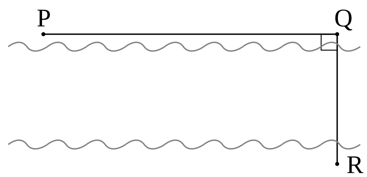
Selected Solutions — Chapter 8

Modelling for Measuring Inaccessible Heights, page 456

Answers may vary. It is assumed that the tree is vertical, and the terrain is flat. The model cannot be used on a hill. The model can be used to measure the width of a river. In the diagram below, assume QR is the width. I am standing at Q. R is a tree on the opposite bank. P is a tree on the same bank as I am. I can measure PQ and ∠QPR, then use the trigonometric ratio to calculate QR, since I chose Q so that $\angle PQR = 90^\circ$.



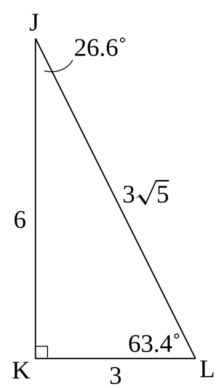
8.1 Exercises, page 457

4. Answers may vary. For part i: In △ABC, the side opposite ∠A is BC. The adjacent side is AB. So, $\tan A = \frac{2}{3}$

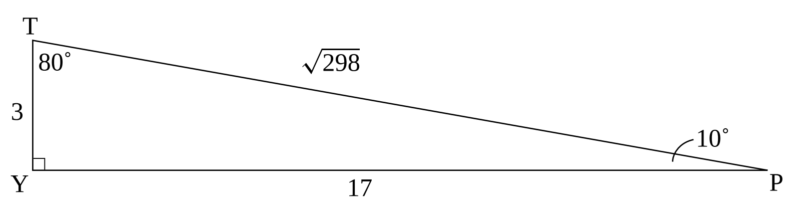
   I used my calculator. I keyed: 2 [÷] 3 [=] [TAN⁻¹] to display 33.69006753. To the nearest whole number, $\angle A = 34^\circ$

7. Triangles may vary.

   a)



   b)



8. Answers may vary. For part a: Since $\tan J = \frac{1}{2}$, then

   $\frac{KL}{JK} = \frac{1}{2}$

   I let $KL = 3$, then $JK = 6$

   I constructed $KL = 3$ cm, used a protractor to construct $\angle K = 90^\circ$, then marked the length $KJ = 6$ cm. I joined JL to form the triangle.

   Since $\tan J = \frac{1}{2}$, then $\angle J \doteq 26.6^\circ$

   Since the sum of the acute angles in a right triangle is 90°,

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$\angle L = 90^\circ - \angle J$
$= 63.4^\circ$

I used the Pythagorean Theorem to calculate JL.

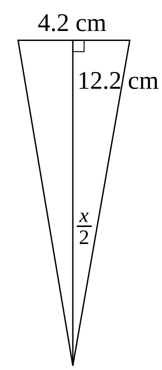
$JL^2 = JK^2 + KL^2$
$= 6^2 + 3^2$
$= 45$
$JL = \sqrt{45}$
$= 3\sqrt{5}$

11. Assumptions may vary. I assume the building and I are on level ground. I also assume the angle of elevation is measured from ground level.

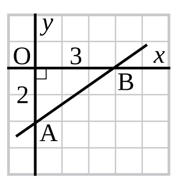
18. Draw a cross section of the cone and the can. Draw the vertical line through the vertex. This forms two right triangles, with legs $\frac{8.4}{2}$ cm, or 4.2 cm; and 12.2 cm.

Then, $\tan\frac{x}{2} = \frac{4.2}{12.2}$
$\frac{x}{2} \doteq 19^\circ$
$x \doteq 38^\circ$

The vertex angle is approximately 38°.



19. **a)** Consider the right triangle formed by the line $y = \frac{2}{3}x - 2$, the x-axis, and the y-axis. Label it △OBA. The lengths of the legs are indicated.

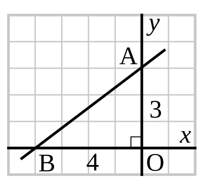


Then ∠OBA is the required angle.

$\tan B = \frac{2}{3}$
$= 0.\overline{6}$
$\angle B \doteq 33.7^\circ$

The angle is approximately 33.7°.

b) Consider the right triangle formed by the line $3x - 4y = -12$, the x-axis, and the y-axis. Label it △OBA. The lengths of the legs are indicated.



Then $\angle OBA$ is the required angle.

$\tan B = \frac{3}{4}$

$= 0.75$

$\angle B \doteq 36.9°$

The angle is approximately 36.9°.

c) In parts a and b, the tangent of the angle is equal to the slope of the line since both are calculated from $\frac{\text{rise}}{\text{run}}$.

For $5x + 2y = 8$, solve for y to determine the slope.

$2y = 8 - 5x$

$y = \frac{8}{2} - \frac{5}{2}x$

The slope is the coefficient of x, which is $-\frac{5}{2}$.

Let the required angle be B.

The tangent of the angle is $\frac{5}{2}$. We can ignore the negative sign because we simply want the numerical value of the $\frac{\text{rise}}{\text{run}}$.

$\tan B = \frac{5}{2}$

$= 2.5$

$\angle B \doteq 68.2°$

The angle is approximately 68.2°.

8.2 Exercises, page 464

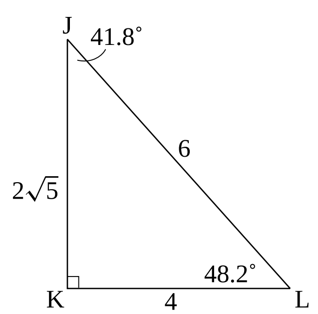
2. Answers may vary. For part i: In $\triangle MTA$, the side opposite $\angle A$ is MT. The hypotenuse is AT. So, $\sin A = \frac{8}{13}$

I used my calculator. I keyed: 8 [÷] 13 [=] [SIN⁻¹] to display 37.97987245.

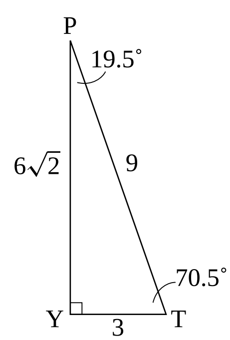
To the nearest whole number, $\angle A = 38°$

6. Answers may vary.

a)



b)



7. Answers may vary. For part a: Since $\sin J = \frac{2}{3}$,

then $\frac{KL}{JL} = \frac{2}{3}$

I let $KL = 4$, then $JL = 6$

Since $\sin J = \frac{2}{3}$

Then $\angle J \doteq 41.8°$

$\angle L = 90° - \angle J$

$= 48.2°$

I constructed $KL = 4$ cm, used a protractor to construct $\angle K = 90°$, and to construct $\angle L = 48°$. I drew the hypotenuse from L and

