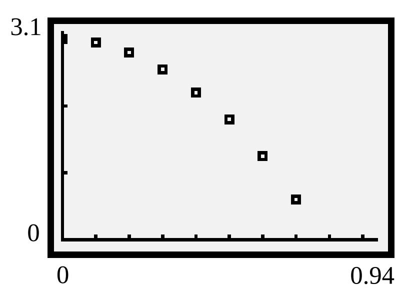
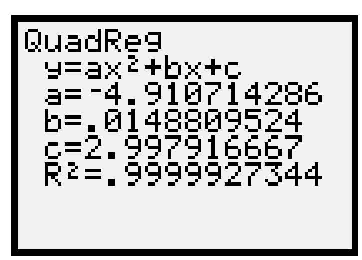


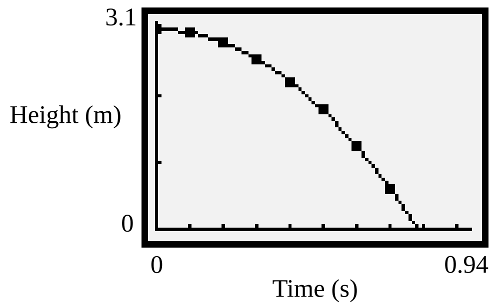


2. a)



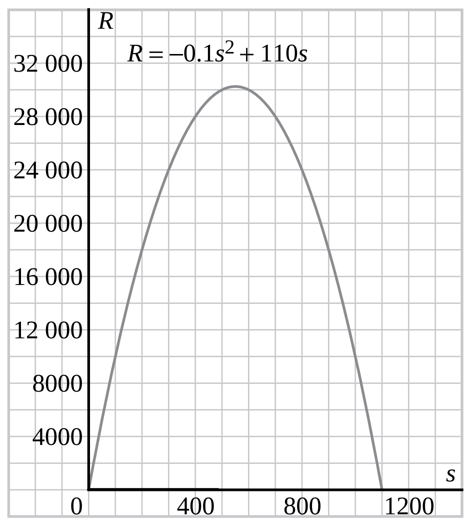


Selected Solutions — Chapter 2

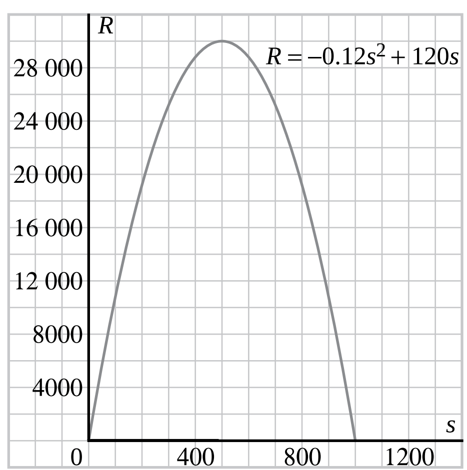


2.2 Exercises, page 105

2. a)

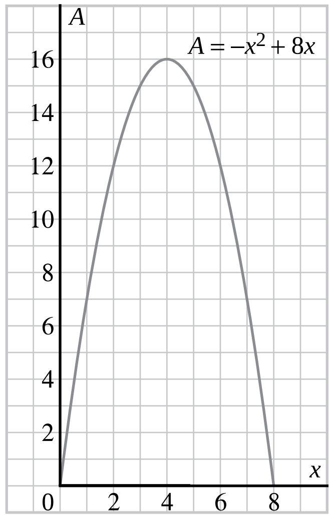


b)

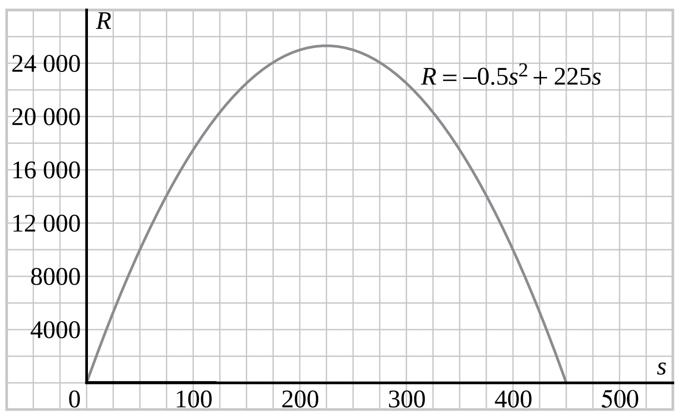


Selected Solutions — Chapter 2

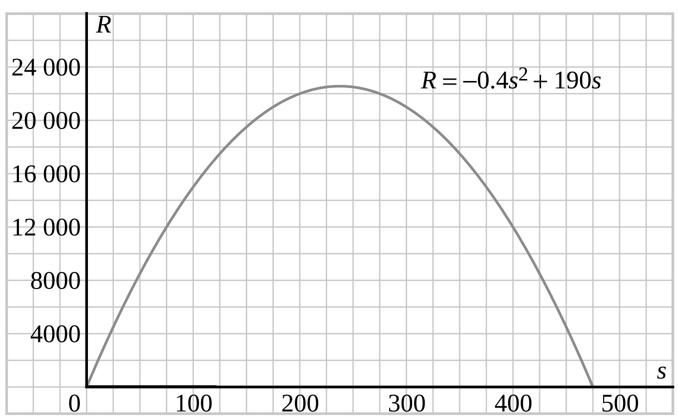
3. b)



4. c)

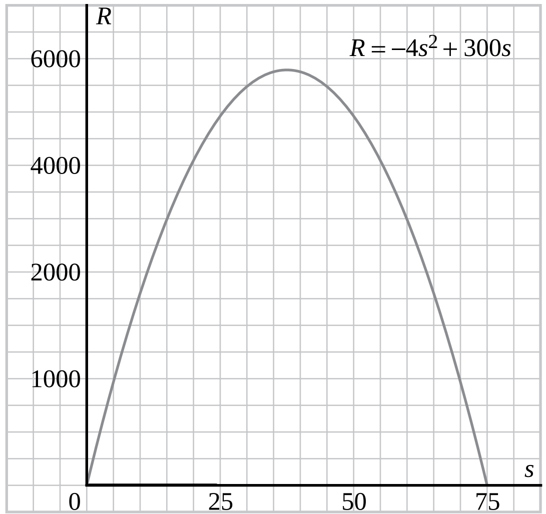


5. a)

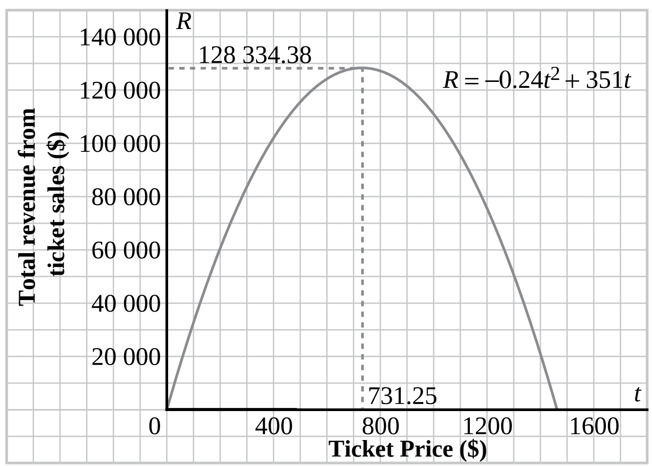


Selected Solutions — Chapter 2

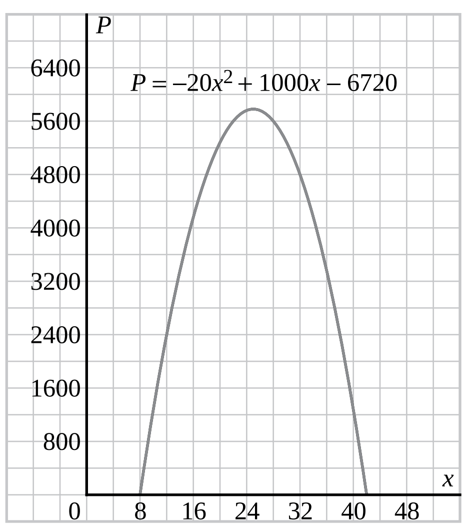
6. a)



7. d) **Revenue vs. ticket price for Continental A30084 flights**



8. c)

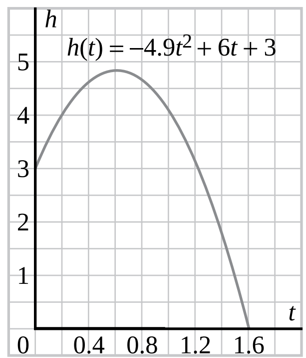


Selected Solutions — Chapter 2

Modelling T-Shirt Price Increases

- Answers may vary. They may have taken a survey to determine how many people would buy T-shirts that sell for different prices, then found a parabola of best fit.
- Answers may vary. One possibility is $8 \le x \le 42$.
- The x-intercepts represent selling prices for which the profit is zero. The y-intercept is the money lost if the T-shirts are given away.
- A parabola opening down is a reasonable model because: if prices are too low, the profit is low; as prices increase, so does profit, but only up to a point. Once prices pass a certain point, sales begin to drop enough that profit begins to decrease.

