Selected Solutions — Chapter 1

Investigate, page 4

2. To calculate simple interest, multiply the principal by the time in years and the interest rate expressed as a decimal.

1.1 Exercises, page 9

1. It is better to have a daily interest savings account if your balance fluctuates a lot. If your balance is fairly steady, for example if it never decreases and only increases once per month, then the regular savings account is better.

4. Answers may vary. For part a:
I used the formula $I = Prt$ and substituted $P = 627$, $r = 0.045$, and $t = \frac{27}{365}$. I divided 4.5% by 100 to express the rate as a decimal. (Remember, the interest rate, r, must be written as a decimal, and the time, t, must be expressed in years, not days.)

$$I = 627 \times 0.045 \times \frac{27}{365}$$
$$= 2.09$$

Thus, the simple interest earned is \$2.09.

11. Answers may vary. For exercise 8:
I used the formula, divided both sides by Pt to solve for r, to get $r = \frac{I}{Pt}$.

Then I substituted the values of I, $P = 350$, and $t = \frac{5}{12}$, to get

$$r = \frac{12.76}{350 \times \frac{5}{12}}$$
$$= 0.0875$$

Thus, the interest rate is 8.75%.
To calculate the interest, I subtracted the principal, \$350, from the accumulated amount, \$362.76, and got \$12.76.

13. **b)** Answers may vary. Marcel is not using his account wisely. His unpaid balance is growing, as are his credit charges, since each month Marcel's payments are less than the purchases he made the previous month. Furthermore, on a yearly basis, the interest rate works out to be 30.6%, which is extremely high. (Remember, interest compounds every month on unpaid balances.) Department stores are notorious for the high interest rates they charge; Marcel would be much better off obtaining a credit card that has a lower interest rate (and, of course, minimizing his spending).

c) Predictions may vary. Marcel's total monthly payments for the next 6 months will be 6 × \$300 or \$1800. But his balance of \$1718.46 and credit charges for the next 6 months will exceed \$1800. Therefore, he will not reduce his balance to zero by the end of the year.

Selected Solutions — Chapter 1

d) As the following spreadsheet shows, Marcel would have \$16.72 left to pay.

	A	B	C	D	E	F	G
1	Continuing Marcel's Account Record, Systematic Payment						
2	Month	Prev. bal.	Payment	Unpaid balance	Credit charge	Purchases	New balance
3	July	\$1 718.46	\$300.00	\$1 418.46	\$ 31.92	\$0.00	\$1 450.38
4	August	\$1 450.38	\$300.00	\$1 150.38	\$ 25.88	\$0.00	\$1 176.26
5	September	\$1 176.26	\$300.00	\$876.26	\$ 19.72	\$0.00	\$895.98
6	October	\$895.98	\$300.00	\$595.98	\$ 13.41	\$0.00	\$609.39
7	November	\$609.39	\$300.00	\$309.39	\$ 6.96	\$0.00	\$316.35
8	December	\$316.35	\$300.00	\$16.35	\$ 0.37	\$0.00	\$16.72
9				Total credit charge	\$ 98.26		

14. d) Answers may vary. Whenever possible pay the amount due in full to avoid interest charges.

15. b) Credit card interest is based on the number of days between statements; depending on the length of the month, this could be 28, 29, 30, or 31 days. In solving this problem one assumes that the executive will be charged interest on \$41 000 for 30 days, and then on \$6000 for one month (28, 29, 30, or 31 days). Since we aren't told how many days are in the next month, we can't be sure exactly how much interest will be charged.

16. $A = P(1 + rt)$

$A = 650$, $r = 0.055$, $t = \frac{10}{12} = \frac{5}{6}$

$$P = \frac{A}{1 + rt}$$

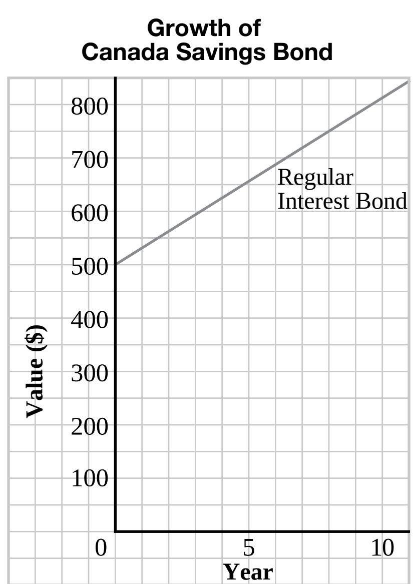
$$= \frac{650}{1 + (0.055)\left(\frac{5}{6}\right)}$$

$$= 621.51$$

Invest \$621.51 now to meet the payment.

Investigate, page 13

2.

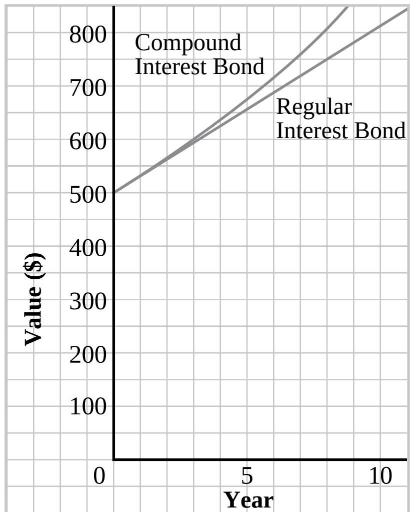


3. The value increases by the same amount every year. It is an arithmetic sequence.

Selected Solutions — Chapter 1

4. The graph would be a straight line with a steeper slope.

6.



7. The graph would curve up faster for a higher interest rate, and slower for a lower interest rate.

8. The value increases geometrically every year. $A = P(1 + i)^n$, where A is the accumulated amount, P is the principal, i is the annual interest rate expressed as a decimal, and n is the time in years.

1.2 Exercises, page 17

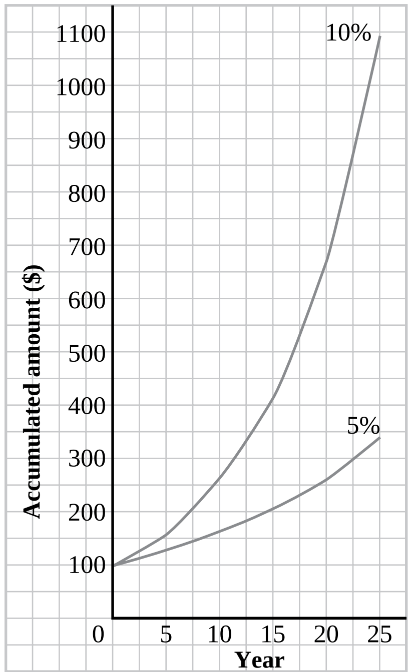
6. Answers may vary. For part b:
I used the formula $A = P(1 + i)^n$. From the advertisement, a three-year debenture has an interest rate of 4.5%, so $i = 0.045$. I substituted this value and the values $P = 2000$ and $n = 3$ into the formula for A to get $A = 2000(1 + 0.045)^3$. This simplifies to 2282.33.
Thus, the maturity value is $2282.33.

8. Answers may vary. For part a:
Since we are given the accumulated amount A and are asked to find the present value P that must be invested today, I solved the formula $A = P(1 + i)^n$ for P dividing both sides of the equation by $(1 + i)^n$.
$P = \frac{A}{(1 + i)^n}$
I then substituted $A = 3980$, $n = 5$, and $i = 0.0575$, to get
$P = \frac{3980}{(1.0575)^5}$, which simplifies to 3009.41.
You need to invest $3009.41 today to have $3980 in 5 years.