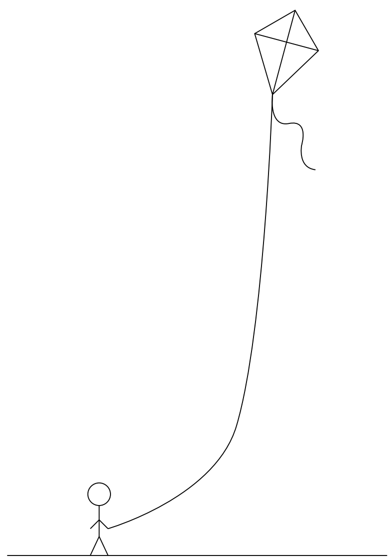
Selected Solutions — Chapter 8

ensured it was long enough to intersect the vertical line from K, at point J. I checked that JL was approximately 6 cm. I used the Pythagorean Theorem to calculate JK.

$$\begin{aligned} JK^2 &= JL^2 - KL^2 \\ &= 6^2 - 4^2 \\ &= 20 \\ JK &= 2\sqrt{5} \end{aligned}$$

Modelling the Height of a Kite

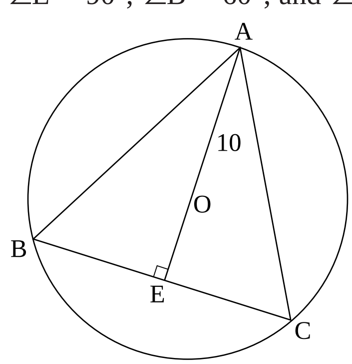
Answers may vary. The string curves away from the ground, as shown.



Since the string is not a straight line, the height of the kite will be less than that calculated. That is, the end of a curve of length 176 m will not reach as high as a straight line of length 176 m. The calculation of the height of the kite is the height measured from the hand holding it. For the correct height, the height of the hand above the ground must be added to the calculated height.

21. **a)** In a right triangle with angles 30˚, 60˚, and 90˚, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

23. Copy the diagram. Label AO, and extend AO to meet the side opposite A at E. By symmetry, △ABE is a 30-60-90 triangle with ∠E = 90˚, ∠B = 60˚, and ∠A = 30˚. Join AB.



OB = OA = 10 cm, so $\triangle$OAB is isosceles, with $\angle B = \angle A = 30^\circ$.
$\triangle$OBE is also a 30-60-90 triangle, with $\angle B = 30^\circ$, $\angle O = 60^\circ$, and $\angle E = 90^\circ$.

In $\triangle$OBE

$$\begin{aligned} \cos B &= \frac{BE}{OB} \\ \cos 30^\circ &= \frac{BE}{10} \\ BE &= 10\cos 30^\circ \\ &\doteq 10(0.8660) \\ &\doteq 8.660 \end{aligned}$$

But

$$\begin{aligned} BC &= 2BE \\ &\doteq 2(8.660) \\ &\doteq 17.3 \end{aligned}$$

The side length of the triangle is 17.3 cm.

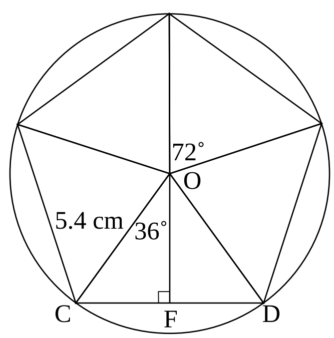
24. Sketch a regular pentagon in a circle. Join each vertex to the centre O. The angle at the centre is 360°.

Each central angle is $\frac{360^\circ}{5} = 72^\circ$.

Each triangle is isosceles.

Consider $\triangle$OCD. Drop the perpendicular from O on to CD at F.

Then, $\angle COF = \frac{72^\circ}{2}$
$= 36^\circ$



In $\triangle$OCF,

$$\begin{aligned} \sin 36^\circ &= \frac{CF}{OC} \\ CF &= OC\sin 36^\circ \\ &= 5.4\sin 36^\circ \\ CD &= 2 \times CF \\ &= 2(5.4\sin 36^\circ) \end{aligned}$$

The perimeter is

$$\begin{aligned} 5 \times CD &= 5(2)(5.4\sin 36^\circ) \\ &\doteq 31.7 \end{aligned}$$

The perimeter is 31.7 cm.

Selected Solutions — Chapter 8

25. From the diagram,

$$\sin 87.73^\circ = \frac{r}{r+5}$$
$$(r+5)\sin 87.73^\circ = r$$
$$r\sin 87.73^\circ + 5\sin 87.73^\circ = r$$
$$r - r\sin 87.73^\circ = 5\sin 87.73^\circ$$
$$r(1 - \sin 87.73^\circ) = 5\sin 87.73^\circ$$
$$r = \frac{5\sin 87.73^\circ}{1 - \sin 87.73^\circ}$$
$$\doteq 6366.6$$

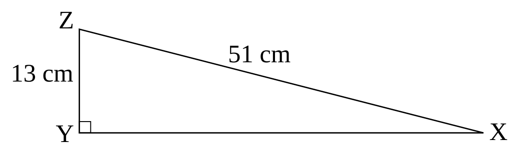
The radius is about 6367 km. Use the formula $C = 2\pi r$ to calculate the circumference. Substitute $r \doteq 6366.6$.

$$C = 2\pi(6366.6)$$
$$\doteq 40\,002.7$$

The circumference is about 40 000 km.

8.3 Exercises, page 474

5. Answers may vary. For part c: I drew a diagram and labelled it with the given information.



I used the Pythagorean Theorem to calculate the length of the leg YX.

$$YX^2 = XZ^2 - ZY^2$$
$$= 51^2 - 13^2$$
$$= 2432$$
$$YX = \sqrt{2432}$$
$$\doteq 49.3$$

YX is approximately 49 cm.

To determine the measure of ∠X, I used the sine ratio.

$$\sin X = \frac{13}{51}$$
$$\doteq 0.2549$$
$$\angle X \doteq 14.7678^\circ$$

∠X is approximately 15°.

I know that the two acute angles in a right triangle have a sum of 90°. So, $\angle Z = 90^\circ - 15^\circ$
$$= 75^\circ$$

∠Z is approximately 75°.

10. The road changes in altitude 4 m for every 100 m of road.

Selected Solutions — Chapter 8

Modelling the Inclination of a Road

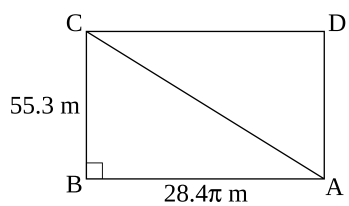
Answers may vary. The inclination of a road is the angle between the road and the horizontal. In exercise 8, we knew the change in altitude and the length of the road. So, we used the sine ratio to calculate the angle.

Exercise 9 was similar; here we knew the length of the road and the angle of inclination. So, we could use the sine of that angle to calculate the change in altitude.

In exercise 10, the change in altitude divided by the length of the road is multiplied by 100%, to express the gradient as a percent.

The slope of a line is the rise divided by the run, which is the tangent of the angle the line makes with the horizontal. If we know the change in altitude and the change in horizontal distance, for a stretch of road, we could use the tangent ratio to calculate the angle. However, the inclination of a road is always positive; whereas the slope of a line may be negative.

17. Visualize the cylinder flattened into a rectangle. The width of the rectangle is the height of the cylinder, or 55.3 m. The length of the rectangle is the circumference of the cylinder, which is π times the diameter of the cylinder, or 28.4π m. The diagonal of the rectangle is the staircase.