9. a)



10. Since the perimeter of the pen is 40 m, $2l + 2w = 40$. Thus $l + w = 20$, which means $l = 20 - w$. The graph of $l = 20 - w$ is a straight line, with domain $0 < w < 10$, since $l > w$ and l must be positive.

11. For the graph to have a maximum value, the parabola must open down, so $a < 0$. The vertex must be on the x-axis, which means there is exactly one x-intercept. The x-intercept is found by equating $f(x)$ to 0.

$ax^2 + bx + c = 0$ ①

For exactly one x-intercept, this equation must have equal roots. Let the equal roots be $x = k$. Then, the equation can be written

$a(x - k)(x - k) = 0$

$a(x^2 - 2kx - k^2) = 0$

$ax^2 - 2akx - ak^2 = 0$

Compare this equation with equation ①: $ax^2 + bx + c = 0$

Equate coefficients of x: $b = -2ak$ or $k = \frac{b}{-2a}$ ②

Equate the constant terms: $c = ak^2$ ③

Substitute for k from ② into ③.

$$c = a\left(\frac{b}{-2a}\right)^2$$

$$c = \frac{ab^2}{4a^2}$$

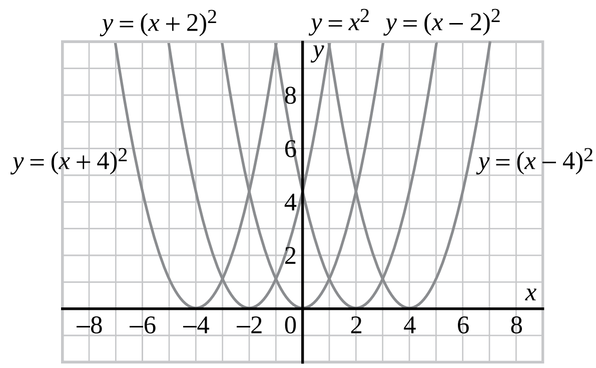
$$b^2 = 4ac$$

Thus, the conditions that must be satisfied are $b^2 = 4ac$ and $a < 0$.

Selected Solutions — Chapter 2

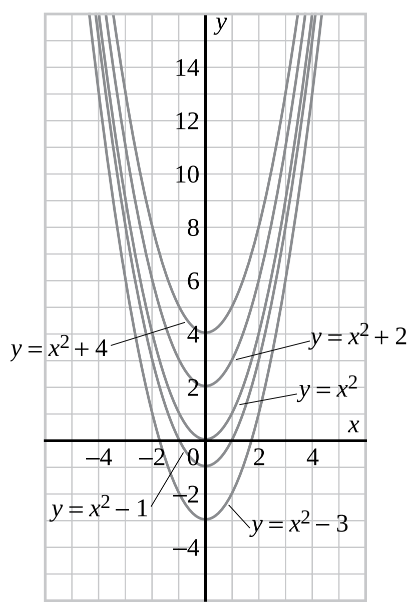
Investigate, page 109

1.



3. c) Consider the function $y = (x - 3)^2$. If you were to graph this function using a table of values, you would start with values of x, subtract 3, then square the results. To give the same y-coordinates as for $y = x^2$, the values of x must be 3 greater than they were for $y = x^2$. This means that the x-coordinates of all points of the graph of $y = (x - 3)^2$ are 3 units greater than those on the graph of $y = x^2$. Therefore, the graph of $y = (x - 3)^2$ is translated 3 units to the right of the graph of $y = x^2$.

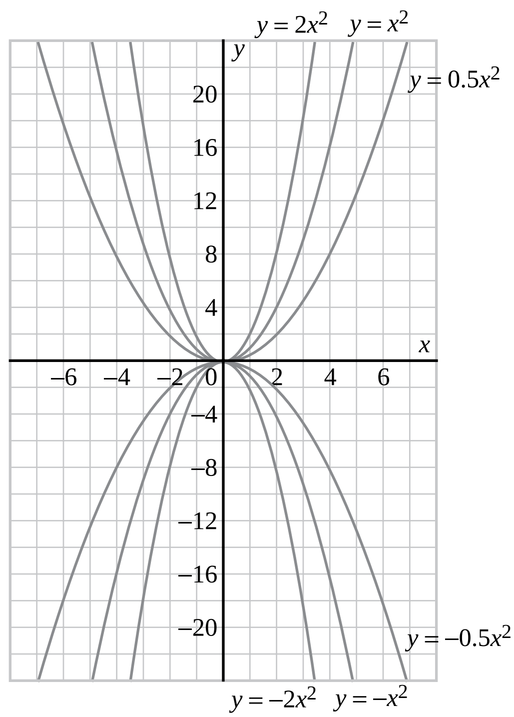
4. a)



b) q provides information about the vertex. If q is positive, the graph is translated up. If q is negative, the graph is translated down. Consider the function $y = x^2 + 2$. The y-coordinates of all points on the graph of $y = x^2 + 2$ are 2 greater than those on the graph of $y = x^2$. Therefore, the graph of $y = x^2 + 2$ is 2 units above the graph of $y = x^2$.

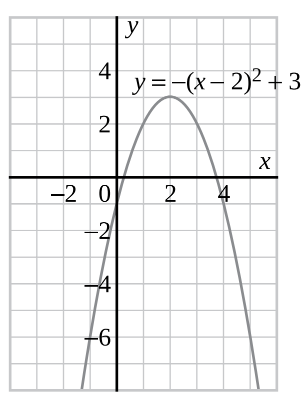
Selected Solutions — Chapter 2

5. a)



b) The sign of a indicates the direction of opening of the graph. If a is positive, the parabola opens up. If a is negative, the parabola opens down. The magnitude of a indicates the shape of the graph compared to the graph of $y = x^2$. If the magnitude of a is greater than 1 (that is, if $a > 1$ or $a < -1$), then the parabola is narrower than $y = x^2$ (that is, expanded vertically). If the magnitude of a is less than 1 (that is, if $-1 < a < 1$), then the parabola is wider than $y = x^2$ (that is, compressed vertically). If $a = -1$, then the parabola is congruent to $y = x^2$. Consider the function $y = 2x^2$. The y-coordinates of all points on the graph of $y = 2x^2$ are 2 times those on the graph of $y = x^2$. Therefore, the graph of $y = 2x^2$ is expanded vertically relative to the graph of $y = x^2$. If a is negative, as in $y = -2x^2$ or $y = -x^2$, there is a reflection in the x-axis.

6. b)



Selected Solutions — Chapter 2

2.3 Exercises, page 115

2. Imagine the parabola is made of wire. Translating the parabola means moving the wire without altering its shape. Expanding the parabola vertically means attaching the wire to the page where it touches the *x*-axis, then pulling vertically on the wire elsewhere to stretch the parabola so that its shape changes.

A parabola translated vertically is congruent to the original parabola. A parabola expanded vertically is not congruent to the original parabola, but has the same vertex and axis of symmetry.

5. a) All the graphs are congruent to $y = x^2$. All their vertices have *x*-coordinate 5. They all have axis of symmetry $x = 5$. The *y*-coordinates of their vertices are 4, 2, 0, –2, –4 respectively.

b) All the graphs are congruent to $y = x^2$. All their vertices have *y*-coordinate 4. Their axes of symmetry are $x = 5$, $x = 3$, $x = 1$, $x = -1$, $x = -3$ respectively. The *x*-coordinates of their vertices are 5, 3, 1, –1, –3 respectively.

