Selected Solutions — Chapter 7

7.1 Exercises, page 402

1. d) $\frac{\sqrt{a}}{3}$ is not a rational expression because there is the square root of a variable in the numerator.

g) $\frac{x}{\sqrt{x}}$ is not a rational expression because there is the square root of a variable in the denominator.

h) $\frac{\sqrt{3x}}{2}$ is not a rational expression because there is the square root of a variable in the numerator.

2. b) $\frac{\sqrt{x}}{x-2}$ is not a rational expression because there is the square root of a variable in the numerator.

e) $\frac{x-\sqrt{x}}{3}$ is not a rational expression because there is the square root of a variable in the numerator.

f) $\frac{y}{\sqrt{2x}}$ is not a rational expression because there is the square root of a variable in the denominator.

k) $\frac{3\sqrt{a}+1}{a+5b}$ is not a rational expression because there is the square root of a variable in the numerator.

3. A rational number written in fraction form, such as $\frac{3}{5}$, is a rational expression because both the numerator, 3, and denominator, 5, are polynomials. A rational number is a special case of a rational expression.

7. For part g: The rational expression $\frac{x^3+1}{y-3}$ cannot be evaluated for $y = 3$ because that would produce a denominator 0.

For part i: The rational expression $\frac{x^2-9}{x^2-x-6}$ can be written as $\frac{x^2-9}{(x-3)(x+2)}$. This cannot be evaluated for $x = -2$ because that would produce a denominator 0.

10. c) When $x = 5$, the denominator is 0, and the expression is not defined. When $x = -4$, the denominator is not 0.

12. Answers may vary. For part h: To find the values of x for which the expression, $\frac{5x-4}{x^3+1}$, is not defined, equate the denominator to 0, and solve for x.

$$x^3 + 1 = 0$$
$$x^3 = -1$$
$$x = \sqrt[3]{-1}$$
$$x = -1$$

The denominator is 0 if $x = -1$. Hence, the expression is not defined when $x = -1$.

13. Since x does not occur in the denominator, there is no value of x that will make the denominator 0.

Selected Solutions — Chapter 7

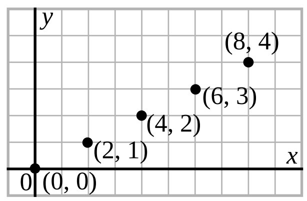
14. a) Since there is no variable in the denominator, there is no restriction on the variable.

d) Since the denominator contains an x^2 term, the denominator is positive for all values of x. That is, there is no value of x for which the denominator is 0.

e) Since the denominator contains an x^2 term, the denominator is positive for all values of x. That is, there is no value of x for which the denominator is 0.

15. Answers may vary. For part e: A rational expression has a restriction on the variable if it is possible that the denominator can be zero. The denominator $x^2 + 5$ can never be zero because x^2 is always positive, as is +5.

17. d)



20. Answers may vary. For part c: For values of $x = 0$ and $x = 4$ not to be permitted, the denominator must be 0 when either value of x is substituted. The denominator is, therefore, $x(x - 4)$. The numerator can be any expression. For example, $\frac{5x - 1}{x(x - 4)}$

Investigate, page 407

3. To write a fraction in lowest terms, divide the numerator and denominator by their greatest common factor.

7.2 Exercises, page 407

5. Answers may vary. For part i: To simplify $\frac{x^2 - 10x + 25}{5 - x}$, I attempted to factor the numerator. I looked for two integers with a sum of −10 and a product of 25. The integers are −5 and −5. The expression becomes $\frac{(x - 5)(x - 5)}{5 - x}$. I recognized that the denominator $5 - x$ was the opposite of $x - 5$, so I wrote $5 - x$ as $-(x - 5)$.

$\frac{(x - 5)(x - 5)}{-(x - 5)}$

If x is not equal to 5, I divide numerator and denominator by $x - 5$.

The expression becomes $\frac{x - 5}{-1}$, or $5 - x$.

The nonpermissible value of x is that which makes the denominator 0 in the original expression. This is $x = 5$.

6. e) Substitute a value for the variable, other than the nonpermissible one, into the simplified and unsimplified expressions. If the results are equal, you have likely simplified the expression correctly.

13. Answers may vary. For part g: To simplify $\frac{2m^2 + 4m - 16}{3m^2 - 48}$, I first attempted to factor the numerator and denominator. The numerator has a common factor 2 and becomes $2(m^2 + 2m - 8)$. I then looked for two integers that add to make 2 and multiply to make –8. The integers are –2 and 4. The numerator is $2(m - 2)(m + 4)$. The denominator has common factor 3 and becomes $3(m^2 - 16)$.
I recognize the binomial as a difference of squares that factors to $3(m - 4)(m + 4)$. The expression is $\frac{2(m-2)(m+4)}{3(m-4)(m+4)}$. The nonpermissible values of m are 4 and –4. If $m \neq -4$, divide numerator and denominator by $m + 4$ to get $\frac{2(m-2)}{3(m-4)}$.