The angle of inclination of the stairway is ∠BAC.
Use the tangent ratio in △ABC.

$$\tan A = \frac{55.3}{28.4\pi}$$
$$\doteq 0.619\ 808$$
$$\angle A \doteq 31.8^\circ$$

The angle of inclination of the staircase is about 32°.

Problem Solving: Calculating the Speed of Earth's Rotation, page 480

1. a) Use the formula for the circumference C of a circle with radius r:

$C = 2\pi r$

Substitute $C = 40\ 076$.

$40\ 076 = 2\pi r$

Solve for r.

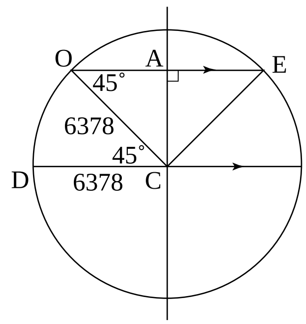
$$r = \frac{40\ 076}{2\pi}$$
$$\doteq 6378$$

Earth's radius is approximately 6378 km.

Selected Solutions — Chapter 8

b) If we consider the cross section of Earth, at Ottawa, parallel to the equator, the figure is a circle with diameter 9020 km. This circle is a circle of latitude.

c)



Consider the diagram on page 480. Draw a vertical line through the centre of Earth. It meets the diameter of the circle of latitude through Ottawa at A, and the centre of Earth at C. Label the horizontal radius CD, and the radius at Ottawa, CO. Then AO ∥ CD, and CO is a transversal. Therefore, alternate angles are equal and $\angle AOC = \angle OCD = 45^\circ$.

Since $\triangle AOC$ is a right triangle, then $\angle ACO = 45^\circ$. This means that $\triangle ACO$ is isosceles, with AC = AO.

Use the Pythagorean Theorem.

$$AO^2 + AC^2 = OC^2$$
$$2AO^2 = 6378^2$$
$$AO^2 = \sqrt{20\ 339\ 442}$$
$$AO \doteq 4509.9$$

The diameter of the circle of latitude through Ottawa is

$$2AO = 2(4509.9)$$
$$\doteq 9020$$

The distance 9020 km is correct.

d) Use $C = \pi d$ to determine the circumference of the circle of latitude through Ottawa.

$$C = \pi(9020)$$
$$\doteq 28\ 337$$

Ottawa travels about 28 337 km in 24 h.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Speed of Ottawa} = \frac{28\ 337 \text{ km}}{24 \text{ h}}$$
$$= 1181 \text{ km/h}$$

Ottawa is travelling at about 1180 km/h.

2. Answers may vary. J.D. is incorrect. The Pythagorean Theorem can be used, as illustrated in exercise 1d. D.N. should have expanded his explanation before using the Pythagorean Theorem.

3. a) Latitude lines are circles on Earth, parallel to the equator. The quote refers to the circle of latitude that passes through Ottawa. Its diameter is the segment OD, in the diagram in exercise 1c.

b) Look at the diagram in exercise 1c.
By symmetry, $\angle AEC = \angle AOC = 45^\circ$

So, in $\triangle OCE$, $\angle C = 90^\circ$

In $\triangle OCE$, $\cos O = \frac{OC}{OE}$

$\cos 45^\circ = \frac{6378}{OE}$

Use the shortcut method.

$OE = \frac{6378}{\cos 45^\circ}$

OE is the diameter of the circle of latitude at Ottawa. This is equal to the radius of Earth divided by $\cos 45^\circ$.

c) From part b, $OE = \frac{6378}{\cos 45^\circ}$

$\doteq 9020$

The circumference of the circle through Ottawa is $\pi(9020) \doteq 28\ 337$.

The speed of Ottawa is $\frac{28\ 337 \text{ km}}{24 \text{ h}} \doteq 1081$ km/h.

Note that J.D. did not round correctly, to get $OE = 9020$ km, and this made his value for circumference different.

4. Answers may vary.

5. Answers may vary.

Modelling for Measuring Inaccessible Heights, page 483

Answers may vary. The model assumes the cliff is vertical, and the river shore is horizontal, with the segment measured along the shore at right angles to the measured width of the river. Nature is rarely perfectly straight. This model does measure the width of the river as explained in Example 2.

8.4 Exercises, page 484

3. b) $\angle CAB = \angle DAB + \angle EAB$

c) The sides of the triangles would still be in the same ratios, so the angles would be the same.

Suppose the squares had side length x centimetres.

Then $\tan \angle CAB = \frac{x}{x}$

$= 1$

$\angle CAB = 45^\circ$

$\tan \angle DAB = \frac{x}{2x}$

$= \frac{1}{2}$

$\angle DAB \doteq 26.6^\circ$

and $\tan \angle EAB = \frac{x}{3x}$

$= \frac{1}{3}$

$\angle EAB = 18.4^\circ$

5. c) For part a: I let the length of the guy wire be l metres. In the smaller right triangle, I used the cosine ratio to calculate l.

$$\cos 39° = \frac{7.0}{l}$$
$$l = \frac{7.0}{\cos 39°}$$
$$\doteq 9.007$$

I let the height of the tower be h metres.
In the larger right triangle, I used the tangent ratio to calculate h.

$$\tan 53° = \frac{h}{7.0}$$
$$h = 7.0 \tan 53°$$
$$\doteq 9.2893$$

The guy wire is approximately 9.0 m and the towers approximately 9.3 m.
For part b: I let the distance from the ground to the point on the tower where the wire is attached be y metres.
In the smaller right triangle, I used the tangent ratio to calculate y.

$$\tan 39° = \frac{y}{7.0}$$
$$y = 7.0 \tan 39°$$
$$= 5.6685$$

I subtracted the distance, 5.6685 m, from the height of the tower, 9.2893 m, to calculate the distance from the top of the tower to the wire. This distance is $9.2893 \text{ m} - 5.6685 \text{ m} \doteq 3.6 \text{ m}$.

14. c) Answers may vary. The carton is not uniform. The stick has a width and thickness but, in the calculation, we assume it has no width or thickness. It also depends on how the metre stick is placed in the carton—with width touching the edge of the carton or with thickness touching the edge.

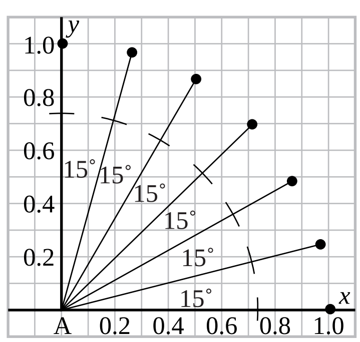
15. b) The angle of inclination depends on the horizontal distance of the lower end of the stick from the wall, and the height of the carton. The diagonal of the carton is longer than the length or width of the carton, so the angle of inclination of the stick is smaller in exercise 15a.

Linking Ideas: Mathematics and Technology
Sines and Cosines of Obtuse Angles, page 489

4. a) $\cos 130° \doteq -0.642\ 787\ 609$; there is no acute angle that has the same cosine because all acute angles have positive cosines.