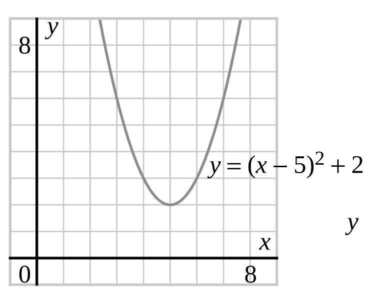
c) All the graphs have vertex (5, 4) and axis of symmetry $x = 5$. The first graph is expanded vertically by a factor of 3 relative to the second graph. The third graph is compressed vertically by a factor of $\frac{1}{2}$ relative to the second graph. The fourth graph is compressed vertically by a factor of $\frac{1}{2}$, and reflected in the *x*-axis relative to the second graph. The fifth graph is reflected in the *x*-axis relative to the second graph. The sixth graph is expanded vertically by a factor of 3, and reflected in the *x*-axis relative to the second graph.

6. a) If $a > 0$ the parabola opens upward, and if $a < 0$ it opens downward. As the magnitude $|a|$ increases, the parabola becomes narrower and "closes up"; that is, the parabola is expanding vertically with increasing $|a|$.

b) As *p* increases, the parabola is translated to the right. As *p* decreases, the parabola is translated to the left.

c) As *q* increases, the parabola is translated up. As *q* decreases, the parabola is translated down.

8. a)



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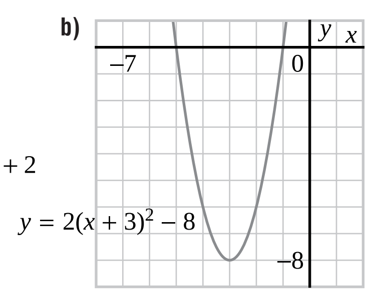
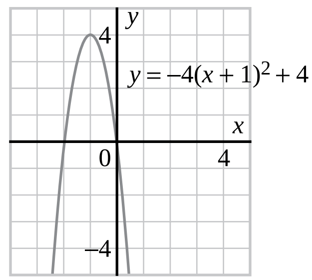
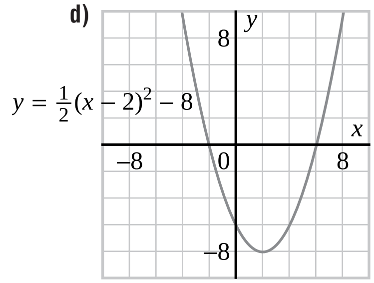


Selected Solutions — Chapter 2

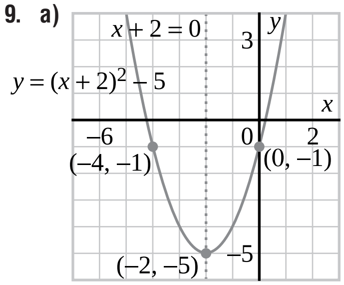
c)



d)



9. a)



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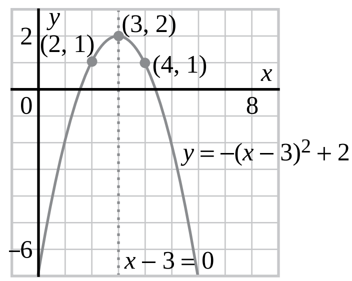
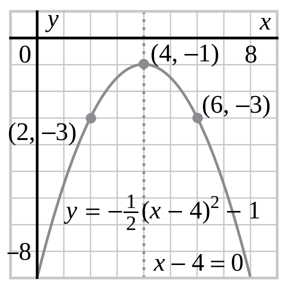


c)



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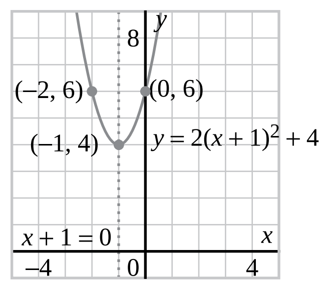
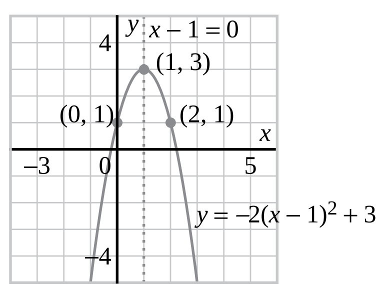


e)



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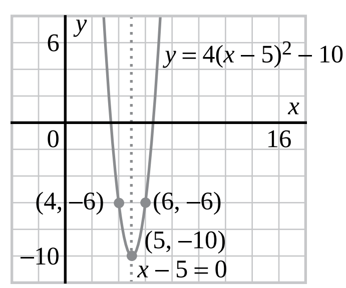
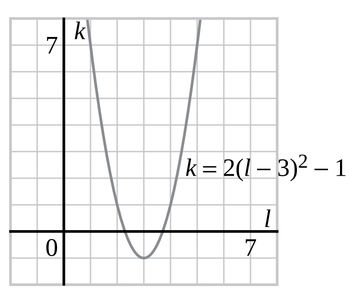


10. a)



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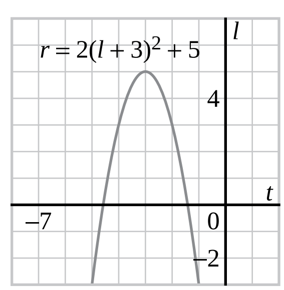
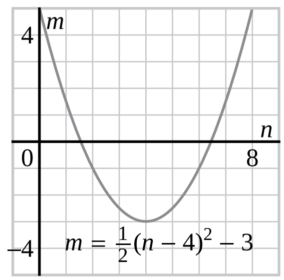
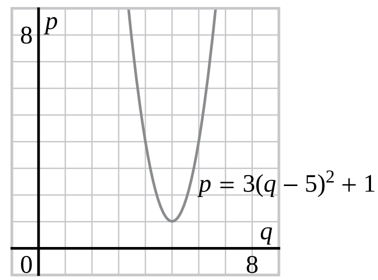


Selected Solutions — Chapter 2

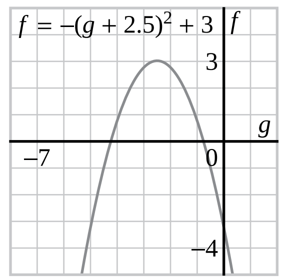
c)



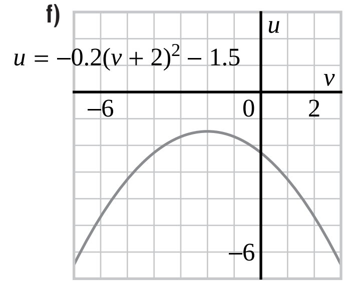
d)



e)



f)



16. b) Answers may vary.

i) The equation has the form $y = a(x - p)^2 + q$. I need to determine the values of a, p, and q. I know the vertex is (3, –1), so that means $p = 3$ and $q = -1$. So the equation has the form $y = a(x - 3)^2 - 1$. To determine the value of a, I used the information that the x-intercepts are 2 and 4. This means that the points (2, 0) and (4, 0) are on the graph. I substituted one of these points ((2, 0)) into the equation to get $0 = a(2 - 3)^2 - 1$, which simplified to $a = 1$.
Thus, the equation of the parabola is $y = (x - 3)^2 - 1$. Then I checked that (4, 0) also satisfies the equation. I substituted $x = 4$ and $y = 0$ in the equation.
L.S. = 0 and R.S. = $(4 - 3)^2 - 1$, or 0. So L.S. = R.S.

17. a) From the graph, one parabola has vertex (0, –1), and x-intercepts –1 and 1. This is the same kind of data that was given in exercise 16b, so the same method can be used here.

$y = a(x - p)^2 + q$

Since the vertex is at (0, –1), then $p = 0$, and $q = -1$.

$y = a(x - 0)^2 + (-1)$

$y = ax^2 - 1$ ①

The point (1, 0) is on the parabola. Substitute in equation ①.

$0 = a - 1$

$a = 1$

$y = x^2 - 1$

The other parabolas are congruent to this one but translated horizontally. Their equations are obtained by replacing x by $x - p$, for the appropriate values of p, which can be read from the graph. Thus the equations of the other parabolas are

$y = (x - 1)^2 - 1$, $y = (x - 2)^2 - 1$, $y = (x - 3)^2 - 1$,
$y = (x - 4)^2 - 1$, and so on, and
$y = (x + 1)^2 - 1$, $y = (x + 2)^2 - 1$, $y = (x + 3)^2 - 1$,
$y = (x + 4)^2 - 1$, and so on

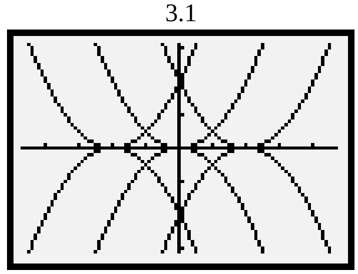
Selected Solutions — Chapter 2

b) From the graph, one parabola has vertex (0, 0), and passes through the points (3, 2) and (–3, 2). Use the same method as in part a.
The equation has the form $y = a(x - p)^2 + q$.
Since the vertex is (0, 0), then $p = 0$ and $q = 0$. Thus, the equation reduces to $y = ax^2$. Substituting the point (3, 2) into this equation,
$2 = 9a$
$a = \frac{2}{9}$
Thus, the equation of this parabola is $y = \frac{2}{9}x^2$.