Selected Solutions — Chapter 1

10. b)

Accumulated amounts for interest rates of 5% and 10%

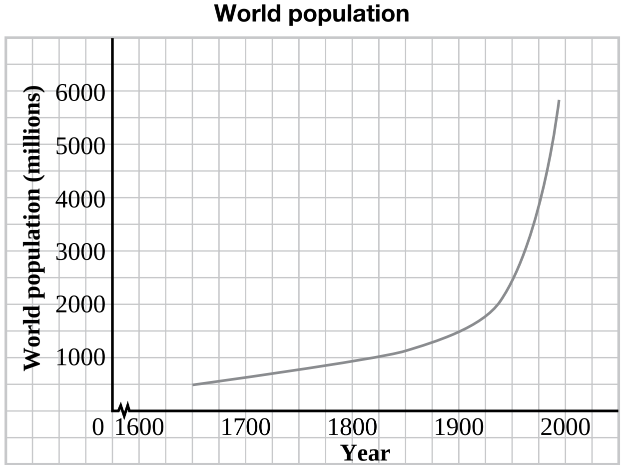


c) You can estimate the values from the graph in part b or interpolate from the tables. The approximate values are **i)** 22.5 years, and **ii)** 11.5 years.

d) Refer to the table in part a. Until about 15 years the investment at 10% has less than double the value of the investment at 5%. At about 24 years the investment at 10% reaches triple the value of the investment at 5%, and the ratio continues to increase as time passes.

15. a)

World population



d) They are similar in that the growth is extremely rapid. The compound interest bond growth represents a geometric sequence; every year the value of the bond increases by the same *factor*. (However, the *amount* of increase each year is greater and greater as time passes.) The difference is that the interest rate for the

Selected Solutions — Chapter 1

compound interest bond is constant, whereas the growth rate for the world's population is actually increasing. This means that the world's population is growing even faster than a geometric series.

21.

	A	B	C	D	E	F	G
1	Growing $100 with Compound Interest (compounded annually)						
2	Years	4%	5%	6%	7%	8%	9%
3	1	$ 104.00	$ 105.00	$ 106.00	$ 107.00	$ 108.00	$ 109.00
4	2	$ 108.16	$ 110.25	$ 112.36	$ 114.49	$ 116.64	$ 118.81
5	3	$ 112.49	$ 115.76	$ 119.10	$ 122.50	$ 125.97	$ 129.50
6	4	$ 116.99	$ 121.55	$ 126.25	$ 131.08	$ 136.05	$ 141.16
7	5	$ 121.67	$ 127.63	$ 133.83	$ 140.26	$ 146.93	$ 153.86
8	6	$ 126.54	$ 134.01	$ 141.86	$ 150.08	$ 158.68	$ 167.71
9	7	$ 131.60	$ 140.71	$ 150.37	$ 160.59	$ 171.37	$ 182.80
10	8	$ 136.86	$ 147.75	$ 159.39	$ 171.83	$ 185.08	$ 199.25
11	9	$ 142.33	$ 155.14	$ 168.95	$ 183.86	$ 199.89	$ 217.18
12	10	$ 148.02	$ 162.90	$ 179.09	$ 196.73	$ 215.88	$ 236.73

22. The bonds earn 6.75% interest from November 1 until the end of February, which is a total of 4 months. For the other 8 months of the year (March 1 until October 31), the interest earned is 7.5%. To avoid writing lots of zeros, lets work in billions of dollars, then convert to dollars at the end. Thus, the principal amount P is 6, in billions of dollars.

Thus, the interest earned for the first 4 months, in billions of dollars, is:

$$\begin{aligned} I &= Prt \\ &= 6 \times 0.0675 \times \frac{4}{12} \\ &= 0.135 \end{aligned}$$

The interest earned for the last 8 months, in billions of dollars, is:

$$\begin{aligned} I &= Prt \\ &= 6 \times 0.075 \times \frac{8}{12} \\ &= 0.3 \end{aligned}$$

The total interest earned for the whole year, in billions of dollars, is:
$0.135 + 0.3 = 0.435$

If the interest rate had remained at 6.75% for the whole year, then the total interest earned for the year would have been:

$$\begin{aligned} I &= Prt \\ &= 6 \times 0.0675 \times 1 \\ &= 0.405 \end{aligned}$$

Thus, the additional interest that the government has to pay, in billions of dollars, is:

$0.435 - 0.405 = 0.03$

To convert this amount to dollars, multiply by a billion (1 000 000 000) to get:

$0.03 \times (1\ 000\ 000\ 000) = 30\ 000\ 000$

Thus, the government has to pay an additional $30 000 000.

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Investigate, page 21

1. a) 2% per annum means 2% per year. In this case, the interest is compounded every six months, so on April 30, 1% of the current balance is credited as interest, and again on October 31, 1% of the current balance is credited as interest.

2. To find the new amount A of an investment P after interest is added, multiply by $(1 + i)$, where i is the interest rate:
$A = P(1 + i)$

In this case, the interest rate is 2% and it is added twice, so we use $i = 0.01$ each time. Thus, the first time we add interest, we multiply by (1.01):

$A = 1000(1.01)$

The second time we add interest, we again multiply by (1.01). Thus, after we add interest the second time, the amount is:

$$\begin{aligned} A &= 1000(1.01) \times (1.01) \\ &= 1000(1.01)^2 \end{aligned}$$

3. As in exercise 2, every time interest is added, we multiply by $(1 + i)$. Since interest is compounded monthly, we add interest 12 times per year, so for i we use $\frac{0.02}{12}$. Thus, by the end of a year, the balance is:

$$\begin{aligned} A &= 1000 \times (1 + i) \times (1 + i) \times \cdots \times (1 + i) \quad \text{(12 factors of } (1 + i)) \\ &= 1000 \times (1 + i)^{12} \\ &= 1000 \times \left(1 + \frac{0.02}{12}\right)^{12} \end{aligned}$$

4. The balance at the end of a year with monthly compounding is \$1020.18; with semi-annual compounding the balance is \$1020.10. In general, when there is a single initial deposit and there are no withdrawals, the more frequent the compounding, the higher the final balance. This is true because with compound interest, you get interest on your interest, not just on the initial principal amount. For example, with semi-annual compounding, you get your first interest payment after six months, and you don't get a chance to earn any more interest on this interest until another 6 months have passed. However, with monthly compounding, you get your first interest payment after a month, and at the end of the second month, you get interest on the principal and also interest on the first interest payment. Then at the end of the third month, you get another interest payment, which is made up of interest on the principal and interest on the first two interest payments.

In this case, the difference between monthly compounding and semi-annual compounding is very small — just \$0.08. However, as time passes, the difference can grow dramatically, especially for higher interest rates.

Selected Solutions — Chapter 1

1.3 Exercises, page 24

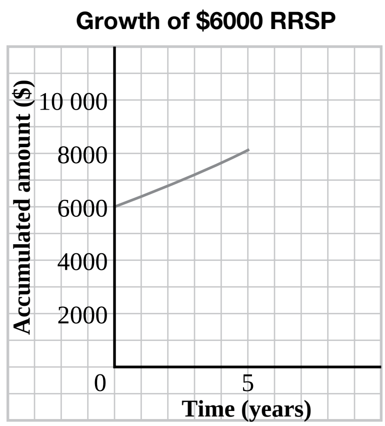
5. Answers may vary. For exercise 4a:

I used the formula $A = P(1 + i)^n$. Since the 5% interest is compounded monthly, I used $\frac{0.05}{12}$ for i. Since the investment is for 3 years, and interest is compounded monthly, there are $3 \times 12 = 36$ compounding periods, so I used 36 for n. The principal is \$250, so I used 250 for P. I substituted these values to get

$A = 250\left(1 + \frac{0.05}{12}\right)^{36}$, which simplified to 290.37.