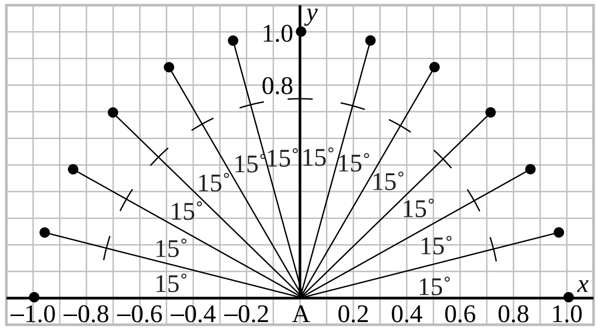
Investigate, page 490

4..



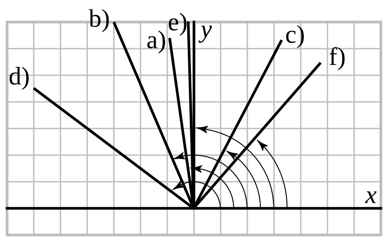
Selected Solutions — Chapter 8

6.



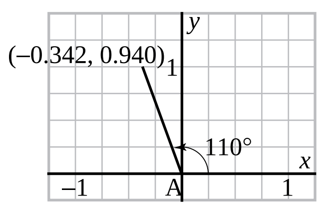
8.5 Exercises, page 496

2.

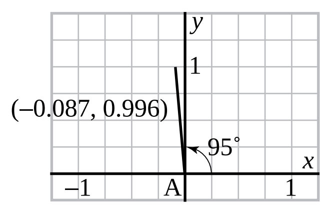


3. Obtuse angles have negative cosines and positive sines. Acute angles have positive cosines and positive sines.

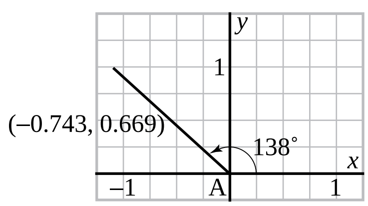
4. a)



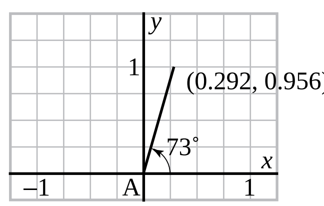
b)



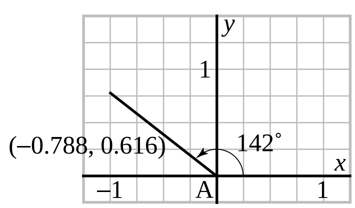
c)



d)



e)



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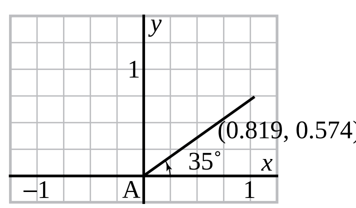

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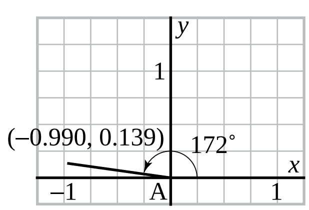

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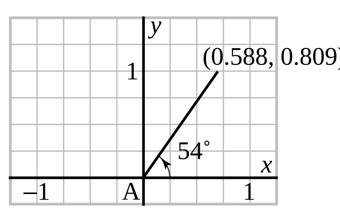


Selected Solutions — Chapter 8

6. Each value of sine corresponds to 2 angles between 0˚ and 180˚.
Each value of cosine corresponds to 1 angle between 0˚ and 180˚.

8. Answers may vary. For part c: $\cos C = -0.7321$; since this equation involves the cosine, I know there is only one value of ∠C, and it lies between 90˚ and 180˚. I used my calculator and keyed:
0.7321 [+/−] [COS⁻¹] to display 137.0627346.
So, $\angle C \doteq 137°$
For part d: $\sin C = 0.4283$; since this equation involves sine, I know there are two values of ∠C. I used my calculator and keyed:
0.4283 [SIN⁻¹] to display 25.35972216.
One value of ∠C is 25.4˚; the other value is 180˚ – 25.4˚, or 154.6˚.

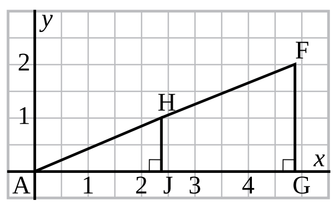
9. **a)** $(\cos 30°)^2 + (\sin 30°)^2 = 0.75 + 0.25$
$= 1$

b) $(\cos 72°)^2 + (\sin 72°)^2 = 1$

c) $(\cos 115°)^2 + (\sin 115°)^2 = 1$

d) $(\cos 164°)^2 + (\sin 164°)^2 = 1$

10. Draw two triangles from page 460, on a grid.



The length AH is 2.6 cm.
Consider 2.6 cm as representing 1 unit.
Then HJ, which is 1 cm, will represent $\frac{1}{2.6}$ units, or 0.3846; and AJ, which is 2.4 cm, will represent $\frac{2.4}{2.6}$ units, or 0.9231.
The coordinates of H are (0.9231, 0.3846).
Consider △AHJ and the ratios of the lengths of the sides.

$$\sin A = \frac{HJ}{AH} = \frac{1}{2.6} \doteq 0.3846 \qquad \cos A = \frac{AJ}{AH} = \frac{2.4}{2.6} \doteq 0.9231$$

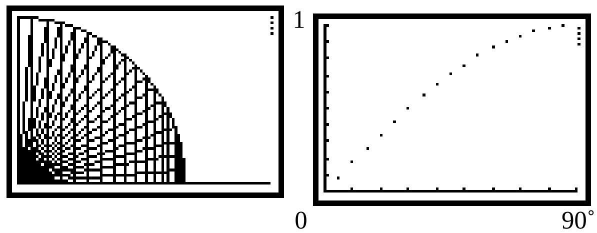
Similarly, in △AFG, the length AF is 5.2 cm. Consider 5.2 cm as representing 1 unit.
Then FG, which is 2 cm, will represent $\frac{2}{5.2}$ units, or 0.3846; and AG, which is 4.8 cm, will represent $\frac{4.8}{5.2}$ units, or 0.9231.
The coordinates of F are (0.9231, 0.3846).
Consider △AFG and the ratios of the lengths of the sides.

$$\sin A = \frac{FG}{AF} = \frac{4.8}{5.2} \doteq 0.9231 \qquad \cos A = \frac{AG}{AF} = \frac{2}{5.2} \doteq 0.3846$$

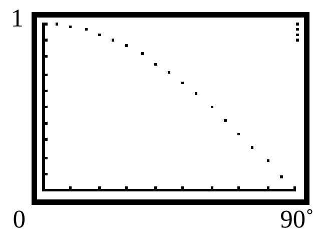
Selected Solutions — Chapter 8

Exploring with a Graphing Calculator: Graphing Sines and Cosines, page 498

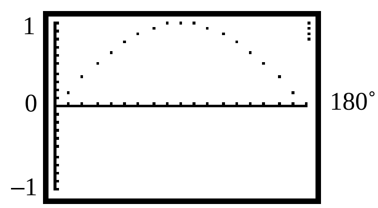
2. a) Predictions may vary. For increments of 5˚



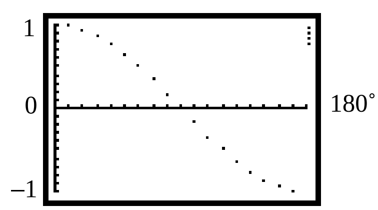
b) For increments of 5˚



3. a) Predictions may vary. For increments of 10˚



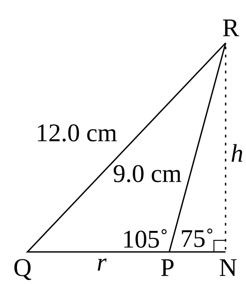
b) For increments of 10˚



8.6 Exercises, page 503

3. b) Add 8 + DM instead of subtracting 8 – AN, since the perpendicular was constructed outside the triangle.