From the graph, another parabola is the reflection in the x-axis of the parabola whose equation was determined above. Thus, its equation is $y = -\frac{2}{9}x^2$.

A third parabola has vertex (0, –2), and passes through the points (3, 2) and (–3, 2). Using the same method as before, the equation of this parabola has the form $y = a(x - p)^2 + q$. Substitute the coordinates of the vertex to get $y = ax^2 - 2$. Then substitute the point (3, 2) in this equation.
$2 = 9a - 2$
$4 = 9a$
$a = \frac{4}{9}$
Thus, the equation of this parabola is $y = \frac{4}{9}x^2 - 2$. Reflecting this parabola in the x-axis gives the fourth parabola, which has equation $y = -\frac{4}{9}x^2 + 2$.

18. b) Answers may vary.



19. Since the axis of symmetry is the y-axis, the equations have the form $y = ax^2 + q$.

a) The points (2, 9) and (3, 14) are on the parabola. Substitute these points into the equation.
$9 = 4a + q$
Solve for q.
$q = 9 - 4a$
$14 = 9a + q$
Solve for q.
$q = 14 - 9a$
Compare the expressions for a.
$9 - 4a = 14 - 9a$
Solve for a.
$5a = 5$
$a = 1$

Substitute $a = 1$ into $q = 9 - 4a$ to get the value of q.

$q = 9 - 4(1)$
$= 5$

Thus the equation of the parabola is

$y = x^2 + 5$

b) Use the method from part a. The points (–2, 1) and (4, –5) are on the parabola. Substitute these points into the equation $y = ax^2 + q$.

$1 = 4a + q$

Solve for q.

$q = 1 - 4a$

$-5 = 16a + q$

Solve for q.

$q = -5 - 16a$

Compare the expressions for q.

$1 - 4a = -5 - 16a$

Solve for a.

$12a = -6$

$a = -\frac{6}{12}$
$= -\frac{1}{2}$

Substitute $a = -\frac{1}{2}$ into $q = 1 - 4a$.

$q = 1 - 4\left(-\frac{1}{2}\right)$
$= 3$

Thus, the equation of the parabola is

$y = -\frac{1}{2}x^2 + 3$

20. The x-intercepts of the parabola are 0 and 10. Thus, its maximum occurs at $x = 5$. Thus, its vertex is (5, 120).
Substitute $p = 5$ and $q = 120$ in the general equation
$h = a(x - p)^2 + q$.
$h = a(x - p)^2 + 120$
(0, 0) is on the parabola. Substitute $h = 0$ and $x = 0$.
$0 = 25a + 120$
Solve for a.
$a = -4.8$
Then, the equation is $h = -4.8(x - 5)^2 + 120$.

21. Yes, it is always possible to find the equation of the parabola. When the two points have different x-coordinates, we can use the formula $y = a(x - p)^2 + q$, as we did in exercise 19. When the two points have the same x-coordinate, the points lie on the same vertical line, and a possible parabola has its axis of symmetry parallel to the x-axis. The general equation is then $x = a(y - p)^2 + q$ and the particular equation can be determined, using the method of exercise 19.

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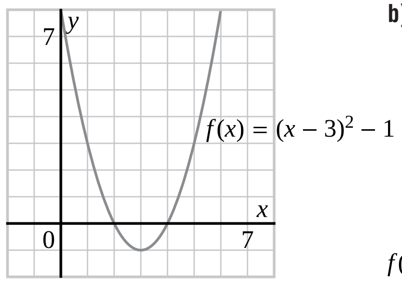
Linking Ideas: Mathematics and Technology
Dynamic Graphs Part I, page 121

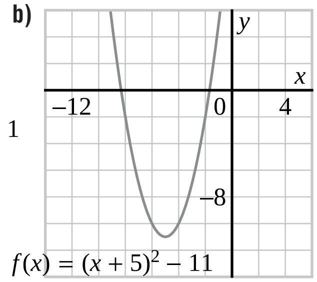
3. The graphs at the tops of both pages would not change, since $b = 0$. The graphs at the bottoms of both pages would be reflected in the y-axis.

2.4 Exercises, page 124

2. a)



b)



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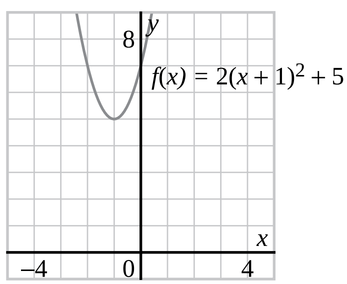

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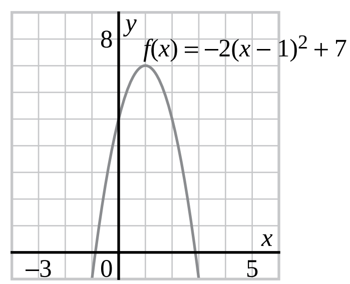

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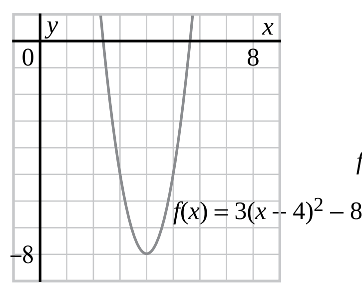

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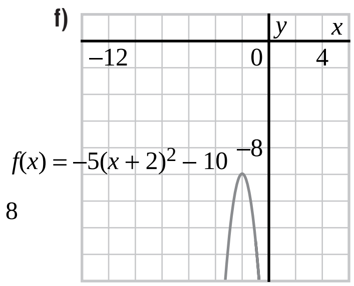
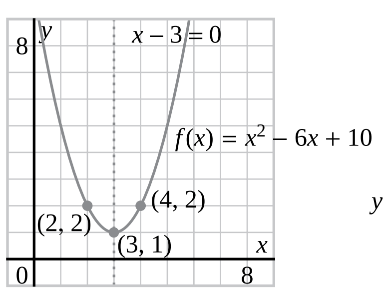


6. They are identical. When you complete the square, the value of a is not affected.

7. a)



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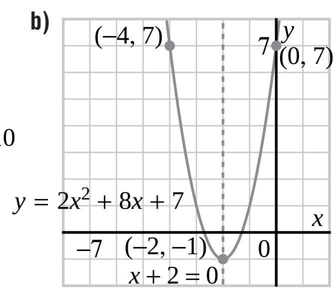

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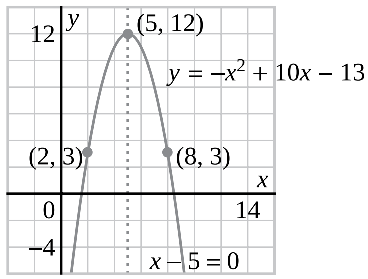
c)



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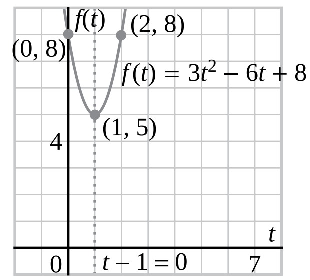

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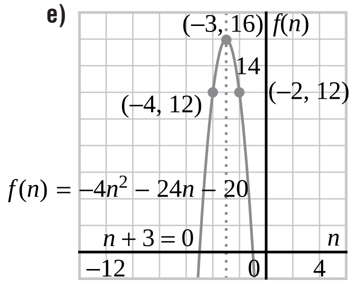

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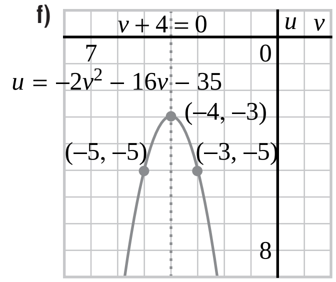
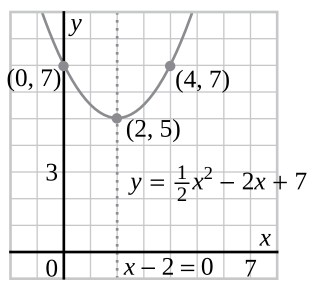


8. a)