Thus, the value of the investment after 3 years is \$290.37.

9. a)



b) The RRSP grows at 6.25% compounded annually.

12. a)

	A	B	C	D	E
1	Comparing the Effect of Changing Compounding Periods				
2	Investment:	1000	Interest rate:	0.0475	
3					
4	Time (years)	Annually	Semi-annually	Monthly	Daily
5	0	1000.00	1000.00	1000.00	1000.00
6	1	1047.50	1048.06	1048.55	1048.64
7	2	1097.26	1098.44	1099.45	1099.65
8	3	1149.38	1151.23	1152.83	1153.14
9	4	1203.97	1206.57	1208.80	1209.23
10	5	1261.16	1264.56	1267.48	1268.06

b) cell B6: =$B$5*(1+$D$2)^A6

$B$5 is the principal; $D$2 is the annual interest rate; the dollar signs mean the cell is fixed; A6 is the number of years. This formula calculates the value of the investment for each year when the interest is compounded annually.

cell C6: =$C$5*(1+$D$2/2)^(A6*2)

$C$5 is the principal; divide the interest rate $D$2 by the number of compounding periods in a year (2); multiply the number of years A6 by the number of compounding periods in a year (2). This formula calculates the value of the investment for each year when the interest is compounded semi-annually.

cell D6: =$D$5*(1+$D$2/12)^(A6*12)

$D$5 is the principal; divide the interest rate $D$2 by the number of compounding periods in a year (12); multiply the number of years A6 by the number of compounding periods in a year (12). This formula calculates the value of the investment for each year when the interest is compounded monthly.

cell E6: =$E$5*(1+$D$2/365)^(A6*365)

$E$5 is the principal; divide the interest rate $D$2 by the number of compounding periods in a year (365); multiply the number of years A6 by the number of compounding periods in a year (365). This formula calculates the value of the investment for each year when the interest is compounded daily.

d) Answers may vary.

14. Change "Investment" to "Yield." Change the formulas to

cell B6: =$B$5/[(1+$D$2)^A6]

cell C6: =$C$5/[(1+$D$2/2)^(A6*2)]

cell D6: =$D$5/[(1+$D$2/12)^(A6*12)]

cell E6: =$E$5/[(1+$D$2/365)^(A6*365)]

You will notice that the bracket factors [...] are the same as the ones in exercise 12, only here we are dividing by them instead of multiplying by them. So, in cells B5 – E5 you put the future value of the investment. Then the formulas give you the present values of the investments for the various compounding frequencies. A6 is the number of years in the future you have to wait until you reach the specified future value.

16. Answers may vary. For part a:

I used the formula $A = P(1 + i)^n$, divided both sides by $(1 + i)^n$ to solve for P, to get $P = \frac{A}{(1 + i)^n}$. Then I substituted $A = 800$, $i = \frac{0.035}{12}$, and $n = 4 \times 12 = 48$ to get $P = \frac{800}{\left(1 + \frac{0.035}{12}\right)^{48}}$, which simplifies to 695.63.

Thus, the present value is $695.63.

19. I would choose the second schedule. It yields $1304.17 after 1 year. The first schedule yields only $1256.71 after 1 year, which is $47.46 less than the first schedule.

Modelling Saving Situations

Answers may vary.

For exercise 20, the doubling times may change. If the rates increase, the doubling times decrease, and vice versa.
In this case, if rates go down, there would be no change. But if rates go up, the doubling time decreases.

21. Answers may vary. For part a:

For the money to double, the accumulated amount must equal twice the principal. I substituted $A = 2P$ in the formula $A = P(1 + i)^n$, to get $2P = P(1 + i)^n$, or $2 = (1 + i)^n$. Then I substituted $i = \frac{0.07}{2}$, or 0.035.

Selected Solutions — Chapter 1

I used a graphing calculator or a spreadsheet to determine a value for n for which $2 = (1.035)^n$.

n is about 20. This represents 20 six-month periods, so it takes about 10 years for money to double in value.

22. **c)** Answers may vary. There is not enough difference between the yields, even for large amounts of time. Also, the calculations would be a nightmare: records would have to be kept not only of the day of transactions, but of their hour or minute as well.

23. Let x dollars represent the equal deposits. Let A_1 represent the accumulated amount of the first deposit after 6 months.

$$\begin{aligned} A_1 &= x\left(1 + \tfrac{0.075}{2}\right) \\ &= 1.0375x \end{aligned}$$

Another deposit of x dollars is then made. Let A_2 represent the sum of A_1 and the second equal deposit.

$$\begin{aligned} A_2 &= 1.0375x + x \\ &= 2.0375x \end{aligned}$$

After another 6 months, the yield is $1000.

$$\begin{aligned} 1000 &= A_2(1.0375) \\ &= 2.0375x(1.0375) \\ x &= \frac{1000}{(2.0375)(1.0375)} \\ &= 473.06 \end{aligned}$$

The deposits are equal to $473.06 each.

Now check the result by starting with a principal of $473.06 and see if at the end you really do accumulate $1000.

24. Let x dollars represent the down payment. The merchant owes the supplier $(3000 - x)$ dollars. Let A_1 represent the amount the merchant owes the supplier after 6 months.

$$\begin{aligned} A_1 &= (3000 - x)\left(1 + \tfrac{0.095}{2}\right) - 1000 \\ &= (3000 - x)(1.0475) - 1000 \\ &= 2142.5 - 1.0475x \end{aligned}$$

After another 6 months, she pays off the last $1000.

$$\begin{aligned} A_1(1.0475) - 1000 &= 0 \\ (2142.5 - 1.0475x)(1.0475) - 1000 &= 0 \\ 2142.5 - 1.0475x &= \frac{1000}{1.0475} \\ 1.0475x &= 2142.5 - \frac{1000}{1.0475} \\ x &= \frac{2142.5 - \frac{1000}{1.0475}}{1.0475} \\ &= 1133.98 \end{aligned}$$

The down payment is $1133.98. Once again, check the result.

Selected Solutions — Chapter 1

Problem Solving: Investigating Financial Calculations on the TI-83, page 28

1. Using a financial calculation package, one finds that the amount that must be invested today is $6190.43. Alternatively, one can use the formula $A = P(1 + i)^n$.

We wish to calculate P, the amount that must be invested now, so we solve this formula for P by dividing both sides of the equation by the bracket factor:

$$P = \frac{A}{(1 + i)^n}$$

Since the interest rate is 6.5%, compounded semi-annually, $i = \frac{0.065}{2}$ or 0.0325, and therefore, $1 + i = 1.0325$. Now, the accumulated amount is $7500, and the time is 3 years, so $n = 6$. Substituting these values into the formula for P gives:

$$P = \frac{7500}{(1.0325)^6}$$
$$= 6190.43$$

Now check the result by starting out with this principal and making sure that the calculated accumulated amount after 3 years really is $7500.