14. **g)** The expression cannot be simplified because the numerator and denominator have no common factors.

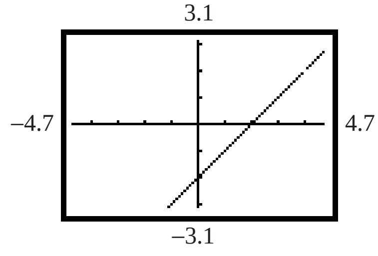
h) The expression cannot be simplified because the numerator and denominator have no common factors.

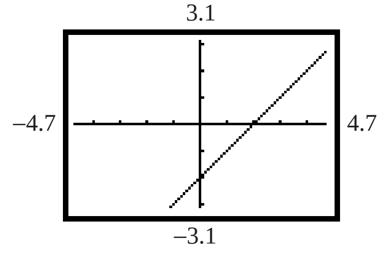
15. Answers may vary. For $x \neq 0$, x must occur as a factor in the denominator, which it does.
For $x \neq 4$, $x - 4$ must occur as a factor in the denominator.
To ensure the expression simplifies to $\frac{x+2}{2x}$, multiply numerator and denominator of this expression by $(x - 4)$ to get the expression $\frac{(x-4)(x+2)}{2x(x-4)}$, which can be written $\frac{x^2 - 2x - 8}{2x^2 - 8x}$.

16. Answers may vary. George is wrong because he cannot divide the numerator by monomials that are not factors of both the numerator and the denominator.
Saleha is correct.
Roberta is wrong because she cannot divide by 2, since it is not a common factor of the terms in the denominator.

Exploring with a Graphing Calculator: Verifying Rational Simplifications, page 410

1. **a)** $y = \frac{x^2 - 6x + 8}{x - 4}$ $\qquad$ $y = x - 2$





$$\frac{x^2 - 6x + 8}{x - 4} = \frac{(x-2)(x-4)}{x-4}$$
$$= x - 2, \text{ if } x \neq 4$$

b) The graph of $y = \frac{x^2 - 6x + 8}{x - 4}$ has a hole at $x = 4$. Otherwise, it is identical to the graph of $y = x - 2$. This confirms that $\frac{x^2 - 6x + 8}{x - 4}$ is equal to $x - 2$, except when $x = 4$.

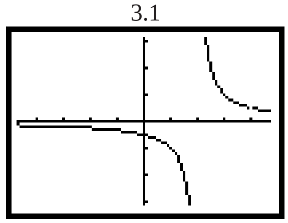
c) For the graph of $y = x - 2$, when you use the trace at X = 4, the display is Y = 2.

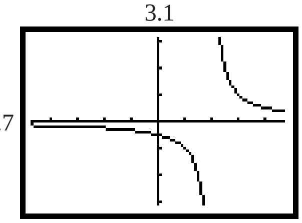
For the graph of $y = \frac{x^2 - 6x + 8}{x - 4}$, when you use the trace at X = 4, the display is Y = ; that is, no value is indicated for y.

d) The hole in the graph of $y = \frac{x^2 - 6x + 8}{x - 4}$ confirms that the value $x = 4$ is not permitted.

e) When we scroll through the table for $y = \frac{x^2 - 6x + 8}{x - 4}$, and get to X = 4, the word ERROR is displayed for the Y-value. When we scroll through the table for $y = x - 2$, and get to X = 4, the Y-value is 2.

2. a) $y = \frac{x - 4}{x^2 - 6x + 8}$ $\qquad$ $y = \frac{1}{x - 2}$





$$\frac{x - 4}{x^2 - 6x + 8} = \frac{x - 4}{(x - 2)(x - 4)}$$
$$= \frac{1}{x - 2}, \text{ if } x \neq 2 \text{ and } x \neq 4$$

b) The graph of $y = \frac{x - 4}{x^2 - 6x + 8}$ has a hole at $x = 4$. Otherwise, it is identical to the graph of $y = \frac{1}{x - 2}$. This confirms that $\frac{x - 4}{x^2 - 6x + 8}$ is equal to $\frac{1}{x - 2}$, except when $x = 4$.

c) For the graph of $y = \frac{x - 4}{x^2 - 6x + 8}$: As you trace to the right from $x = 0$, y increases until you get to X = 1.9, Y = –10, then X = 2, Y = , then X = 2.1, Y = 10. As you continue to the right, y decreases until you get to X = 4, Y = ; then y continues to decrease as x increases.