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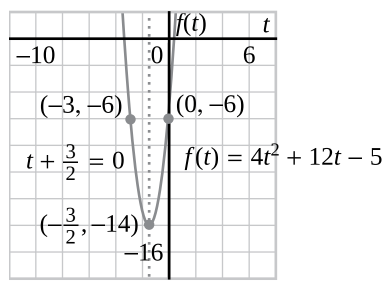

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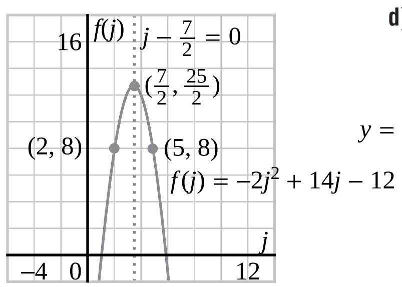

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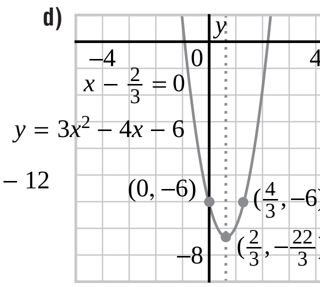

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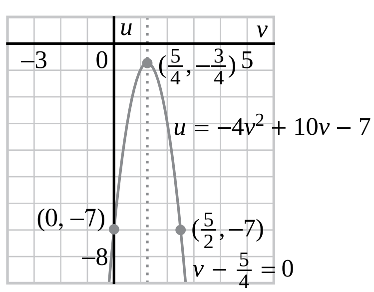

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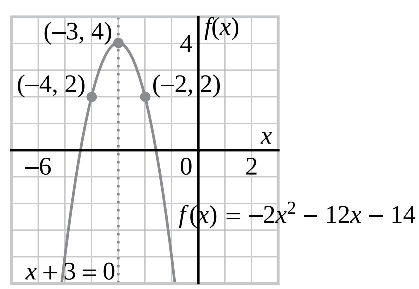


9. Explanations may vary. For exercise 8e:
For $u = -4v^2 + 10v - 7$, I first completed the square.
To do this, I removed –4 as a common factor to get
$u = -4\left(v^2 - \frac{5}{2}v\right) - 7$, then added and subtracted
$\left[\frac{1}{2}\left(-\frac{5}{2}\right)\right]^2$, or $\frac{25}{16}$ to get $u = -4\left(v^2 - \frac{5}{2}v + \frac{25}{16} - \frac{25}{16}\right) - 7$

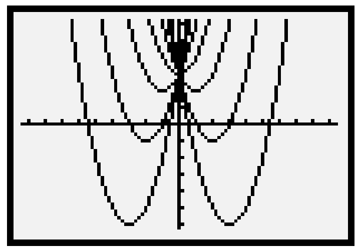
Selected Solutions — Chapter 2

This simplifies to $u = -4\left(v - \frac{5}{4}\right)^2 - \frac{3}{4}$

To graph this equation, I plotted the vertex at $\left(\frac{5}{4}, -\frac{3}{4}\right)$, then used the method of differences to complete the graph.

11. a)

6.2

−9.4  9.4

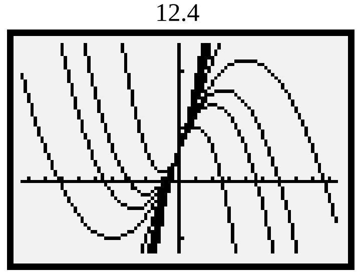
−6.2

b) As b decreases from 6 to 0, the parabola moves up and to the left. As b continues to decrease to negative values, the parabola moves down and to the left.

c) When $b = 0$ the parabola has its vertex on the y-axis.

12. a)

12.4

−9.4  9.4

−6.2

b) As a decreases to 0, the parabola is compressed vertically and translated down and to the left. When $a = 0$, the graph is no longer a parabola but a straight line. All the parabolas with positive values of a lie on the upper left side of the line and open up. For negative values of a, as a decreases, the parabola is expanded vertically and translated down and to the left. All the parabolas with negative values of a lie on the lower right side of the line and open down.

c) When $a = 0$, the graph is a line, not a parabola. This line divides the family of parabolas into two groups; the parabolas with positive values of a lie on one side of the line and the parabolas with negative values of a lie on the other side of the line.

14. To find the x-intercepts, substitute $y = 0$, then solve for x.

$$ax^2 + c = 0$$
$$ax^2 = -c$$
$$x^2 = -\frac{c}{a}$$

For real x-intercepts, $-\frac{c}{a} \geq 0$ and $a \neq 0$.
For $-\frac{c}{a}$ to be ≥ 0, $\frac{c}{a}$ must be ≤ 0
For $\frac{c}{a}$ to be ≤ 0, $c \geq 0$ when $a < 0$ or $c \leq 0$, when $a > 0$.

There are real x-intercepts if $a > 0$ and $c \leq 0$, or $a < 0$ and $c \geq 0$.

Selected Solutions — Chapter 2

15. a) $f(x) = ax^2 + bx + c$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) - a\left(\frac{b}{2a}\right)^2 + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b^2}{4a^2}\right) + c$$

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

b) From the equation in part a, the axis of symmetry is $x = -\frac{b}{2a}$. The coordinates of the vertex are $\left(-\frac{b}{2a}, -\frac{b^2}{4a} + c\right)$. When $x = 0$, the y-intercept is c.

c) For the x-intercepts, $f(x) = 0$. Equate the expression for $f(x)$ from part a to 0, then solve for x.

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\text{or } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The x-intercepts are $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

2.5 Exercises, page 130

4. Let x and y be the two numbers, and let C be their sum.
$x + y = C$ or $y = C - x$
Let P represent their product.
$P = xy$
Substitute for y from above.
$P = x(C - x)$
$\quad = -x^2 + Cx$
Since the coefficient of x^2 is negative, the graph opens down and has a maximum point. Complete the square.

$$P = -x^2 + Cx$$
$$= \left(x^2 - Cx + \left(\frac{C}{2}\right)^2 - \left(\frac{C}{2}\right)^2\right)$$
$$= -\left(x - \frac{C}{2}\right)^2 + \left(\frac{C}{2}\right)^2$$

The maximum value is $\left(\frac{C}{2}\right)^2$, and it occurs when $x = \frac{C}{2}$.
But, $y = C - x$
Substitute for x.

$$y = C - \frac{C}{2}$$
$$= \frac{C}{2}$$

Therefore, $x = y$ when the product is a maximum.

17. Let x represent the number. The amount by which this number exceeds its square is $x - x^2$.
Determine the value of x for which this amount is as large as possible; in other words, determine the value of x for which the function $f(x) = x - x^2$ has its maximum value. Complete the square.

$$f(x) = -(x^2 - x)$$
$$= -\left(x^2 - x + \frac{1}{4} - \frac{1}{4}\right)$$
$$= -\left(x^2 - x + \frac{1}{4}\right) + \frac{1}{4}$$
$$= -\left(x - \frac{1}{2}\right)^2 + \frac{1}{4}$$

Since the parabola opens down, it has a maximum value. The vertex of the parabola is at $\left(\frac{1}{2}, \frac{1}{4}\right)$. Thus, the maximum value of the function occurs when $x = \frac{1}{2}$. This means that the number that exceeds its square by the greatest possible amount is $\frac{1}{2}$.

18. Let x and y be the dimensions of the rectangle. Then the perimeter of the rectangle is $P = 2x + 2y$, and its area is $A = xy$. The rectangle has a given perimeter; therefore, P is a constant.
$P = 2x + 2y$
Solve for y.

$$2y = P - 2x$$
$$y = \frac{P}{2} - x$$

Substitute this expression for y into the formula for A.

$$A = x\left(\frac{P}{2} - x\right)$$
$$= -x^2 + \frac{P}{2}x$$

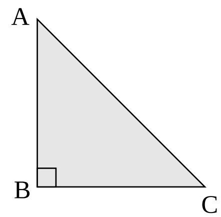
Determine the maximum value of A by completing the square.

$$A = -\left(x^2 - \frac{P}{2}x\right)$$
$$= -\left(x^2 - \frac{P}{2}x + \frac{P^2}{16} - \frac{P^2}{16}\right)$$
$$= -\left(x^2 - \frac{P}{2}x + \frac{P^2}{16}\right) + \frac{P^2}{16}$$
$$= -\left(x^2 - \frac{P}{4}\right)^2 + \frac{P^2}{16}$$

Thus, the maximum area of a rectangle with given perimeter P is $\frac{P^2}{16}$.