7. a) Sketch △PQR. Drop the perpendicular from R on to QP extended to N. Label the length of RN as *h*. Since QPN is a straight line, ∠RPN = 180˚ – 105˚, or 75˚.



In △RPN, use the sine ratio to calculate h.

$$\sin 75^\circ = \frac{h}{9}$$
$$h = 9\sin 75^\circ$$
$$h \doteq 8.6933$$

In △RQN, use the sine ratio to calculate ∠Q.

$$\sin Q = \frac{h}{12}$$
$$\doteq \frac{8.6933}{12}$$
$$\angle Q \doteq 46.4^\circ$$

In △RQP, use the sum of the angles in a triangle to calculate ∠R.

$$\angle R = 180^\circ - (105^\circ + 46.4^\circ)$$
$$\angle R = 28.6^\circ$$

In △RQN, use the tangent ratio to calculate QN.

$$\tan Q = \frac{h}{QN}$$
$$\tan 46.4^\circ \doteq \frac{8.6933}{QN}$$
$$QN \doteq \frac{8.6933}{\tan 46.4^\circ}$$
$$\doteq 8.2717$$

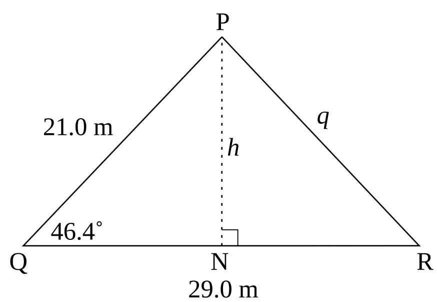
In △RPN, use the cosine ratio to calculate PN.

$$\cos 75^\circ = \frac{PN}{9}$$
$$PN = 9\cos 75^\circ$$
$$\doteq 2.3294$$

Then, $QP = QN - PN$
$$= 8.2717 - 2.3294$$
$$= 5.9423$$

QP is 5.9 cm.

b) Sketch △PQR. Drop the perpendicular from P on to QR at N. Label the length of PN as h.



In △PQN, use the sine ratio to calculate h.

$$\sin 46.4^\circ = \frac{h}{21.0}$$
$$h = 21.0\sin 46.4^\circ$$
$$h \doteq 15.208$$

In △PQN, use the Pythagorean Theorem to calculate QN.

$$QN^2 = QP^2 - PN^2$$
$$QN^2 \doteq 21.0^2 - (15.208)^2$$
$$\doteq 209.7286$$
$$QN \doteq \sqrt{209.7286}$$
$$QN \doteq 14.482$$

Since QR = QN + NR, then

$NR = 29.0 - QN$

$= 14.518$

In △PNR, use the Pythagorean Theorem to calculate PR.

$PR^2 = PN^2 + NR^2$

$= (15.208)^2 + (14.518)^2$

$\doteq 442.043$

$PR = \sqrt{442.043}$

$\doteq 21.025$

PR is 21.0 m.

In △PNR, use the cosine ratio to calculate ∠R.

$\cos R = \frac{NR}{PR}$

$= \frac{14.518}{21.025}$

$\angle R \doteq 46.3^\circ$

In △PQR, use the sum of the angles in a triangle to calculate ∠P.

$\angle P = 180^\circ - (46.4^\circ + 46.3^\circ)$

$\angle P = 87.3^\circ$

Modelling for Measuring Inaccessible Heights, page 508

Answers may vary. Triangle BTC could have been used. Use the Sine Law in △ABC to calculate BC, then use the sine ratio in △BTC to calculate CT. The model would be slightly different, but it would still apply. The tower and the two points from which the angles of elevation are measured must lie on the same line. The model assumes the street is straight, and level; that is, zero slope.

8.7 Exercises, page 509

2. Answers may vary. For part c: To calculate the length of AB, I intended to use the Sine Law. I first calculated ∠C, using the sum of the angles in a triangle is 180°.

$\angle C = 180^\circ - (40^\circ + 100^\circ)$, or 40°

Since $\angle C = \angle A = 40^\circ$, △ABC is isosceles.

So, AB = BC = 5

5. Answers may vary. For part c: To calculate the measure of ∠B, I used the Sine Law. I wrote $\frac{\sin B}{b} = \frac{\sin A}{a}$, since I know the measures of ∠A and *a*. I substituted ∠A = 60°, *a* = 3, and *b* = 1.

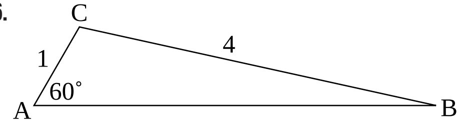
$\frac{\sin B}{1} = \frac{\sin 60^\circ}{3}$

I used my calculator.

I keyed: 60 [SIN] [÷] 3 [=] [SIN⁻¹] to display 16.77865488.

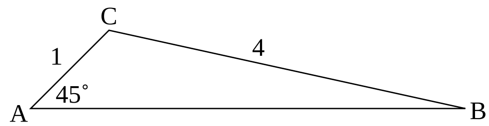
∠B is approximately 16.8°.

6.



Selected Solutions — Chapter 8

7.



19. a) For Example 1, Section 8.3: Use the diagram on page 470.

Use the Sine Law to calculate ∠A.

$\frac{\sin A}{a} = \frac{\sin B}{b}$

Substitute the known measures.

$\frac{\sin A}{2} = \frac{\sin 90^\circ}{5}$

$\frac{\sin A}{2} = \frac{1}{5}$

Use the shortcut method to solve this equation.

$\sin A = 0.4$

$\angle A \doteq 23.5782$

∠A is approximately 24°.

$\angle C = 90^\circ - 23.5782^\circ$

$= 66.4218^\circ$

∠C is approximately 66°.

Use the Sine Law to calculate *c*.

$\frac{c}{\sin C} = \frac{a}{\sin A}$

Substitute the known measures.

$\frac{c}{\sin 66.4218^\circ} = \frac{2}{\sin 23.5782^\circ}$

Use the shortcut method to solve the equation.

$c = \frac{2 \sin 66.4218^\circ}{\sin 23.5782^\circ}$

$\doteq 4.5826$

c is approximately 4.6 cm.

For Example 2, Section 8.3: Use the diagram on page 470.

Use the Sine Law to calculate *y*.