For the graph of $y = \frac{1}{x - 2}$: As you trace to the right, the values of x and y are as explained above for $y = \frac{x - 4}{x^2 - 6x + 8}$, except for X = 4, Y = 0.5.

d) The hole in the graph of $y = \frac{x - 4}{x^2 - 6x + 8}$ confirms that the value $x = 4$ is not permitted, and the way the graph goes to positive and negative infinity at $x = 2$ confirms that the value $x = 2$ is not permitted.

Selected Solutions — Chapter 7

e) When we scroll through the table for $y = \frac{x-4}{x^2-6x+8}$, and get to X = 2, the word ERROR is displayed for the Y-value.
When we get to X = 4, the word ERROR is displayed for the Y-value.
When we scroll through the table for $y = \frac{1}{x-2}$, and get to X = 2, the word ERROR is displayed for the Y-value.

3. Each graph is the reciprocal of the other graph. Where $y = x - 2$ is large and negative, $y = \frac{1}{x-2}$ is small and negative. Where $y = x - 2$ is small and negative, $y = \frac{1}{x-2}$ is large and negative. Where $y = x - 2$ is large and positive, $y = \frac{1}{x-2}$ is small and positive. Where $y = x - 2$ is small and positive, $y = \frac{1}{x-2}$ is large and positive. When $x = 2$, $y = x - 2$ is 0 and $y = \frac{1}{x-2}$ is undefined.

Investigate, page 411

2. a) The numerator of the first fraction is the square of the denominator of the second fraction. The first fractions have the same denominator. The second fractions have the same numerator.

b) Answers may vary. To simplify $\frac{x^2}{10} \times \frac{7}{x}$; divide numerator and denominator by x to get $\frac{x}{10} \times \frac{7}{1}$, then multiply the numerators, and multiply the denominators to get $\frac{7x}{10}$.

4. a) The denominator of the second fraction is the square of the denominator of the first fraction. Corresponding numerators are the same.

b) Answers may vary. To simplify $\frac{2}{x} \div \frac{5}{x^2}$, write the quotient as the product $\frac{2}{x} \times \frac{x^2}{5}$, then divide numerator and denominator by x to get $\frac{2}{1} \times \frac{x}{5}$, then multiply the numerators and multiply the denominators to get $\frac{2x}{5}$.

7.3 Exercises, page 413

6. Answers may vary. For part e: To simplify
$\frac{-8m^2n^5}{15mn^2} \div \frac{2m^4}{-25n^2}$, I first wrote the quotient as the product
$\frac{-8m^2n^5}{15mn^2} \times \frac{-25n^2}{2m^4}$.
I identified common factors of 2, 5, m^2, and n^2, so I divided the numerators and denominators by these to get $\frac{-4n^3}{3m} \times \frac{-5n^2}{m^2}$. Then I multiplied the numerators and multiplied the denominators to get $\frac{20n^5}{3m^3}$.

The nonpermissible values of the variables are those that would make any denominator 0. These are $m = 0$ and $n = 0$.

8. For each rational expression, I multiplied the numerator and denominator by the same factor, then rewrote the resulting rational expression as a product of two rational expressions.
Answers may vary. For part g: I multiplied $\frac{-1}{abc}$ by $\frac{c(b-1)}{c(b-1)}$ to get $\frac{-c(b-1)}{abc^2(b-1)}$, or $\frac{c(1-b)}{abc^2(b-1)}$. I then wrote this as a product by selecting terms for the numerators and the denominators: $\frac{1-b}{abc^2} \times \frac{c}{b-1}$. I could have written the same expression as different products: $\frac{1-b}{ac^2} \times \frac{c}{b(b-1)}$ or $\frac{1-b}{c^2} \times \frac{c}{ab(b-1)}$, and so on.

9. I used a process similar to that in exercise 8. I multiplied the numerator and denominator of each rational expression by the same factor, then rewrote the resulting rational expression as a product of two rational expressions. I then replaced the second rational expression with its reciprocal, and replaced the multiplication sign with the division sign.
Answers may vary. For part f: I multiplied $\frac{-5}{xy}$ by $\frac{2xy}{2xy}$ to get $\frac{-10xy}{2x^2y^2}$.
I then wrote this as a product by selecting terms for the numerators and the denominators:
$\frac{-5y}{2x^2} \times \frac{2x}{y^2}$. I replaced the second rational expression with its reciprocal and replaced $\times$ with $\div$ to get
$\frac{-5y}{2x^2} \div \frac{y^2}{2x}$.