Selected Solutions — Chapter 2

19. This problem is solved using the following idea: if a varying positive quantity reaches a maximum value, then its square is also maximum.



Let $x = \text{AB}$, let $y = \text{BC}$, and let $k = \text{AC}$. The area of the triangle (using the formula $A = \frac{1}{2}bh$) is $A = \frac{1}{2}xy$. Use the Pythagorean Theorem: $x^2 + y^2 = k^2$, where k is a constant. Solve this relation for y.

$y = \sqrt{k^2 - x^2}$

Substitute this expression for y into the formula for the area

$A = \frac{1}{2}x\sqrt{k^2 - x^2}$

Square the formula for A to eliminate the square root, and analyze A^2 instead of A, knowing that when A^2 is a maximum, so is A.

$A^2 = \frac{1}{4}x^2(k^2 - x^2)$

Let $X = x^2$.

$A^2 = \frac{1}{4}X(k^2 - X)$

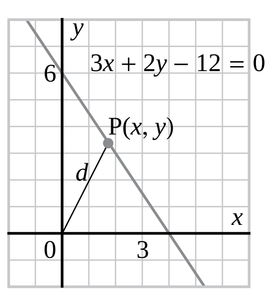
The roots of this quadratic expression are $X = 0$ and $X = k^2$. Thus, the maximum value of A^2 occurs for a value of X halfway between the roots, at $X = \frac{k^2}{2}$. The maximum value of A occurs for the same value of X. Thus, A has a maximum value for $x^2 = X$, or $\frac{k^2}{2}$.

$$\begin{aligned} \text{Since } x^2 + y^2 = k^2,\ y^2 &= k^2 - x^2 \\ &= k^2 - \frac{k^2}{2} \\ &= \frac{k^2}{2} \end{aligned}$$

Thus, A achieves its maximum value when $x^2 = y^2$. Since x and y are both positive, this condition is equivalent to $x = y$. Thus, A is a maximum when $x = y$.

20. Let (x, y) be the coordinates of a point on the line $3x + 2y - 12 = 0$. Then the distance from the origin to a point on the line is

$d = \sqrt{x^2 + y^2}$.



Selected Solutions — Chapter 2

Since distance is a positive quantity, when the distance is a minimum, so is the square of the distance.
Square both sides of the formula for d.

$d^2 = x^2 + y^2$

Every point on the line satisfies the equation $3x + 2y - 12 = 0$. Solve this equation for y.

$$\begin{aligned} 3x + 2y - 12 &= 0 \\ 2y &= -3x + 12 \\ y &= -\frac{3}{2}x + 6 \end{aligned}$$

Substitute this expression into the formula for d^2.

$$\begin{aligned} d^2 &= x^2 + \left(-\frac{3}{2}x + 6\right)^2 \\ &= x^2 + \frac{9}{4}x^2 - 18x + 36 \\ &= \frac{13}{4}x^2 - 18x + 36 \\ &= \frac{13}{4}\left(x^2 - \frac{72}{13}x\right) + 36 \\ &= \frac{13}{4}\left(x^2 - \frac{72}{13}x + \left(\frac{36}{13}\right)^2 - \left(\frac{36}{13}\right)^2\right) + 36 \\ &= \frac{13}{4}\left(x - \frac{36}{13}\right)^2 - \frac{13}{4}\left(\frac{36}{13}\right)^2 + 36 \\ &= \frac{13}{4}\left(x - \frac{36}{13}\right)^2 - \frac{324}{13} + 36 \\ d^2 &= \frac{13}{4}\left(x - \frac{36}{13}\right)^2 + \frac{144}{13} \end{aligned}$$

The minimum value of this function is $\frac{144}{13}$. But this function represents the square of the distance from the origin to the line. Thus, the minimum distance from the origin to the line is

$d = \sqrt{\frac{144}{13}}$, or 3.33.

Problem Solving: Elegance in Mathematics, page 132

1. Answers may vary. The exercises in Section 2.5 that can be solved using the maximum principle are: 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 18. (Note that exercise 4 is just a restatement of the maximum principle.) Here are some examples.

   Exercise 2.
   Since the numbers add up to 12, the maximum product is achieved when they are both equal to 6.

   Exercise 8.
   Let the three sides of the lot have lengths x, x, and y. Then $2x + y = 600$. By the maximum principle, $2x$ and y are the same for the maximum value of the area, so $2x = y = 300$. Thus, $x = 150$ m and $y = 300$ m. The maximum area is therefore $300 \times 150 = 45\ 000$ m^2.

   Exercise 12.
   Let x be the number of riders per day, let y dollars be the fare, and let R dollars be the revenue generated. Thus, $R = xy$.

   Let n represent the number of \$0.05 fare decreases. Find a relation between x and n, and a relation between y and n, then combine these to find a relation between x and y.

$x = 20\ 000 + 2000n$

$y = 0.9 - 0.05n$

Solve these equations for n.

$$n = \frac{x - 20\ 000}{2000}$$

$$n = \frac{0.9 - y}{0.05}$$

Equate the previous two expressions for n.

$$\begin{aligned} \frac{x - 20\ 000}{2000} &= \frac{0.9 - y}{0.05} \\ 0.05(x - 20\ 000) &= 2000(0.9 - y) \\ 0.5x - 1000 &= 1800 - 2000y \\ 0.05x + 2000y &= 2800 \end{aligned}$$

Now apply the maximum principle. The maximum product $R = xy$ is achieved when $0.05x$ and $2000y$ are equal, which means that each quantity is one-half of 2800. Thus, $0.05x = 1400$ and $2000y = 1400$. To determine the fare, solve for y.

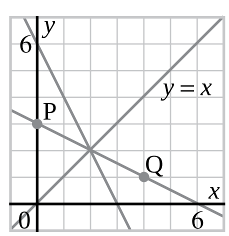
$$\begin{aligned} y &= \frac{1400}{2000} \\ &= 0.7 \end{aligned}$$

Thus a fare of \$0.70 results in maximum revenue.

2. Answers may vary. Elegance in mathematics suggests simplicity, clarity, grace, richness, and beauty. The fact that the maximum principle is based on a simple idea (two quantities are equal) and furthermore that this simple idea is rooted in geometry (among an infinite number of rectangles that satisfy some constraint, the only one of them that is a square has maximum area) lend the maximum principle elegance.

2.6 Exercises, page 136

3. a)



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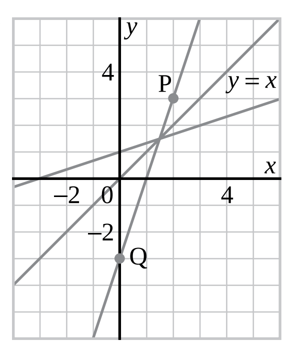

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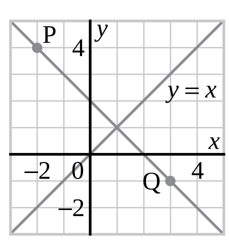

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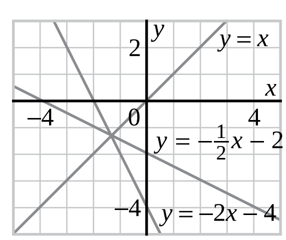
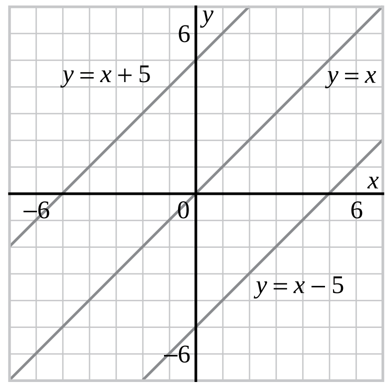


Selected Solutions — Chapter 2

e)



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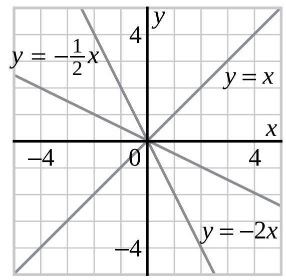


8. To find the function that is the inverse of $y = mx + b$, interchange x and y then solve for y.

$$x = ym + b$$
$$ym = x - b$$
$$y = \frac{x - b}{m}$$

The expression for y is undefined if $m = 0$; otherwise the expression represents the inverse of $y = mx + b$.

9. The inverse of the inverse of a linear function is the same as the original function. Geometrically, reflecting a line twice in the line $y = x$ produces the original line. Algebraically, applying inverse operations, then applying inverse operations again, produces the original function.

Let $y = 2x + 5$

To get the inverse, interchange x and y.

$x = 2y + 5$

Solve for y.