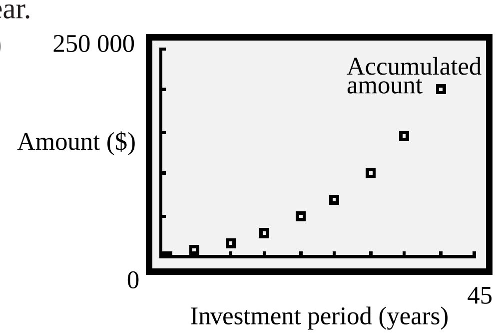
2. a) By using trial and error with a spreadsheet, (or by using a financial calculation package on a calculator or computer) you can determine that the interest rate is about 6.71%.

b) Answers may vary. How realistic this scenario is depends on what the current interest rates are. Sometimes car dealers offer extremely low interest rates when they are very eager to sell cars. At other times interest rates are much higher than 6.71%. In the latter case, Marge would either have to budget for higher monthly payments or try to negotiate for a longer term loan.

3. Using a financial calculation package on a calculator, or using a spreadsheet, you can determine that the time needed is 14 years. (You can check that after 13.5 years the accumulated value is $980.12, but after 14 years the value is $1013.20.)

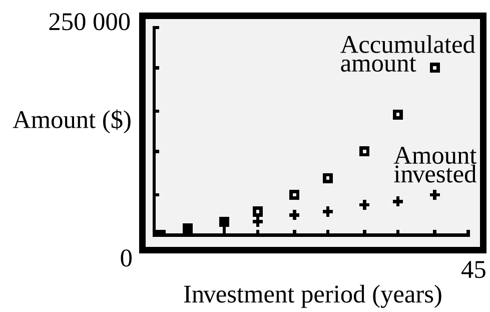
4. The following tables have been constructed assuming that the payments are deposited into the annuity at the beginning of each month. The accumulated amounts are calculated at the ends of each year.

b)



Selected Solutions — Chapter 1

d)



e) In the early years of the investment, the accumulated amount is not much more than the amount invested. However, in the later years of the investment the accumulated amount is many times more than the amount invested, and furthermore, it increases more rapidly with every year that passes. All this you can see by examining the graph in part d. For example, between years 5 and 10, the increase is about $9400, but between years 35 and 40 the increase is nearly $57 000. Thus, the earlier you start an investment, the longer it will last before you die, and therefore the more you will enjoy those later years of rapid growth.

5. Use a calculator's financial calculation package or a spreadsheet and trial-and-error to solve this one. The data are $A = 50\ 000$, $i = \frac{0.045}{12}$, and $n = 36$. The result is that $1294.99 must be deposited monthly. You can check this result by using a spreadsheet.

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2. d) The interest after one year is $72 290.08 for monthly compounding. This is $1065.08 more than for semi-annual compounding, but still $209.92 less than annual compounding at 7.25%. The differences become greater if the investments are followed for a longer time.

3. Answers may vary. There are many factors that influence the interest rates that banks charge for various investments. Some of these are: length of the investment, guesses about how the prime interest rate will change in the future, how eager the bank is to attract investors, and competition with other banks. To help customers decide which investments to choose, they should carefully outline their own needs and desires, compare the offerings of various banks and investment companies, and make their best guess about how interest rates will change in the future.

Modelling Department Store Interest Rates, page 32

- Yes. One assumes that the account is inactive just so that the effective interest rate can be calculated in the simplest way possible. In this way, we have a common standard by which we can compare various accounts.

Selected Solutions — Chapter 1

- Answers may vary. Department store interest rates are comparable to those of credit card companies. High effective interest rates encourage customers to pay off their balances.

Investigate, page 32

2. a)

	A	B	C	D	E	F
1	**Computer loan**					
2		Interest rate:		0.1		
3		Principal:		2007		
4		Monthly payment:		98		
5		Amount			Less	Revised
6	Month	owing	Interest	Payment	interest	amount owing
7	1	2007.00	16.73	98.00	81.28	1925.73
8	2	1925.73	16.05	98.00	81.95	1843.77
9	3	1843.77	15.36	98.00	82.64	1761.14
10	4	1761.14	14.68	98.00	83.32	1677.81
11	5	1677.81	13.98	98.00	84.02	1593.80
12	6	1593.80	13.28	98.00	84.72	1509.08
13	7	1509.08	12.58	98.00	85.42	1423.65
14	8	1423.65	11.86	98.00	86.14	1337.52
15	9	1337.52	11.15	98.00	86.85	1250.66
16	10	1250.66	10.42	98.00	87.58	1163.08
17	11	1163.08	9.69	98.00	88.31	1074.78
18	12	1074.78	8.96	98.00	89.04	985.73
19	13	985.73	8.21	98.00	89.79	895.95
20	14	895.95	7.47	98.00	90.53	805.41
21	15	805.41	6.71	98.00	91.29	714.13
22	16	714.13	5.95	98.00	92.05	622.08
23	17	622.08	5.18	98.00	92.82	529.26
24	18	529.26	4.41	98.00	93.59	435.67
25	19	435.67	3.63	98.00	94.37	341.30
26	20	341.30	2.84	98.00	95.16	246.15
27	21	246.15	2.05	98.00	95.95	150.20
28	22	150.20	1.25	98.00	96.75	53.45
29	23	53.45	0.45	98.00	97.55	-44.11
30	24	-44.11	-0.37	98.00	98.37	-142.47

b) cell B7: =$D$3 is the principal, which is the original amount owing

cell C7: =B7*$D$2*(1/12)

B7 is the amount owing, $D$2 is the interest rate, which is divided by 12, the number of compounding periods: this gives the interest charge for the current month.

cell D7: =$D$4 is the monthly payment.

cell E7: =D7–C7

The part of the monthly payment that goes to paying off the principal is the monthly payment minus the interest charge.

cell F7: =B7–E7

This is the amount owing after the monthly payment.

Selected Solutions — Chapter 1

e) \$214.76, which is \$130.24 less than the finance charge in exercise 1

4. Take the bank loan and save \$130.24 in finance charges.

1.4 Exercises, page 34

1. a) The accumulated amount of \$1 in 1 year at 10% compounded semi-annually is:

$$1\left(1+\frac{0.10}{2}\right)^2 = (1.05)^2$$
$$= 1.1025$$

Subtract the principal of \$1. The effective annual interest rate is 10.25%.

b) The accumulated amount of \$1 in 1 year at 13% compounded semi-annually is:

$$1\left(1+\frac{0.13}{2}\right)^2 = (1.065)^2$$
$$= 1.134\ 225$$

Subtract the principal of \$1. The effective annual interest rate is 13.4225%.

c) The accumulated amount of \$1 in 1 year at 12% compounded semi-annually is:

$$1\left(1+\frac{0.12}{2}\right)^2 = (1.06)^2$$
$$= 1.1236$$

Subtract the principal of \$1. The effective annual interest rate is 12.36%.

d) The accumulated amount of \$1 in 1 year at 9% compounded semi-annually is:

$$1\left(1+\frac{0.9}{2}\right)^2 = (1.045)^2$$
$$= 1.092\ 025$$

Subtract the principal of \$1. The effective annual interest rate is 9.2025%.