13. For each rational expression, I multiplied the numerator and denominator by the same factor, then rewrote the resulting rational expression as a product of two rational expressions.
Answers may vary. For part d: I multiplied $\frac{(x+y)^2}{x}$ by $\frac{xy}{xy}$ to get $\frac{(x+y)^2xy}{x^2y}$. I then rewrote this as a product by selecting terms for the numerators and the denominators: $\frac{x(x+y)^2}{xy} \times \frac{y}{x}$.

14. I used a process similar to exercise 8. I multiplied the numerator and denominator of each rational expression by the same factor, then rewrote the resulting rational expression as a product of two rational expressions. I then replaced the second rational expression with its reciprocal, and replaced the multiplication sign with the division sign.
Answers may vary. For part c: I multiplied $\frac{a}{a+b}$ by $\frac{4b^2}{4b^2}$ to get $\frac{4ab^2}{4b^2(a+b)}$. I then wrote this as a product by selecting terms for the numerators and the denominators: $\frac{2ab^2}{4(a+b)} \times \frac{2}{b^2}$. I replaced the second rational expression with its reciprocal and replaced $\times$ with $\div$ to get $\frac{2ab^2}{4(a+b)} \div \frac{b^2}{2}$.

Selected Solutions — Chapter 7

16. a) $\dfrac{x^2-16y^2}{6x^2y} \div \dfrac{x^2+xy-20y^2}{4x^3y^2} = \dfrac{x^2-16y^2}{6x^2y} \times \dfrac{4x^3y^2}{x^2+xy-20y^2}$

$= \dfrac{(x-4y)(x+4y)}{6x^2y} \times \dfrac{4x^3y^2}{(x+5y)(x-4y)}$

$= \dfrac{2xy(x+4y)}{3(x+5y)}$

b) $\dfrac{a^2+11ab+30b^2}{a^2-25b^2} \times \dfrac{3a^2-15ab}{6a^2+36ab} = \dfrac{(a+6b)(a+5b)}{(a+5b)(a-5b)} \times \dfrac{3a(a-5b)}{6a(a+6b)}$

$= \dfrac{1}{2}$

c) $\dfrac{x^2+5xy+6y^2}{x^2+4xy-5y^2} \times \dfrac{x^2+3xy-10y^2}{x^2+xy-6y^2} = \dfrac{(x+2y)(x+3y)}{(x+5y)(x-y)} \times \dfrac{(x+5y)(x-2y)}{(x+3y)(x-2y)}$

$= \dfrac{x+2y}{x-y}$

d) $\dfrac{m^2-9mn+14n^2}{m^2+7mn+12n^2} \div \dfrac{3m^2-21mn}{4m^3+16m^2n} = \dfrac{m^2-9mn+14n^2}{m^2+7mn+12n^2} \times \dfrac{4m^3+16m^2n}{3m^2-21mn}$

$= \dfrac{(m-2n)(m-7n)}{(m+3n)(m+4n)} \times \dfrac{4m^2(m+4n)}{3m(m-7n)}$

$= \dfrac{4m(m-2n)}{3(m+3n)}$

17. a) $\dfrac{3x^2+3x-6}{x^2y-7xy} \times \dfrac{x^2y-13xy+42y}{6x^2+12x} = \dfrac{3(x^2+x-2)}{xy(x-7)} \times \dfrac{y(x^2-13x+42)}{6x(x+2)}$

$= \dfrac{3(x+2)(x-1)}{xy(x-7)} \times \dfrac{y(x-6)(x-7)}{6x(x+2)}$

$= \dfrac{(x-1)(x-6)}{2x^2}$

b) $\dfrac{x^2+5xy+6y^2}{x^2+7xy+10y^2} \times \dfrac{x^2+6xy+5y^2}{x^2+2xy-3y^2} = \dfrac{(x+2y)(x+3y)}{(x+2y)(x+5y)} \times \dfrac{(x+5y)(x+y)}{(x+3y)(x-y)}$

$= \dfrac{x+y}{x-y}$

c) $\dfrac{x+2y}{x-3y} \times \dfrac{x^2-9y^2}{x^2-4y^2} \div \dfrac{x+3y}{x-2y} = \dfrac{x+2y}{x-3y} \times \dfrac{(x-3y)(x+3y)}{(x-2y)(x+2y)} \times \dfrac{x-2y}{x+3y}$

$= 1$

d) $\dfrac{(3a+7b)^2}{2a-5b} \times \dfrac{4a^2-25b^2}{9a^2-49b^2} \div \dfrac{2a+5b}{3a-7b}$