$2y = x - 5$

$y = \frac{x - 5}{2}$

To get the inverse, interchange x and y.

$x = \frac{y - 5}{2}$

Solve for y.

$y = 2x + 5$

This is the original function.

10. Let $y = ax + b$ ①

To get the inverse, interchange x and y.

$x = ay + b$

Solve for y.

$y = \frac{x}{a} - \frac{b}{a}$ ②

For the function and its inverse to be the same, equations ① and ② must be identical.

The coefficients of x must be equal: $a = \frac{1}{a}$

$$a^2 = 1$$
$$a = \pm 1$$

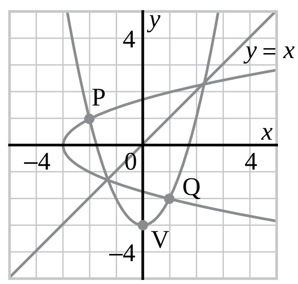
The constant terms must be equal: $b = -\frac{b}{a}$ ③

Substitute each value of a in equation ③.

When $a = 1$, $b = -b$
Hence, $b = 0$
Substitute $a = 1$ and $b = 0$ in equation ①.
$y = x$
When $a = -1$, $b = b$
This is true for all real values of b.
Substitute $a = -1$ in equation ①.
$y = -x + b$
The only linear functions that are their own inverses are $y = x$ and $y = -x + b$.

2.7 Exercises, page 140

2. a)



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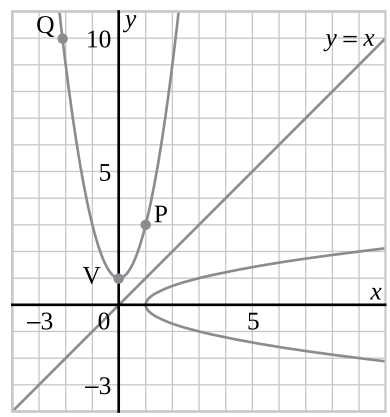

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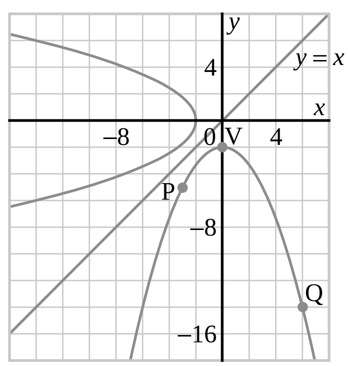

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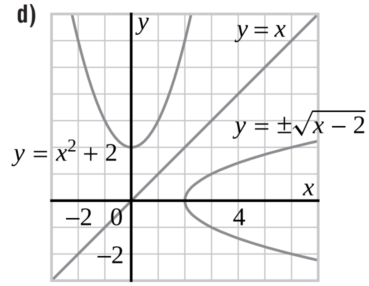

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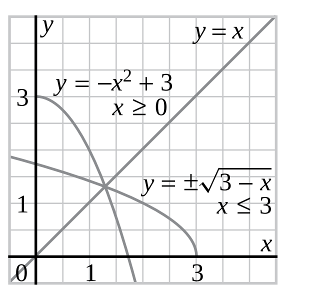

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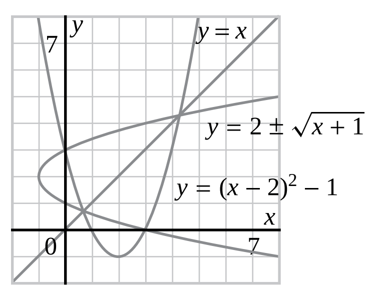
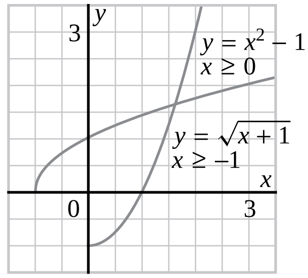


Selected Solutions — Chapter 2

4. a)



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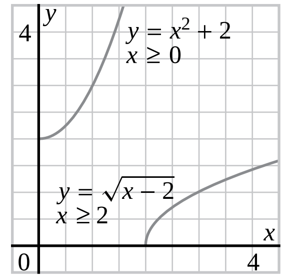

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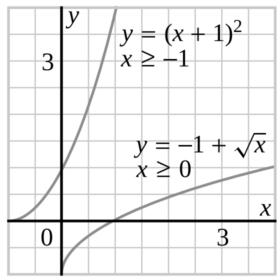

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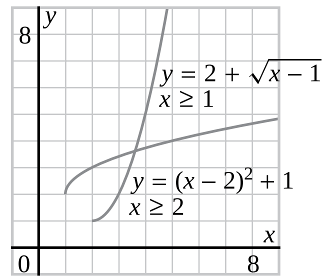

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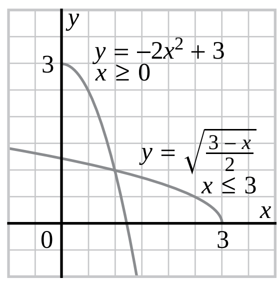

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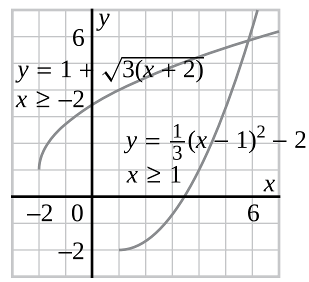


7. a) $y = 1 + \sqrt{x}$

Recognize that this function is only true for $x \geq 0$ and $y \geq 1$.

Interchange x and y to get the inverse.

$x = 1 + \sqrt{y}$

Solve for y.

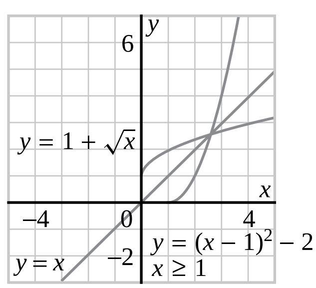
$x - 1 = \sqrt{y}$

Square each side.

$(x - 1)^2 = y$

$y = (x - 1)^2$

Hence, this function is only true for $x \geq 1$ and $y \geq 0$.



b) $y = 3 - 2\sqrt{x}$

This function is only true for $x \geq 0$ and $y \leq 3$.

Interchange x and y.

$x = 3 - 2\sqrt{y}$

Solve for y.

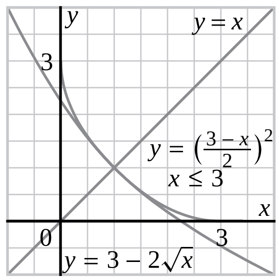
$x - 3 = -2\sqrt{y}$

Square each side.

$(x - 3)^2 = 4y$

$y = \frac{1}{4}(x - 3)^2$

Hence, this function is only true for $x \leq 3$ and $y \geq 0$.



Mathematical Modelling: Modelling Golf Ball Trajectories, page 142

1. a) The maximum height is achieved when half the time of flight has passed. Thus, the maximum height occurs at $t = \frac{11.4}{4.9}$. Substitute this value of t into the $h(t)$ equation.

$$h(t) = -4.9\left(\frac{11.4}{4.9}\right)^2 + 22.8\left(\frac{11.4}{4.9}\right)$$
$$= 26.52 \text{ m}$$

b) When the ball hits the ground, $h = 0$. Thus, set $h = 0$ in equation ① and solve for t:

$$h(t) = -4.9t^2 + 22.8t$$
$$0 = t(-4.9t + 22.8)$$

Thus, $t = 0$ (which is the time when the ball is hit) or $t = \frac{22.8}{4.9}$ or 4.65 s. Thus, the ball hits the ground about 4.65 s after it is hit.

2. $d(t) = 29.2t$

$$= 29.2\left(\frac{22.8}{4.9}\right)$$
$$= 135.87 \text{ m}$$

5. c) Rewrite the equations using x in place of d and y in place of h.

$y = -4.9t^2 + 22.8t$

$x = 29.2t$

Solve the second equation for t and substitute into the first equation.

Selected Solutions — Chapter 2

$$t = \frac{x}{29.2}$$

$$y = -4.9\left(\frac{x}{29.2}\right)^2 + 22.8\left(\frac{x}{29.2}\right)$$

$$y = -\frac{4.9}{29.2^2}x^2 + \frac{22.8}{29.2}x$$

7. **a)** The ball would not go as far or as high.

b) The trajectories would have approximately the same shapes, but they would no longer be parabolas.

8. Answers may vary. The club angle and initial speed might change, and that would affect both formulas 1 and 2.

9. Answers may vary. Different golfers swing their clubs at different speeds, and this would affect the initial velocity. Also, different golfers would put different spins on the ball (for example, hooks and slices), and that may take the balls off-course, or affect how high the balls go.