5. Answers may vary. For part a:
For both parts of this exercise I used the formula $A = P(1+i)^n$. I calculated A for each part then compared the two values.

i) Since the interest rate is 12.5% and it is compounded semi-annually, $i = \frac{0.125}{2} = 0.0625$. The investment is for 5 years, so $n = 2 \times 5 = 10$. Finally, $P = 2500$. I substituted these numbers into the formula $A = P(1+i)^n$ to get $A = 2500(1.0625)^{10}$, which simplifies to 4583.84.

ii) Since the interest rate is 11.75% and it is compounded monthly, $i = \frac{0.1175}{12}$. The investment is for 5 years, so $n = 12 \times 5 = 60$. Finally, $P = 2500$. I substituted these numbers into the formula $A = P(1+i)^n$ to get $A = 2500\left(1+\frac{0.1175}{12}\right)^{60}$, which simplifies to 4485.87.

Thus, the investment in part i is better by \$97.97.

Selected Solutions — Chapter 1

7. Answers may vary. For part a:
I calculated the accumulated amount of $1 at 8% compounded semi-annually:
$1\left(1+\frac{0.8}{2}\right)^2$, which simplifies to 1.0816.
I subtracted the principal of $1 to get the interest $0.0816. This represents an effective annual interest rate of $0.0816 \times 100\% = 8.16\%$.

8. Part a yields $1862.12 in one year, and part b yields $1918.90 in one year. The plan in part b is better.

10. **b)** The last payment is $187.75 to bring the balance to 0.

c) The finance charge is $752 in exercise 9 and $530.15 in exercise 10. The interest rate is 17.1% in exercise 9 and 12.25% in exercise 10. Both the interest rate and the finance charge are lower in exercise 10.

11. **c)** Answers may vary. If Krista can only afford small monthly payments, the second option is better. If she wants to save money overall, the first option is better.

12. **c)** Answers may vary. If Ashley can only afford small monthly payments, the last option is better. If she wants to save money overall, the first option is better.

13. **a)** The accumulated amount of $1 in 6 months at 12% compounded monthly is:

$$\begin{aligned}1\left(1+\frac{0.12}{12}\right)^6 &= (1.01)^6\\ &= 1.061\ 52\end{aligned}$$

The effective semi-annual interest rate is 6.152%.

b) The accumulated amount of $1 in 6 months at 12% compounded annually is:

$$\begin{aligned}1(1+0.12)^{\frac{6}{12}} &= (1.12)^{\frac{1}{2}}\\ &= 1.0583\end{aligned}$$

The effective semi-annual interest rate is 5.83%.

14. The accumulated amount of $1 in one year at 0.050 94% is:

$$1\left(1+\frac{0.000\ 509\ 4\times 365}{12}\right)^{12} = 1.202\ 62$$

The effective annual interest rate is 20.262%, not 18.6%. (The bank has just multiplied 0.050 94 by 365 to get 18.6.)

15. **a)** The accumulated amount of $1 in one year at i compounded semi-annually, assuming i is the rate expressed as a decimal, is:

$$1\left(1+\frac{i}{2}\right)^2$$

Subtract the principal of $1 to get the effective annual rate, r:

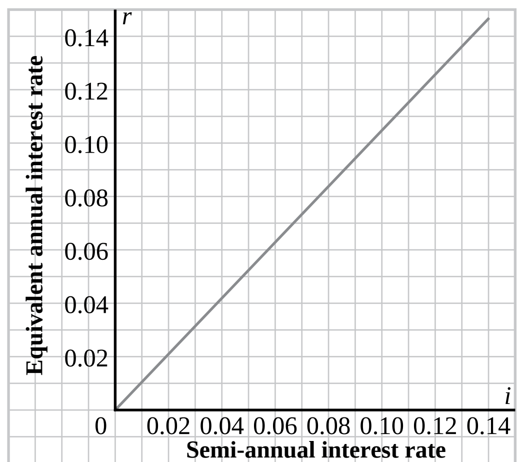
$$r = \left(1+\frac{i}{2}\right)^2 - 1$$

Selected Solutions — Chapter 1

b) The graph can be plotted after making a table of values.

i	*r*
0	0
0.02	0.0201
0.04	0.0404
0.06	0.0609
0.08	0.0816
0.10	0.1025
0.12	0.1236
0.14	0.1449

Semi-annual interest rate
and its equivalent annual interest rate



Investigate, page 37

1. The wage per hour increases after 40 h due to overtime.
2. The slopes are 6 and 9. They represent the rate of pay in dollars per hour.
3. She earned \$6/h for the first 40 h, and \$9/h for the remaining 30 h.

1.5 Exercises, page 40

1. The employee will work harder to earn a greater salary. Good salespeople will earn more money.
2. Bi-weekly is 26 times a year, semi-monthly is 24 times a year.
3. If the first pay of a year is on January 1, there will be 53 paydays. It is not possible for there to be fewer than 52 paydays.
4. Her commission is $5\% \times \$26\ 324 = \$1316.20$, so her total salary is $\$900 + \$1316.20 = \$2216.20$.

Selected Solutions — Chapter 1

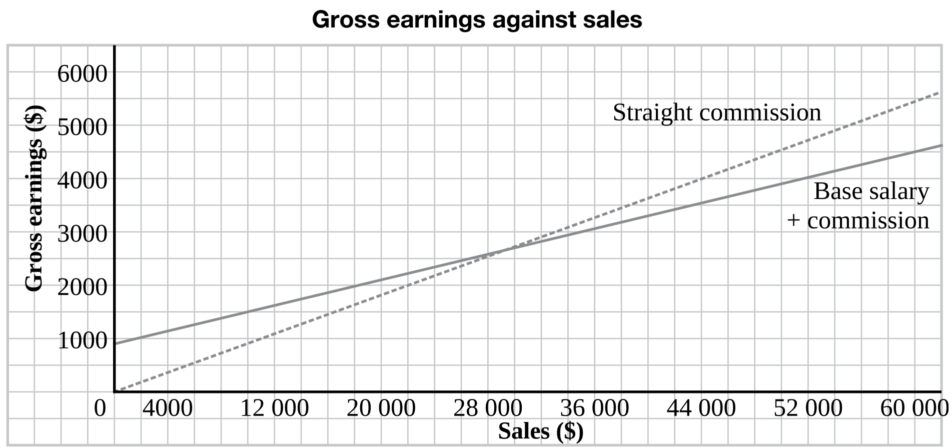
5. Making 4 flower arrangements per hour at $2.50 per arrangement, she could make $10 per hour. The flat rate of $12 per hour is higher.

7. Answers may vary. For part a:
I multiplied the number of hours worked by the hourly rate:
$27 \times$ $7.50, then I added $134 tips to get $336.50.

9. For an 8-hour day, the gross pay is the same. For less than 8 hours, the Fish House pays more. For more than 8 hours, the Tea House pays more. Eric might also consider which job would be more enjoyable or a better learning experience.

10. Answers may vary. For exercise 9:
For each restaurant, I assumed Eric would work 8 h/day. I multiplied the number of hours worked by the hourly rate, then added the tips.
At the Fish House: $8 \times 6.50 + 34 = 86$
At the Tea House: $8 \times 7.25 + 28 = 86$

11. a)



b) If Jay can make sales of over $30 000, he should take the straight commission. If he cannot make sales of over $30 000, he should take the salary plus commission.

c) Jay needs to know how much he can sell. Perhaps he can find out how much the other salespeople sell.

13. Let s represent his base salary, and c his rate of commission expressed as a decimal.
$s + 11\,500c = 1440$
$s + 17\,900c = 1824$
From the first equation, $s = 1440 - 11\,500c$. Substitute for s in the second equation.
$1440 - 11\,500c + 17\,900c = 1824$
$6400c = 384$
$c = 0.06$
Substitute this into the equation for s.
$s = 1440 - 11\,500(0.06)$
$= 750$
Raud's base salary is $750, and his rate of commission is 6%.