$= \dfrac{(3a+7b)(3a+7b)}{2a-5b} \times \dfrac{(2a-5b)(2a+5b)}{(3a-7b)(3a+7b))} \times \dfrac{3a-7b}{2a+5b}$

$= 3a+7b$

18. Simplify each expression before substituting.

a) $\dfrac{x^2-xy-12y^2}{x^2-2xy-3y^2} \times \dfrac{x^2+5xy+4y^2}{x^2-16y^2} = \dfrac{(x-4y)(x+3y)}{(x+y)(x-3y)} \times \dfrac{(x+4y)(x+y)}{(x+4y)(x-4y)}$

$= \dfrac{x+3y}{x-3y}$

Substitute $x = a+b$ and $y = a-b$.

$= \dfrac{a+b+3(a-b)}{a+b-3(a-b)}$

$= \dfrac{4a-2b}{-2a+4b}$

$= \dfrac{2(2a-b)}{2(2b-a)}$

$= \dfrac{2a-b}{2b-a}$

Selected Solutions — Chapter 7

b) $\left(\frac{3x-21y}{6x+12y}\right)^2 \div \frac{x^2-49y^2}{2x^2+8xy+8y^2} = \left(\frac{3(x-7y)}{6(x+2y)}\right)^2 \times \frac{2(x^2+4xy+4y^2)}{(x-7y)(x+7y)}$

$$= \frac{3(x-7y)3(x-7y)}{6(x+2y)6(x+2y)} \times \frac{2(x+2y)(x+2y)}{(x-7y)(x+7y)}$$

$$= \frac{x-7y}{2(x+7y)}$$

Substitute $x = a + b$ and $y = a - b$.

$$= \frac{a+b-7(a-b)}{2[a+b+7(a-b)]}$$

$$= \frac{-6a+8b}{2[8a-6b]}$$

$$= \frac{2(-3a+4b)}{2[8a-6b]}$$

$$= \frac{4b-3a}{8a-6b}$$

Mathematical Modelling:
Should Pop Cans Be Redesigned?, page 416

1. a) $355 = \pi r^2 h$

Divide each side by πr^2.

$\frac{355}{\pi r^2} = h$

b) $A = 2\pi r^2 + 2\pi rh$

Substitute $h = \frac{355}{\pi r^2}$.

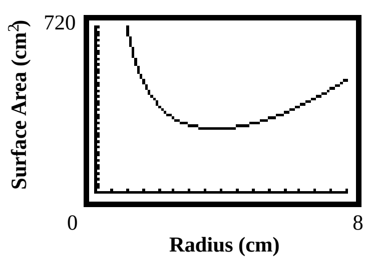
$A = 2\pi r^2 + 2\pi r\left(\frac{355}{\pi r^2}\right)$

$A = 2\pi r^2 + \frac{710}{r}$

3. a) From the table, the least surface area occurs when the radius is 4.0 cm and the height is about 7.1 cm.

b) From the table, the least surface area is about 278 cm^2.

4. a) $y = 2\pi x^2 + \frac{710}{x}$



b) From the graph, the radius is 3.84 cm for minimum surface area.

c) Substitute $r = 3.84$ in the formula $h = \frac{355}{\pi r^2}$ to get

$h = \frac{355}{\pi(3.84)^2}$

$\doteq 7.66$

For minimum surface area, the height is about 7.66 cm.

5. The height is twice the radius, which is equal to the diameter. When the surface area is a minimum, the diameter and the height must be equal.

Selected Solutions — Chapter 7

6. a) Use the formula for the volume of a cylinder,
$V = \pi r^2 h$.
Substitute $r = 3.2$ and $h = 11.0$.
$V = \pi(3.2)^2(11.0)$
$\doteq 354$
The volume is about 354 cm^3. Since 1 mL = 1 cm^3, the volume is about 354 mL.

b) Use the formula from exercise 1b,
$A = 2\pi r^2 + \frac{710}{r}$.
Substitute $r = 3.2$ and $h = 11.0$.
$A = 2\pi(3.2)^2 + \frac{710}{3.2}$
$\doteq 286.2$
The surface area is about 286 cm^2.

c) The difference between the surface area of the can and the minimum surface area is about
$286\ cm^2 - 278\ cm^2 = 8\ cm^2$

d) $\frac{\text{Difference in surface areas}}{\text{Surface area of can}} = \frac{8}{286}$
As a percent, this is $\frac{8}{286} \times 100\% \doteq 2.8\%$.

e) From page 398, each 1% decrease in the surface area of the can reduces the cost by $20 million. A 2.8% decrease reduces the cost by 2.8 × $20 million = $56 million.