$\frac{y}{\sin Y} = \frac{z}{\sin Z}$

Substitute the known measures.

$\frac{y}{\sin 90^\circ} = \frac{9.0}{\sin 36^\circ}$

$\frac{y}{1} = \frac{9.0}{\sin 36^\circ}$

$y \doteq 15.3117$

XZ is approximately 15.3 m.

Since the triangle is a right triangle, the acute angles have a sum of 90°.

$\angle X = 90^\circ - 36^\circ$

$= 54^\circ$

Use the Sine Law to calculate *x*.

$\frac{x}{\sin X} = \frac{z}{\sin Z}$

Substitute the known measures.

$\frac{x}{\sin 54^\circ} = \frac{9.0}{\sin 36^\circ}$

$x = \frac{9.0 \sin 54^\circ}{\sin 36^\circ}$

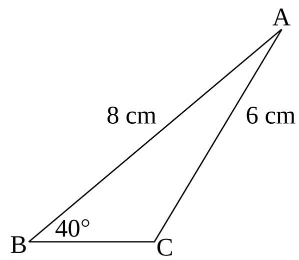
$\doteq 12.3874$

YZ is approximately 12.4 m.

Selected Solutions — Chapter 8

b) You can use the Sine Law to solve a right triangle. Since the sine of 90° is 1, if the Sine Law uses the right angle to hypotenuse ratio, then the Sine Law reduces to the sine ratio.

20. a) Sketch the triangle.



Use the Sine Law to calculate ∠C.

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

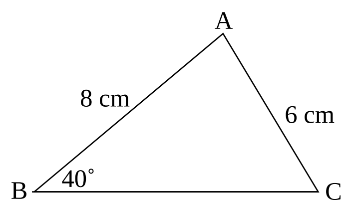
Substitute the known measures.

$$\frac{\sin C}{8} = \frac{\sin 40^\circ}{6}$$
$$\sin C = \frac{8 \sin 40^\circ}{6}$$
$$\doteq 0.8571$$
$$\angle C \doteq 59^\circ$$

But in the diagram, ∠C is obtuse.

So, $\angle C = 180^\circ - 59^\circ$, or 121°

However, a triangle can be drawn with the given measures, where ∠C is 59°.



When $\angle C = 59^\circ$,
$$\angle A = 180^\circ - (40^\circ + 59^\circ)$$
$$= 81^\circ$$

When $\angle C = 121^\circ$,
$$\angle A = 180^\circ - (40^\circ + 121^\circ)$$
$$= 19^\circ$$

Calculate the measure of a for each measure of ∠A.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Substitute $\angle A = 19^\circ$, $b = 6$, and $\angle B = 40^\circ$.

$$\frac{a}{\sin 19^\circ} = \frac{6}{\sin 40^\circ}$$
$$a = \frac{6 \sin 19^\circ}{\sin 40^\circ}$$
$$\doteq 3.039$$

a is approximately 3 cm.

Substitute $\angle A = 81^\circ$, $b = 6$, and $\angle B = 40^\circ$.

$$\frac{a}{\sin 81^\circ} = \frac{6}{\sin 40^\circ}$$
$$a = \frac{6 \sin 81^\circ}{\sin 40^\circ}$$
$$\doteq 9.219$$

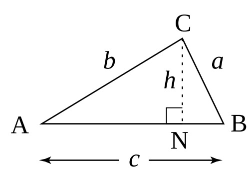
a is approximately 9 cm.

Selected Solutions — Chapter 8

b) I realized that two different triangles satisfy the given conditions, after the first step to calculate ∠C, since there are two possible values for ∠C, when sin C is known.

c) It is possible for two different triangles to have two equal angles and two pairs of equal sides because the positive sine ($\sin C = 0.8571$) indicates that ∠C could be acute or obtuse. There are two solutions for ∠C, and consequently two solutions for ∠A and *a*. When we draw the triangle, we draw line BC, mark a 40° angle at B, then measure BA as 8 cm. When we put the compasses point on A and draw an arc with radius 6 cm, the arc cuts the line through BC at two points, giving two possible positions for C.

21. Copy the diagram from page 505. Drop the perpendicular from C onto AB at N. Label its length *h*.



In △CNB, use the sine ratio to determine *h*.

$$\sin B = \frac{h}{a}$$
$$h = a \sin B$$

In △ACN, use the sine ratio to determine *b*.

$$\sin A = \frac{h}{b}$$

Substitute for *h* from above.

$$\sin A = \frac{a \sin B}{b}$$

Divide each side by *a*.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Similarly, it can be shown that $\frac{\sin B}{b} = \frac{\sin C}{c}$

The Sine Law states: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Modelling for Measuring Inaccessible Distances, page 516

Answers may vary. The tunnel will be straight and the terrain surrounding the hill is flat. Tunnels are not always straight, and terrain is not always flat. It may not be possible to find points A, B, and C such that C is visible from points A and B. It is not possible to use this model to measure heights. We need to know two sides of the triangle and the included angle. This information is not used to find a height, where we are dealing with a vertical distance and, hence, a right triangle.

Selected Solutions — Chapter 8

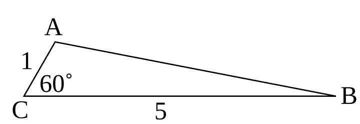
8.8 Exercises, page 517

2. Answers may vary. For part c: I used the Cosine Law to calculate the length of AC. In $\triangle$ABC, $b^2 = c^2 + a^2 - 2ac\cos B$. I substituted the given measures to get $b^2 = 3^2 + 4^2 - 2(3)(4)\cos 120^\circ$.
I simplified this to $b^2 = 9 + 16 - 24\cos 120^\circ$
$= 25 - 24\cos 120^\circ$
I used my scientific calculator and keyed:
120 [cos] [×] 24 [+/−] [+] 25 [=] to display 37; so, $b^2 = 37$.
I didn't clear the calculator; I keyed [$\sqrt{x}$] to display 6.08276253.
The length of AC is about 6.1 units.

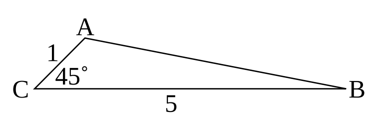
4. **b)** Answers may vary. It is required for accurate digging. If both ends of the tunnel are dug to meet in the middle, the angles must be accurate.

8. Answers may vary. For part b: I could not use the Cosine Law directly to calculate the measure of $\angle$B because I did not know the measures of 3 sides. I used the Cosine Law to calculate the length of AB first. I wrote $c^2 = a^2 + b^2 - 2ab\cos C$. I substituted the given measures to get $c^2 = 3^2 + 1^2 - 2(3)(1)\cos 60^\circ$.
I simplified this to $c^2 = 10 - 6\cos 60^\circ$.
I used my scientific calculator and keyed:
60 [cos] [×] 6 [+/−] [+] 10 [=] to display 7; so, $c^2 = 7$, and $c = \sqrt{7}$.
I do not need to find the square root because I do not need to know c.
I then used the Cosine Law to calculate the measure of $\angle$B.
I wrote $b^2 = a^2 + c^2 - 2ac\cos B$ and substituted the known measures to get $1^2 = 3^2 + 7 - 2(3)(\sqrt{7})\cos B$. I simplified this to get $1 = 16 - 6\sqrt{7}\cos B$, then $6\sqrt{7}\cos B = 15$, and $\cos B = \frac{15}{6\sqrt{7}}$.
I keyed: 15 [÷] 6 [÷] 7 [$\sqrt{x}$] [=] [$\cos^{-1}$] to display 19.10660535.
So, $\angle$B is approximately 19.1°.