Selected Solutions — Chapter 1

Mathematical Modelling: Should You Buy or Lease a Car?, page 42

For B.C.:

1. a)

Basic Cost:	$ 18 253.00
7% GST:	$ 1 277.71
7% PST:	$ 1 277.71
Total:	$ 20 808.44

b) Since payments are made monthly for 3 years, there are 36 payments in total. Thus, the total payments made are $\$8800 + 36 \times \$365.81$ or $21 969.16.

c) The finance charge is $\$21\,969.16 - \$20\,808.44$ or $1160.72.

d) The cost per kilometre is $21 969.16/60 000 km or $0.366/km.

2. a)

Each lease payment:	$ 93.55
7% GST:	$ 6.55
7% PST:	$ 6.55
Total:	$106.65

The total of the monthly payments is $36 \times \$106.65$ or $3839.40.
The total lease payment is $\$8800 + \$3839.40$ or $12 639.40.

b)

You pay:	$ 8531.00
7% GST:	$ 597.17
7% PST:	$ 597.17
Total:	$ 9725.34

c) The total payment is:

$$\begin{aligned}\text{lease payment + buyout cost} &= \$12\,639.40 + \$9725.34 \\ &= \$22\,364.74\end{aligned}$$

d) The cost per kilometre is $22 364.74/60 000 km or $0.373/km.

3. a) For this situation, buying is cheaper in the long run, the difference in cost being $\$22\,364.74 - \$21\,969.16$ or $395.58.

b) Answers may vary. The decision to buy or lease depends on many factors, such as whether or not you own a business (in which case you would have to think about how buying and leasing would affect your income tax payable), whether you can afford the monthly payments, and so on.

4. a) The number of excess kilometres is $75\,230 - 60\,000 = 15\,230$. The charge for the excess kilometres is $\$0.09 \times 15\,230$ or $1370.70.

b) The total amount to lease the car is $\$22\,364.74 + \$1370.70$ or $23 735.44.

c) The dealer may want to resell the car if you decide not to buy it. More kilometres lower the value of the car.

For Alberta:

1. a)

Basic Cost:	$ 18 253.00
7% GST:	$ 1 277.71
Total:	$ 19 530.71

Selected Solutions — Chapter 1

b) Since payments are made monthly for 3 years, there are 36 payments in total. Thus, the total payments made are $\$8800 + 36 \times \$343.35$ or \$21 160.60.

c) The finance charge is $\$21\,160.60 - \$19\,530.71$ or \$1629.89.

d) The cost per kilometre is \$21 160.60/60 000 km or \$0.35/km.

2. a)

Each lease payment:	\$ 93.55
7% GST:	\$ 6.55
Total:	\$100.10

The total of the monthly payments is $36 \times \$100.10$ or \$3603.60.
The total lease payment is $\$8800 + \$3603.60$ or \$12 403.60.

b)

You pay:	\$ 8531.00
7% GST:	\$ 597.17
Total:	\$ 9128.17

c) The total payment is:

$$\begin{aligned}\text{lease payment + buy out cost} &= \$12\,403.60 + \$9128.17\\ &= \$21\,531.77\end{aligned}$$

d) The cost per kilometre is \$21 531.77/60 000 km or \$0.36/km

3. a) For this situation, buying is cheaper in the long run, the difference in cost being $\$21\,531.77 - \$21\,160.60$ or \$371.17.

b) Answers may vary. The decision to buy or lease depends on many factors, such as whether or not you own a business (in which case you would have to think about how buying and leasing would affect your income tax payable), whether you can afford the monthly payments, and so on.

4. a) The number of excess kilometres is $75\,230 - 60\,000 = 15\,230$. The charge for the excess kilometres is $\$0.09 \times 15\,230$ or \$1370.70.

b) The total amount to lease the car is $\$21\,531.77 + \$1370.70$ or \$22 902.47.

c) The dealer may want to resell the car if you decide not to buy it. More kilometres lower the value of the car.

1.6 Exercises, page 48

2. A TD1 form is filled out by every employee that earns income. Based on information on the form, federal and provincial governments deduct income tax from each pay cheque.

3. Different provinces have different income tax rates.

4. a) As income increases the CPP deduction also increases, up to a yearly maximum.

b) As income increases so does the income tax deduction.

c) As gross income increases so does the EI contribution, up to a yearly maximum.

d) Income tax deduction decreases as the TD1 claim code increases.

Selected Solutions — Chapter 1

6. Contributions to RRSP or RPP reduce taxable income, so income tax decreases. CPP and EI are calculated on gross income, so are not affected.

7. **d)** Answers may vary. Jackie earns more per year, so she should pay more CPP, EI, and income tax.

9. **c)** His CPP and EI contributions increased by $0.29 and his income tax deduction remained the same. His net income increased by $5.00 – $0.29 or $4.76 per week, which is $9.52 per pay period.

10. **b)** Her earnings are closer to the lower end of the scale, so the estimate is probably lower than her actual net earnings.

c) Her taxable income would drop into a lower tax bracket, of only 29% as compared to 39%.

11. **b)** His earnings are closer to the higher end of the scale, so his actual net earnings will be lower than the estimate.

12. Answers may vary. For exercise 10:
EI and CPP deductions are each approximately 3% of $1320.
$0.03 \times \$1320 = \$39.60$
Taxable income is gross income minus RRSP contributions.
$\$1320 - \$100 = \$1220$

$$\begin{aligned}\text{Annual taxable income} &= 52 \times \$1220 \\ &= \$63\,440\end{aligned}$$

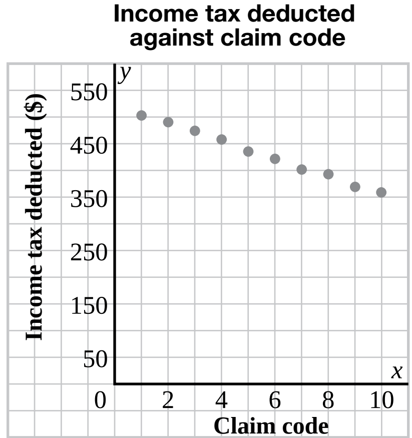
Alina's taxable income is above $60 000. She can expect to pay approximately 39% income tax in each pay period.
$0.39 \times \$1220 = \$475.80$

$$\begin{aligned}\text{Total deductions} &= \$100 + \$10 + \$39.60 + \$39.60 + \$475.80 \\ &= \$665\end{aligned}$$

$$\begin{aligned}\text{Estimated net earnings} &= \text{gross earnings} - \text{total deductions} \\ &= \$1320 - \$665 \\ &= \$655\end{aligned}$$

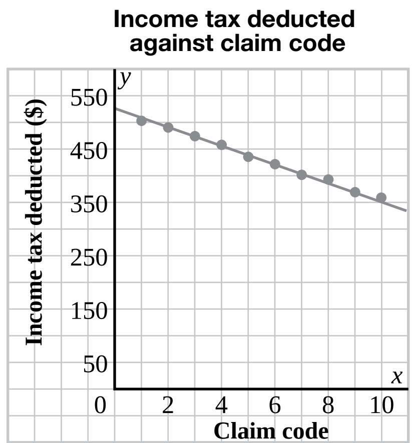
Alina's net earnings for a week can be estimated at $655.