7. Answers may vary. The present can is very well known. If it were to change shape, the soft drink manufacturers would have to promote the new can, and advertising is expensive. Also, to set up the production lines to produce the new can would be expensive.

8. When we consider that the top and bottom of the can are thicker, this does not appreciably affect the volume and surface area of the can. It does affect the amount of aluminum used. If the radius were changed from 3.2 cm to 3.8 cm, more aluminum would be used for the top and bottom of the can. So, this might negate the savings in aluminum if the can were shaped for minimum surface area.

Investigate, page 418

2. a) The denominator of the second fraction is $\frac{2}{3}$ the denominator of the first fraction. Corresponding numerators are the same.

b) Answers may vary. To calculate $\frac{5}{3a} + \frac{1}{2a}$, I identified the common denominator, which is $6a$; that is, the smallest monomial that has $3a$ and $2a$ as factors. I then multiplied each denominator by the monomial that produces a common denominator of $6a$. I multiplied the numerator by the same monomial:
$\frac{5}{3a} \times \frac{2}{2} + \frac{1}{2a} \times \frac{3}{3} = \frac{10}{6a} + \frac{3}{6a}$
Since the denominators are the same, I added the numerators to get $\frac{13}{6a}$.

Selected Solutions — Chapter 7

4. a) The denominator of the second fraction is the square of the denominator of the first fraction. Corresponding numerators are the same.

b) Answers may vary. To calculate $\frac{2}{a} - \frac{1}{a^2}$, I identified the common denominator, which is a^2. I multiplied the denominator of the first fraction by a to get the common denominator a^2. I multiplied the numerator by the same monomial: $\frac{2}{a} \times \frac{a}{a} - \frac{1}{a^2} = \frac{2a}{a^2} - \frac{1}{a^2}$
Since the denominators are the same, I subtracted the numerators to get $\frac{2a - 1}{a^2}$.

7.4 Exercises, page 412

8. Answers may vary. For part h: To simplify $\frac{6}{7b} - \frac{5}{8b}$, I identified the common denominator as $56b$, since both $7b$ and $8b$ are factors. I multiplied $\frac{6}{7b}$ by $\frac{8}{8}$ and $\frac{5}{8b}$ by $\frac{7}{7}$ to get $\frac{48}{56b} - \frac{35}{56b}$. Then I subtracted the numerators to get $\frac{13}{56b}$. The nonpermissible value of b is the value that makes the numerators zero; that is, $b = 0$.

9. Answers may vary. For part a: I found two rational expressions such that when one is added to the other, the result is $\frac{2}{a}$. I chose both denominators as a, then I found two numbers that add to make 2. These are 1 and 1, the numerators: $\frac{1}{a} + \frac{1}{a} = \frac{2}{a}$

10. Answers may vary. For part a: I found two rational expressions such that when one is subtracted from the other, the result is $\frac{2}{a}$. If I choose both denominators as a, then I find two numbers that differ by 2. These are 3 and 1, the numerators: $\frac{3}{a} - \frac{1}{a} = \frac{2}{a}$

15. Answers may vary. For part e: To simplify $\frac{2a+3}{8} - \frac{5a-4}{6}$, I found the lowest number that is divisible by 8 and 6. This is the lowest common denominator, 24. I multiplied $\frac{2a+3}{8}$ by $\frac{3}{3}$ and $\frac{5a-4}{6}$ by $\frac{4}{4}$ to get $\frac{6a+9}{24} - \frac{20a-16}{24}$. I then subtracted the numerators to get $\frac{6a+9-(20a-16)}{24} = \frac{6a+9-20a+16}{24}$, which simplifies to $\frac{25-14a}{24}$.

22. a) $\left(a - \frac{1}{a}\right)\left(a - \frac{2}{a}\right)$

Expand.

$$\begin{aligned} a^2 - a\left(\frac{2}{a}\right) - \frac{1}{a}(a) + \left(\frac{1}{a}\right)\left(\frac{2}{a}\right) &= a^2 - 2 - 1 + \frac{2}{a^2} \\ &= a^2 - 3 + \frac{2}{a^2} \end{aligned}$$

To write this expression with a common denominator a^2:

$$\begin{aligned} a^2 - 3 + \frac{2}{a^2} &= \frac{a^4}{a^2} - \frac{3a^2}{a^2} + \frac{2}{a^2} \\ &= \frac{a^4 - 3a^2 + 2}{a^2} \end{aligned}$$

Selected Solutions — Chapter 7

b) $\left(k + \frac{3}{k}\right)\left(k - \frac{5}{k}\right)$

A method that is alternative to that used in part a is to write each binomial with a common denominator before expanding. In each binomial, multiply k by $\frac{k}{k}$.