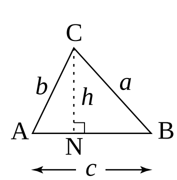
9.



10.



30. Draw the triangle at the bottom of page 513. Drop the perpendicular from C to AB at N. Label its length h.



In $\triangle$ANC, use the sine ratio to determine h.

$\sin A = \frac{h}{b}$

$h = b \sin A$

In $\triangle$ANC, use the cosine ratio to determine the length of AN.

$\cos A = \frac{AN}{b}$

$AN = b \cos A$

Since $AB = AN + NB$, then $NB = AB - AN$, or

$NB = c - AN$

$= c - b \cos A$

In $\triangle$BNC, use the Pythagorean Theorem to determine a.

$a^2 = h^2 + BN^2$

$a^2 = (b \sin A)^2 + (c - b \cos A)^2$

$a^2 = b^2(\sin A)^2 + c^2 - 2bc \cos A + b^2(\cos A)^2$

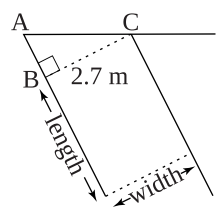
$a^2 = b^2[(\sin A)^2 + (\cos A)^2] + c^2 - 2bc \cos A$

Recall that $(\sin A)^2 + (\cos A)^2 = 1$ for any $\angle$A.

$a^2 = b^2 + c^2 - 2bc \cos A$

This is the Cosine Law for $\triangle$ABC.

31. a) Draw the diagram. Label the vertices as indicated. In $\triangle$ABC, $\angle B = 90^\circ$, since the width is perpendicular to the length.



The curb length per car is AC.
Use the sine ratio in $\triangle$ABC.

Then $\sin A = \frac{2.7}{AB}$

$AB = \frac{2.7}{\sin A}$

The curb length for 20 cars is $20(AB) = \frac{20(2.7)}{\sin A}$, or $\frac{54}{\sin A}$.

i) For $\angle A = 30^\circ$, substitute $A = 30^\circ$.

Curb length $= \frac{54}{\sin 30^\circ}$

$= 108$

The curb length is 108 m.

ii) For $\angle A = 50^\circ$, substitute $A = 50^\circ$.

Curb length $= \frac{54}{\sin 50^\circ}$

$\doteq 70.5$

The curb length is about 70.5 m.

iii) For $\angle A = 60^\circ$, substitute $A = 60^\circ$.

Curb length $= \frac{54}{\sin 60^\circ}$

$\doteq 62.4$

The curb length is about 62.4 m.

Selected Solutions — Chapter 8

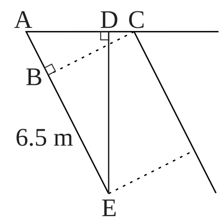
b) The curb length for one car is $\frac{60\text{ m}}{20} = 3$ m. In $\triangle$ABC, AB is 3 m.
Substitute this value for AB, in the formula in part a.

$$\frac{2.7}{3} = \sin A$$

Solve for $\angle$A.

$$\angle A \doteq 64.2^\circ$$

c) Draw the diagram. Drop the perpendicular from the end of the roadway to the curb. Label the vertices as indicated.



The length of roadway is DE.
In $\triangle$ABC, use the tangent ratio to determine AB.

$$\tan A = \frac{BC}{AB}$$

$$\tan A = \frac{2.7}{AB}$$

$$AB = \frac{2.7}{\tan A}$$

Then, $AE = AB + BE$

$$= \frac{2.7}{\tan A} + 6.5$$

In $\triangle$ADE, use the sine ratio to determine DE.

$$\sin A = \frac{DE}{AE}$$

$$DE = AE \sin A$$

$$= \left(\frac{2.7}{\tan A} + 6.5\right) \sin A$$

i) For $\angle A = 30^\circ$, substitute $A = 30^\circ$ in the expression for DE.

$$DE = \left(\frac{2.7}{\tan 30^\circ} + 6.5\right) \sin 30^\circ$$

Key: 2.7 [÷] 30 [TAN] [=] [+]
6.5 [=] [×] 30 [SIN] [=]

$$DE \doteq 5.59$$

The roadway is approximately 5.6 m.

ii) For $\angle A = 50^\circ$, substitute $A = 50^\circ$ in the expression for DE.

$$DE = \left(\frac{2.7}{\tan 50^\circ} + 6.5\right) \sin 50^\circ$$

$$\doteq 6.71$$

The roadway is approximately 6.7 m.

iii) For $\angle A = 60^\circ$, substitute $A = 60^\circ$ in the expression for DE.

$$DE = \left(\frac{2.7}{\tan 60^\circ} + 6.5\right) \sin 60^\circ$$

$$\doteq 6.98$$

The roadway is approximately 7.0 m.

32. Answers may vary. All possible cases for non right triangles can be solved with the Sine Law, the Cosine Law, or both.

Selected Solutions — Chapter 8

Mathematical Modelling: How Far Is the Sun? How Large Is the Sun?, page 522

1. We use the Cosine Law in $\triangle$EVS.

$e^2 = v^2 + s_1^2 - 2vs_1 \cos E$

$e^2 = v^2 + (53.1 \times 10^6)^2 - 2(53.1 \times 10^6)(v) \cos 31.8°$

$e^2 = v^2 + 2.819\ 61 \times 10^{15} - 1.062 \times 10^8 v \cos 31.8°$

2. Use the Cosine Law in $\triangle$EVS.

$e^2 = v^2 + s_2^2 - 2vs_2 \cos E$

$e^2 = v^2 + (210.2 \times 10^6)^2 - 2(210.2 \times 10^6)v \cos 29.3°$

$e^2 = v^2 + 4.418\ 404 \times 10^{16} - 4.204 \times 10^8 v \cos 29.3°$

3. Equate the two right sides of the equations in exercises 1 and 2.

$v^2 + 2.819\ 61 \times 10^{15} - 1.062 \times 10^8 v \cos 31.8°$

$= v^2 + 4.418\ 404 \times 10^{16} - 4.204 \times 10^8 v \cos 29.3°$

$-1.062 \times 10^8 v \cos 31.8° + 4.204 \times 10^8 v \cos 29.3°$

$= 4.418\ 404 \times 10^{16} - 2.819\ 61 \times 10^{15}$

$10^8 v(4.204 \cos 29.3° - 1.062 \cos 31.8°)$

$= 10^{15}(44.184\ 04 - 2.819\ 61)$

$$v = \frac{10^{15}(44.184\ 04 - 2.819\ 61)}{10^8(4.204 \cos 29.3° - 1.062 \cos 31.8°)}$$

Key: 44.18404 [–] 2.81961 [=] [÷] [(] 4.204 [×] 29.3 [cos] [–] 1.062 [×] 31.8 [cos] [)] [=] to display 14.96762631

$v \doteq 10^7(14.967\ 626\ 31)$

$v \doteq 1.50 \times 10^8$

The distance from Earth to the sun is about 1.50×10^8 km, or 150 million kilometres.