14. **b)**



Selected Solutions — Chapter 1

d)



An equation of the line of best fit is $y = -16.4x + 522.6$.

Linking Ideas: Mathematics and Technology
The Future Is Now, page 51

2. As the interest rate decreases, the accumulated amounts become closer together. As the interest rate increases, the accumulated amounts become farther apart.

1.7 Exercises, page 54

2. b) Answers may vary. If you don't need 12 cassettes, it may be better to purchase the smaller amount.

3. a) Answers may vary. In February, more electricity may have been used because of cold. The family may have been away from home for a vacation for part of March, and the April bill is less than the February bill since it is warmer in April than in February.

c) Answers may vary. You would know how much to budget for electricity each month.

4. b) They will run out of money by the time they have to pay the property taxes.

5. d) Answers may vary. The customer doesn't have to pay property taxes in one big lump sum.

7. a) No, at this exchange rate, \$139 U.S. per person is more than \$350 Can.

9. Yes, because the Canadian vacationers can exchange their Canadian dollars for more U.S. dollars than before.

Selected Solutions — Chapter 1

10. These two rates are equivalent.
$1 Can. = $0.71 U.S.
Multiply both sides by $\frac{1}{0.71}$.
$\$\frac{1}{0.71}$ Can. $= \$1$ U.S.
$1 U.S. = $1.408 Can.
This is an exchange rate of 140.8%.

12. Let x represent the assessed value of the property, and let y represent the original mill rate. The difference in the property taxes is:

$$\frac{x(y+2)}{1000} - \frac{xy}{1000} = \frac{xy + 2x - xy}{1000}$$
$$= \frac{x}{500}$$

The increase depends on the assessed value of the property. Thus, not all taxpayers experience the same increase.

13. This is just like calculating compound interest: $A = P(1 + i)^n$.

a) $A = 0.60(1.04)^{60}$
$= 6.31$
1 L of gas will cost $6.31.

b) $A = 8(1.04)^{60}$
$= 84.16$
Admission to a movie will cost $84.16.

c) $A = 22\,000(1.04)^{60}$
$= 231\,431.80$
A new car will cost $231 431.80.

d) $A = 100\,000(1.04)^{60}$
$= 1\,051\,962.74$
A new home will cost $1 051 962.74.

14. The cost of attending university in 18 years will be $\$5000(1.03)^{18} = \$8512.17$
Find the present value.

$$P = \frac{8512.17}{18}$$
$$\doteq 2518.44$$

They should invest $2518.44 today.

Selected Solutions — Chapter 1

15. a)

	A	B	C
1	Interest rate:	0.07	
2	Yearly payment:	$ 233.99	
3			
4	Age of child	Years compounded	Future value of this
5			year's payment
6	0	18	$ 790.87
7	1	17	$ 739.13
8	2	16	$ 690.78
9	3	15	$ 645.59
10	4	14	$ 603.35
11	5	13	$ 563.88
12	6	12	$ 526.99
13	7	11	$ 492.51
14	8	10	$ 460.29
15	9	9	$ 430.18
16	10	8	$ 402.04
17	11	7	$ 375.74
18	12	6	$ 351.16
19	13	5	$ 328.18
20	14	4	$ 306.71
21	15	3	$ 286.65
22	16	2	$ 267.90
23	17	1	$ 250.37
24		Total:	$ 8 512.31

Mathematics File: Keeping Track of Your Money, page 59

1. His balance after the deposit should be $752.56. The cheque to Clothing Express should be $137.64. He forgot to record the $60.00 withdrawal.

2. Chris subtracted the $58.72 cheque incorrectly. The Record Bin cheque should be $33.15.

3. Answers may vary. It allows you to check each transaction in the monthly statement you receive from the bank. Also, after recording each transaction, you know your current balance. If, for example, your balance is a few dollars, you will have to make a deposit to cover any cheques you write and withdrawals you make.

4. Chris saves enough to support his spending habits.

5. Answers may vary. The procedure varies from store to store. If the cash register tallies and the total daily receipts differ by less than a certain amount, say $5, the amounts are considered balanced. If the difference is greater than $5, a check of each receipt and its corresponding register tally is made to find the error.

6. Answers may vary. Enter the information for each transaction during the month. Ensure the amounts are entered in the correct column, credit or debit. Fill Down the Credit, Debit, and Balance columns to the last transaction row to display the balance. Check this balance against the final balance in the bank statement.

Selected Solutions — Chapter 1

1.8 Exercises, page 66

3. Budgets may vary.

New budget for Jodi

Expense category	Actual percent (%)
Housing and utilities	38.0
Food and clothing	16.0
Health and personal care	4.0
Transportation	14.0
Recreation and education	19.0
Savings	5.0
Miscellaneous	4.0

Jodi is spending twice the recommended amount on transportation. I reduced her transportation expense by 12% to 14% and increased her recreation and education expense by 12% to 19%. Her budget for recreation and education is 19% of $1500, or $285. This covers her monthly school costs. Here are some ways Jodi can lower her transportation expense.

- Sell or trade in her car for a cheaper model. This should also lower the cost of car insurance.
- Sell her car and use public transportation.

4. If the government spends more than it takes in for a particular year, then there is a deficit for that year. If the government spends less than it takes in for a particular year, then there is a surplus for that year. If there is a surplus, the government can reduce taxes, or increase services, or invest the money for the benefit of the citizens. If there is a deficit, the government must borrow to cover its operating expenses, and it must pay interest for borrowing the money. If the government has deficits every year for a number of years, then all of the deficits, plus the interest on the borrowed money, combine to form the total government debt. Thus, the debt is the accumulated amount of all of the deficits.

5. **a)** Budgets may vary.

The Zunigas' monthly new budget

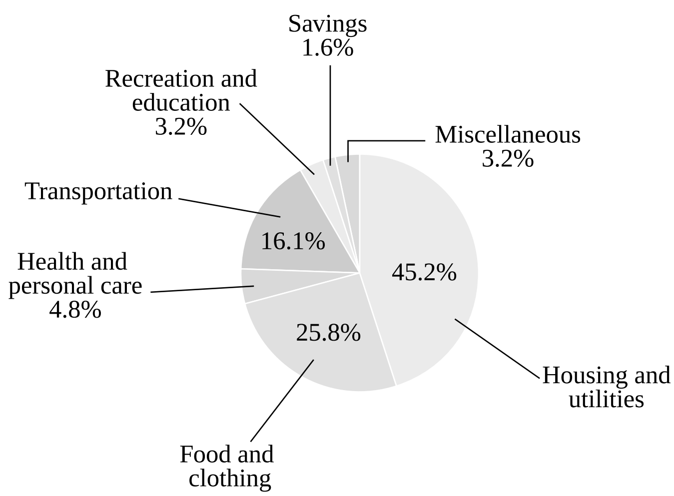
Expense category	Amount ($)
Housing and utilities	1400.00
Food and clothing	800.00
Health and personal care	150.00
Transportation	500.00
Recreation and education	100.00
Savings	50.00
Miscellaneous	100.00

Selected Solutions — Chapter 1

The Zunigas' monthly budget is $3100. The $200 increase in the housing category represents 6.5% of the total monthly budget. Since the transportation category was over the recommended percent guideline by 8.6%, I reduced the transportation expense by 6.5% to 16.1%, or $500. Here are some ways the Zunigas can lower their transportation expense.