$$\begin{aligned}\left(\frac{k^2}{k} + \frac{3}{k}\right)\left(\frac{k^2}{k} - \frac{5}{k}\right) &= \left(\frac{k^2+3}{k}\right)\left(\frac{k^2-5}{k}\right)\\ &= \frac{(k^2+3)(k^2-5)}{k^2}\\ &= \frac{k^4 - 2k^2 - 15}{k^2}\end{aligned}$$

c) $\left(2a - \frac{3}{a}\right)^2$

Write the binomial with a common denominator a.

$$\left(\frac{2a^2}{a} - \frac{3}{a}\right)^2 = \frac{(2a^2-3)^2}{a^2} = \frac{4a^4 - 12a^2 + 9}{a^2}$$

23. a)
$$\begin{aligned}\left(x + \frac{1}{x}\right)^2 - \left(x^2 + \frac{1}{x^2}\right) &= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right) - x^2 - \frac{1}{x^2}\\ &= x^2 + 1 + 1 + \frac{1}{x^2} - x^2 - \frac{1}{x^2}\\ &= 2\end{aligned}$$

b) From part a, we know $\left(x + \frac{1}{x}\right)^2 - \left(x^2 + \frac{1}{x^2}\right) = 2$, which is independent of x.

Substitute 3 for $x + \frac{1}{x}$.

Then, $(3)^2 - \left(x^2 + \frac{1}{x^2}\right) = 2$

$$\begin{aligned}9 - \left(x^2 + \frac{1}{x^2}\right) &= 2\\ 9 - 2 &= x^2 + \frac{1}{x^2}\\ 7 &= x^2 + \frac{1}{x^2}\end{aligned}$$

24. a) Since $\frac{x}{y} = \frac{3}{2}$, then $\frac{y}{x} = \frac{2}{3}$

b) Write the rational expression as the sum of two rational expressions.

$$\begin{aligned}\frac{x+y}{y} &= \frac{x}{y} + \frac{y}{y}\\ &= \frac{x}{y} + 1\end{aligned}$$

Substitute $\frac{x}{y} = \frac{3}{2}$.

$$\begin{aligned}\frac{x}{y} + 1 &= \frac{3}{2} + 1\\ &= \frac{5}{2}\end{aligned}$$

c) Write the rational expression as the sum of two rational expressions.

$$\begin{aligned}\frac{x+y}{x} &= \frac{x}{x} + \frac{y}{x}\\ &= 1 + \frac{y}{x}\end{aligned}$$

Substitute $\frac{y}{x} = \frac{2}{3}$.

$$\begin{aligned}1 + \frac{y}{x} &= 1 + \frac{2}{3}\\ &= \frac{5}{3}\end{aligned}$$

Selected Solutions — Chapter 7

d) $\frac{x+y}{x-y}$

Divide each term by y.

$$\frac{\frac{x}{y}+\frac{y}{y}}{\frac{x}{y}-\frac{y}{y}} = \frac{\frac{x}{y}+1}{\frac{x}{y}-1}$$

Substitute $\frac{x}{y} = \frac{3}{2}$.

$$\frac{\frac{x}{y}+1}{\frac{x}{y}-1} = \frac{\frac{3}{2}+1}{\frac{3}{2}-1} = \frac{\frac{5}{2}}{\frac{1}{2}} = 5$$

25. a) $$\left(x+\frac{1}{x}\right)\left(x+\frac{2}{x}\right) = x^2+2+1+\frac{2}{x^2} = x^2+3+\frac{2}{x^2} = \frac{x^4+3x^2+2}{x^2}$$

b) $$\left(x+\frac{1}{x}\right)\left(x+\frac{2}{x}\right) = \left(\frac{x^2+1}{x}\right)\left(\frac{x^2+2}{x}\right) = \frac{x^4+3x^2+2}{x^2}$$

c) Yes; the order of operations is different, but the end result is the same.

26. $\frac{\frac{1}{x}+2}{\frac{1}{x}-3}$ is not defined for $x = 0$.

$\frac{1+2x}{1-3x}$ is not defined for $x = \frac{1}{3}$.

The statement is not necessarily true for a rational expression that is a simplification of another rational expression. However, we would have to know if the rational expression had been simplified.

Linking Ideas: Mathematics and Science
Exploring the Lens Formula, page 424

1. a) $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

Substitute $f = 40$, $p = 120$, and $q = 60$.