4. The distance from Venus to the sun is e.
Substitute for v in the equation in exercise 2.

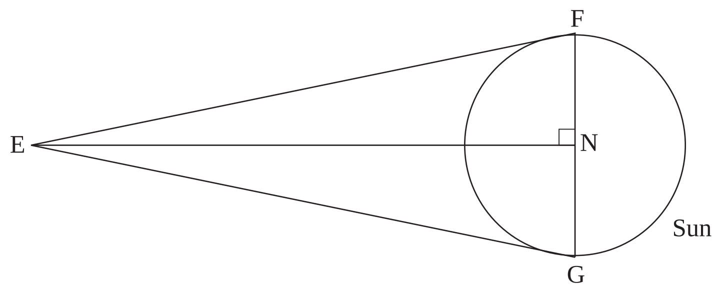
$e^2 = (1.496\ 763 \times 10^8)^2 + 4.418\ 404 \times 10^{16}$
$\quad -(4.204 \times 10^8)(1.496\ 763 \times 10^8) \cos 29.3°$

$e^2 \doteq 1.171\ 301 \times 10^{16}$

$e \doteq 1.082\ 267 \times 10^8$

The distance from Venus to the sun is about 1.08×10^8 km, or 108 million kilometres.

5. The triangle is isosceles. Draw the diagram. Drop the perpendicular from E to the diameter. This segment bisects $\angle$E and bisects the diameter.



Selected Solutions — Chapter 8

In $\triangle EFN$, $EF = 1.497 \times 10^8$ km and $\angle FEN = \frac{0.532^\circ}{2}$, or 0.266°.
Use the sine ratio to calculate the length of FN.

$$\sin 0.266^\circ = \frac{FN}{1.497 \times 10^8}$$
$$FN = 1.497 \times 10^8(\sin 0.266^\circ)$$
$$\doteq 694\,991.1$$

The diameter of the sun is $2 \times FN = 2 \times 694\,999.1$, or $1\,389\,982.2$.
The diameter of the sun is approximately 1.4×10^6 km, or 1.4 million kilometres.

6. Distance = speed × time
Since the speed of light is measured in kilometres per second, convert each time to seconds. For the signals to go from Earth to Venus, and back, in the first diagram:

$$\text{Distance} = 3 \times 10^5 \times 5.90 \times 60$$
$$= 1.062 \times 10^8$$

The distance from Earth to Venus is one-half of this: $\frac{1.062 \times 10^8 \text{ km}}{2}$, or 5.31×10^7 km, which is the value of s_1 in exercise 1.
For the signals to go from Earth to Venus, and back, in the second diagram:

$$\text{Distance} = 3 \times 10^5 \times 23.36 \times 60$$
$$= 4.2048 \times 10^8$$

The distance from Earth to Venus is one-half of this: $\frac{4.2048 \times 10^8 \text{ km}}{2}$, or 2.1024×10^8 km, which is the value of s_2 in exercise 2.

7. When Earth travels in a circular orbit around the sun, the distance from Earth to the sun is the radius of the orbit. The distance that Earth travels around the sun in one year is the circumference of the orbit.

a)
$$\text{Circumference} = \pi d$$
$$= 2\pi r$$
$$= 2\pi v$$
$$= 2\pi(1.497 \times 10^8)$$
$$\doteq 9.406 \times 10^8$$

Earth travels approximately 9.4×10^8 km in one year.

b) $\text{Speed} = \frac{\text{distance}}{\text{time}}$
Convert 1 year to hours.

$$\text{Speed} = \frac{9.406 \times 10^8}{1 \times 365 \times 24}$$
$$\doteq 1.073 \times 10^5$$

Earth's speed is approximately 1.1×10^5 km/h.

8. a) Answers may vary. To consider the orbits as circles has little effect on the results.

b) We used the fact that the orbits are circles when we assumed that *e* and *v* are the same in both diagrams on page 522. We also used this fact in exercise 7.

Selected Solutions — Chapter 8

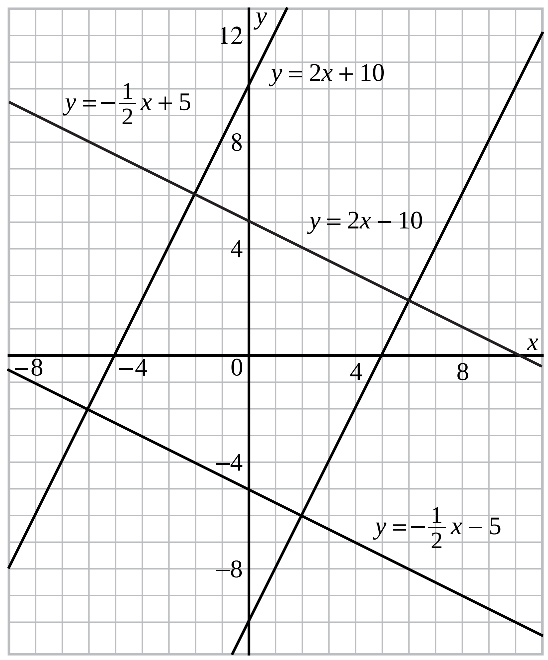
8 Cumulative Review, page 526

3. a) The zero exponent is defined only for bases not equal to zero. That is, 0^0 is undefined.

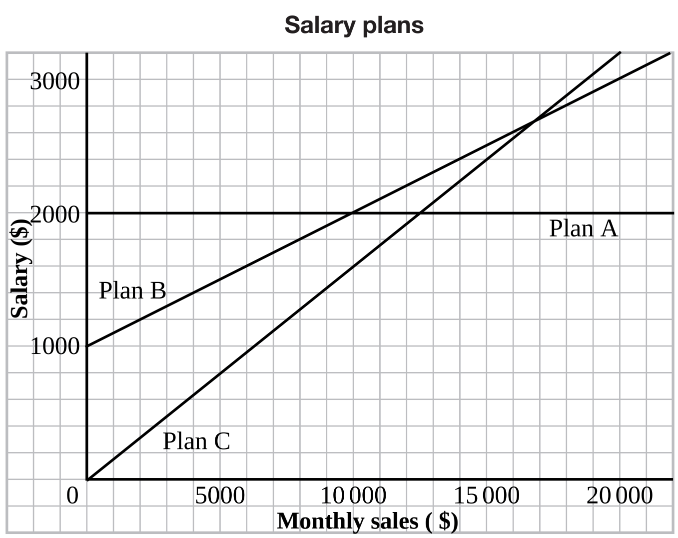
b) The square root of a negative number is undefined.

c) The fourth root of a negative number is undefined.

4.



6. a)



b) Explanations may vary. From the graph:
If monthly sales are less than \$10 000, plan A is best.
For monthly sales between \$10 000 and \$17 000, plan B is best.
For monthly sales greater than \$17 000, plan C is best.