- Sell or trade in their car for a cheaper model. This should also lower the cost of car insurance.
- If they have two cars, they could sell one car and share the other, or one parent can take public transportation to work.

b) **The Zunigas' New Monthly Budget**



6. a) Answers may vary. Yes, this is reasonable because the waiting period to receive unemployment benefits is 30 to 90 days.

7. d) Answers may vary. The categories are Average household size, With children under 18 (%), No full time earner (%), and At least 2 full time earners (%).

1.9 Exercises, page 71

2. a) Using the table on page 76, first locate the entry where the row for 6.5% interest meets the column for 20 years' amortization. The entry is $7.46. This means that the monthly payment for every $1000 of mortgage is $7.46. Since we have a $120 000 mortgage, the monthly payment is $\$7.46 \times 120$ or $895.20. The total cost of the mortgage is the monthly payment times the number of months over which the mortgage is paid. The term is 20 years, which is $20 \times 12 = 240$ months, so the total cost of the mortgage is $240 \times \$895.20$ or $214 848.

4. Some people keep track of current mortgage rates to help them decide which term to choose when obtaining a new mortgage or renewing an existing mortgage.

Selected Solutions — Chapter 1

5. **a)** Using the table on page 76, the monthly payment on Sariah's mortgage is $\$8.74 \times 75 = \$655.50$.

b) ii) Assuming that she re-amortizes for 25 more years (she is not bound to do this, and in the long run she will pay less if she re-amortizes for a shorter length of time), she will now pay \$6.91 per thousand dollars of mortgage. Since her balance is \$72 250 + \$1500 or \$73 750, her monthly payment is now $\$6.91 \times 73.75$ or \$509.61.

iii) If she renegotiates, Sariah will save $\$655.50 - \$509.61$ or \$145.89 per month. After just one year at the new rate, her savings will be $\$145.89 \times 12$ or \$1750.68, which is more than the penalty she had to pay for renewing early. For the rest of her term she will continue to save the same amount each month. So, it seems wise to renew early.

6. **b)** Some people choose long terms, such as 5 years, since it gives them peace of mind. With a short term, they may worry that interest rates will go up by the time they have to renew, and they will have to renew at a higher rate. With a longer term, they will have to face this worry less frequently.

8. Factors may vary. Maria should consider the following:

- Will she be working in 15, 20, or 25 years? If she retires during an amortization period, she might not be able to afford mortgage payments.
- The terms of the mortgage – Can she make a lump sum payment without paying a penalty?

9. **a)**

Amortization period (years)	Monthly payment ($)	Total cost ($)	Percent of original repaid (%)
5	1584.00	95 040	118.80
10	928.80	111 456	139.32
15	719.20	129 456	161.82
20	620.00	148 800	186.00
25	565.60	169 680	212.10

b) As the amortization period increases, the monthly payment decreases.

c) As the amortization period increases, the total cost increases.

10. **a)**

Interest rate (%)	Monthly payment ($)	Total cost ($)	Percent of original repaid (%)
4	666.00	119 880	133.20
6	759.60	136 728	151.92
8	860.40	154 872	172.08
10	967.50	174 150	193.50
12	1080.00	194 400	216.00

Selected Solutions — Chapter 1

b) As the interest rate increases, the monthly payment increases.

c) As the interest rate increases, the total cost increases.

d) No. As you can see from the table in part a, the monthly payment does not double as the interest rate doubles from 4% to 8% or from 6% to 12%.

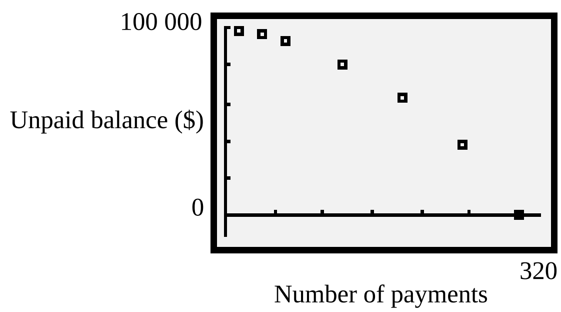
11. a)

Payment period	Payment ($)	Total cost ($)	Percent of original repaid (%)
Monthly	745.57	178 936.80	178.94
Semi-monthly	372.28	178 694.40	178.69
Bi-weekly	343.61	178 677.20	178.68
Weekly	171.70	178 568.00	178.57

b) As payment period decreases, the payments decrease.

c) As the payment period decreases, the total cost of the mortgage decreases. In practice, the more frequent payments (last 3 rows of the table) are made slightly larger, so that the mortgage is paid off faster, resulting in much more significant savings.

12. e)



f) The graph is a curve that eventually reaches the horizontal axis. It is not linear because the unpaid balance is not proportional to the number of payments. Most of the early payments are used to pay interest, and very little of each payment goes to reduce the principal. As time goes on and the unpaid balance decreases, more and more of each payment goes to reducing the principal. Thus, as time passes, the balance reduces faster and faster.

13. f) The earlier you make the extra payments, the more beneficial they will be in reducing the length of your mortgage.

Modelling the Full Amortization of a Mortgage

- Calculations are easier when considering a mortgage this way.
- Whether to "go long" or "go short" depends on how interest rates will change in the future. And who can predict the future? Also, some people invariably take a long term to save themselves from worry about how interest rates will change in the short term.

  Having said all this, in the first case it may be better to select a short term in the first case; because rates have been high and have been increasing, it may be that they will decrease soon. Therefore, we don't want to lock in to a high rate; by going short, we can renew when the rates decrease soon. However, if we are wrong and rates continue to increase, then we will have to renew at a higher rate, at which point we may wish that we had locked in for a longer term.

In the second case, it may be better to select a long term; because rates are low and have been dropping, it may be that they will start to increase. If that happens, we will benefit from being locked in at a lower rate. However, if we are wrong and rates continue to decline, then we would have been better going short, so that we can renew frequently at lower and lower interest rates.

Remember, one cannot make a general rule about what to do in each specific situation. In each case, one must decide based on one's best guess about whether rates will go up or down, and since that depends on many complicated economic and political factors, the decision is not easy.

1 Review, page 77

3. Answers may vary. For exercise 2, part a:
Use the formula $A = P(1 + i)^n$ with the following data:
$P = 500, i = 0.04, n = 4$
Substituting the values in the formula, one obtains
$A = 500(1.04)^4$
$= 584.93$
The accumulated amount is \$584.93.

5. If money is the only consideration, we need to find out which job will pay more. That depends on how many hours of overtime are worked. You could sketch a graph of the two salaries and compare them. Alternatively you could use algebra: let x represent the number of hours of overtime worked, and let A and B represent the salaries for the jobs. If fewer than 40 hours are worked per week, then clearly the first job pays more, since the hourly rate is more and there is no overtime. What if more than 40 hours are worked per week? Then

$$A = 40 \times 17 + (1.5 \times 17)x$$
$$= 680 + 25.5x$$
$$B = 40 \times 15 + (2 \times 15)x$$
$$= 600 + 30x$$

To find out when the salaries are equal, set the expressions for A and B equal and solve for x:

$$A = B$$
$$680 + 25.5x = 600 + 30x$$
$$80 = 4.5x$$
$$x = 17.8$$

Thus, x is about 18. As you can see from the graph, if more than about 18 hours of overtime are worked, then the second job pays more; otherwise the first job pays more. Working that many hours of overtime per week is not likely to lead to a healthy, balanced life for

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most people. And perhaps it is not likely that that much overtime would be available. For those reasons, my choice would be the first job, but other people might choose differently.