10. The trajectories would be straight lines, provided there were no other forces acting.

Linking Ideas: Mathematics and Technology
Dynamic Graphs Part II, page 146

1. **a)** *a* is changing because the shape of the graph changes.

b) *b* is changing because the shape of the graph is constant and its position is changing.

4. **b)** The graphs would all be congruent, and would be in different vertical positions.

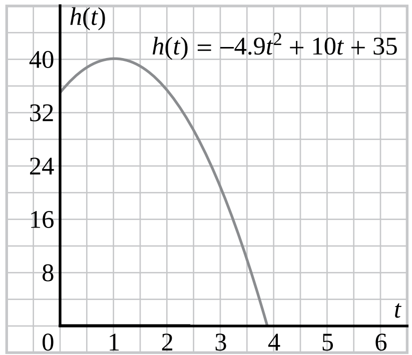
5. Top graphs: in both series, the shapes of the graphs change. However, in the first series the *x*-intercepts and the coordinates of the vertex do not change, whereas in the second series they do. In both series, the *y*-intercepts do not change.

Bottom graphs: In the first series, the *y*-coordinate of the vertex does not change. In the second series, the *y*-intercept does not change. In both series, the shape of the graph does not change.

Selected Solutions — Chapter 2

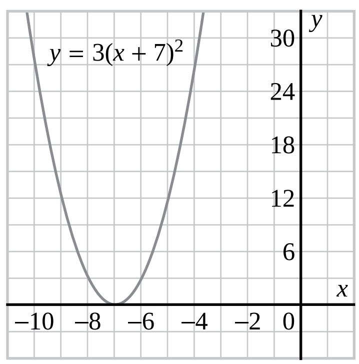
2 Review, page 148

3.



5. a) For values of g larger than 9.8, the vertex will be shifted down and to the left, and the x-intercept will be shifted to the left. For values of g smaller than 9.8, the vertex will be shifted up and to the right, and the x-intercept will be shifted to the right.

6. a)



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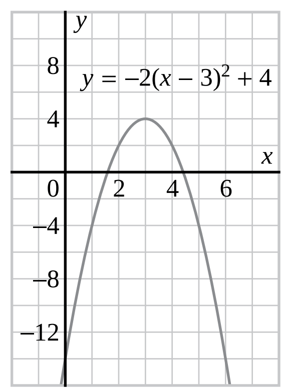
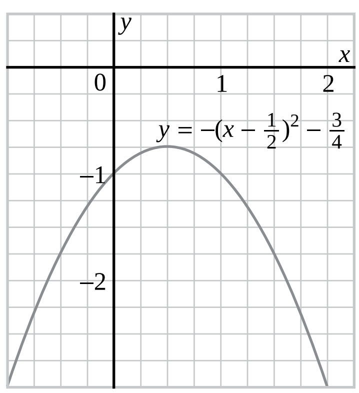
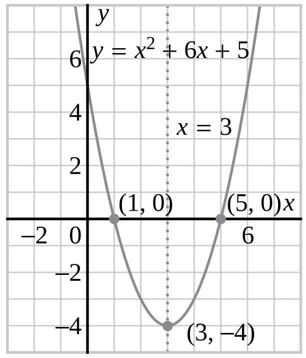


c)



Selected Solutions — Chapter 2

8. a)



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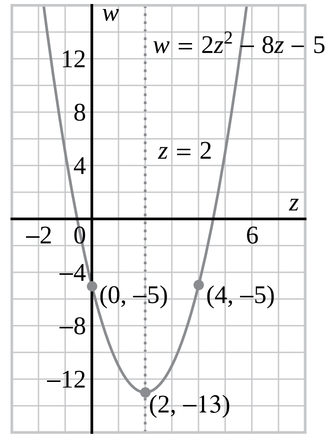

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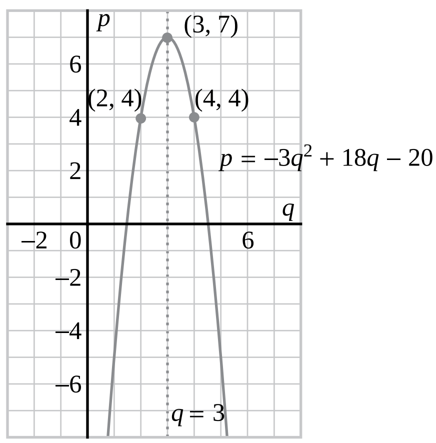
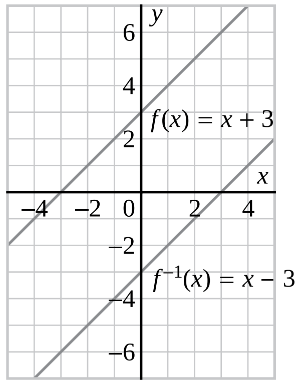


9. Explanations may vary. For part a: I first factored the equation to get $y = (x - 1)(x - 5)$. This tells me the x-intercepts are 1 and 5 (just substitute $y = 0$ into the equation). The axis of symmetry is halfway between the intercepts, so that tells me that $x = 3$ is the axis of symmetry. Substituting $x = 3$ into the formula for y gives me the y-coordinate of the vertex, which is –4; thus, the vertex is at (3, –4). Substituting $x = 0$ into the formula gives me the y-intercept of 5. I notice that the parabola opens up, since the coefficient of x^2 is positive, and all of this information helps me to create the graph. I used the method of differences for a more accurate graph.

12. Explanations may vary. For part c:
I wrote $f(x) = 3(2 - x)$ as $y = 3(2 - x)$.
I interchanged x and y to get $x = 3(2 - y)$, then solved for y to get $y = \frac{6 - x}{3}$. This is the inverse, which can be written $f^{-1}(x) = \frac{6 - x}{3}$.

Selected Solutions — Chapter 2

13. a)



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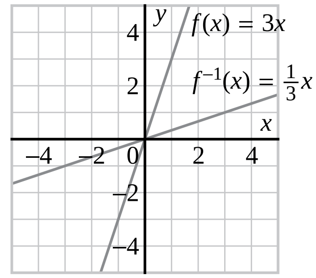

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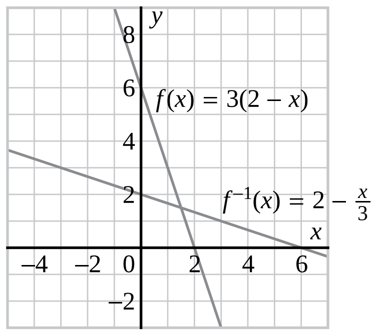


2 Cumulative Review, page 150

4. Explanations may vary. For part a: I used the compound interest formula $A = P(1 + i)^n$.
All the values in the formula are given except P. I substituted all of the given values and then solved for P.

$$A = P(1 + i)^n$$
$$4250 = P(1.07)^4$$
$$P = 4250 \div (1.07)^4$$
$$\doteq 3242.30$$

You would have to invest about \$3242.30.

6. The accumulated amount of \$1 in 1 year at 5% compounded semi-annually is

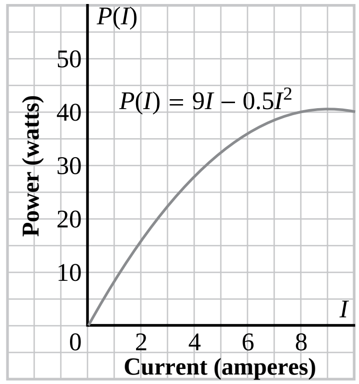
$$1\left(1 + \frac{0.05}{2}\right)^2 = (1.025)^2$$
$$= 1.050\ 625$$

Subtract the principal of \$1. The effective annual interest rate is 5.0625%. This is the same as the given annual compounded rate.

11. Parts a and e are written in the form $y = ax^2 + bx + c$, the other parts are not.

Selected Solutions — Chapter 2

12. a)



15. Explanations may vary. For part a:
I let $f(b) = 0$, and solved the equation by factoring and equating each factor to 0.

$$b^2 + 7b + 12 = 0$$
$$(b + 3)(b + 4) = 0$$
$$b = -3 \text{ or } -4$$

The zeroes of $f(b) = b^2 + 7b + 12$ are –3 and –4.

17. Explanations may vary. For part a: $y = -2(x + 5)^2 - 8$
Since the coefficient of x^2 is negative, the graph opens down and there is a maximum. Since the vertex has coordinates (–5, –8), the maximum value of y is –8.