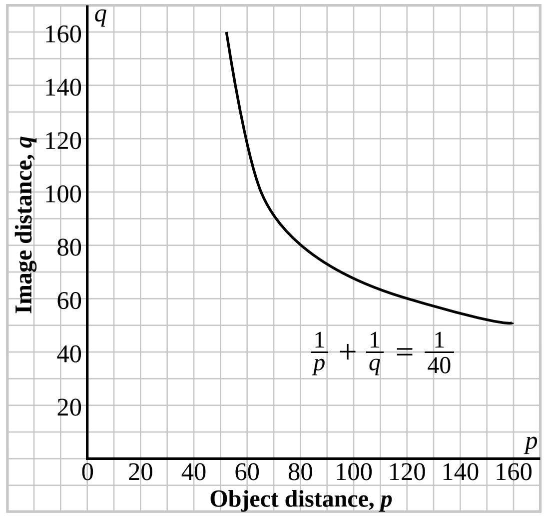
L.S. $= \frac{1}{40}$

R.S. $= \frac{1}{120} + \frac{1}{60} = \frac{1}{120} + \frac{2}{120} = \frac{3}{120} = \frac{1}{40}$

Since L.S. = R.S., the equation is satisfied.

Selected Solutions — Chapter 7

c)



3. a) The graph shows that the image distance approaches 40 mm when the object distance is very great.

b) The graph shows that the image distance approaches infinity as the object distance approaches 40 mm.

c) When $p = 40$, $\frac{1}{40} = \frac{1}{40} + \frac{1}{q}$

$\frac{1}{q} = 0$, so q is infinite.

The image is at infinity.

When $p < 40$ (for example, 30),

$\frac{1}{40} = \frac{1}{30} + \frac{1}{q}$

$\frac{1}{q} = -\frac{1}{120}$

The image is on the same side of the lens as the object. On the graph, the point representing this situation would lie in the fourth quadrant.

4. $\frac{1}{40} = \frac{1}{60} + \frac{1}{q}$

$\frac{1}{q} = \frac{1}{120}$

$q = 120$

The image distance is 120 mm.

5. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

Substitute $f = 40$.

$\frac{1}{40} = \frac{1}{p} + \frac{1}{q}$

Solve for q.

$\frac{1}{q} = \frac{1}{40} - \frac{1}{p}$

$\frac{1}{q} = \frac{p - 40}{40p}$

$q = \frac{40p}{p - 40}$

Answers will vary, depending on the values chosen for p.

The table shows some values of p from 41 mm to 80 mm.

Selected Solutions — Chapter 7

p (mm)	q (mm)
41	1640
42	840
44	440
45	360
48	240
50	200
56	140
60	120
65	104
72	90
80	80

7.5 Exercises, page 428

5. Answers may vary. For part e: To simplify $\frac{5x}{x-1} - \frac{2x}{x+3}$, I multiplied the numerator and denominator of each expression by the denominator of the other expression. In this way, I obtained each expression with the same denominator.

$\frac{5x}{x-1} \times \frac{x+3}{x+3} - \frac{2x}{x+3} \times \frac{x-1}{x-1}$

Then I multiplied to simplify each numerator.

$\frac{5x^2 + 15x}{(x-1)(x+3)} - \frac{2x^2 - 2x}{(x+3)(x-1)}$

Then I wrote the numerators together over the common denominator, remembering to change the signs of the terms in the second numerator because they have been multiplied by −1.

$\frac{5x^2 + 15x - 2x^2 + 2x}{(x-1)(x+3)}$

Then I collected like terms in the numerator.

$\frac{3x^2 + 17x}{(x-1)(x+3)}$

9. Answers may vary. For part a: I found two rational expressions such that when one is added to the other, the result is $\frac{2}{a+1}$. I chose both denominators as $a + 1$. Then I found two expressions that add to 2; I used these as the numerators. They are $2 - a$ and a.

$\frac{2-a}{a+1} + \frac{a}{a+1}$

10. Answers may vary. For part f: I found two rational expressions such that when one is subtracted from the other, the result is $\frac{3a}{4b}$. I chose both denominators as $4b$. Then I found two expressions that subtract to make $3a$. I used these as the numerators. They are $6a$ and $3a$. The expressions are $\frac{6a}{4b} - \frac{3a}{4b}$; the first expression simplifies: $\frac{3a}{2b} - \frac{3a}{4b}$

Selected Solutions — Chapter 7

15. Answers may vary. For part a: To simplify

$\frac{4x}{x^2 - 9x + 18} + \frac{2x - 1}{x - 6}$,

factor each denominator.

$x^2 - 9x + 18 = (x - 3)(x - 6)$

The second denominator does not factor.

$\frac{4x}{(x - 3)(x - 6)} + \frac{2x - 1}{x - 6}$

The common denominator is the simplest expression into which each denominator divides. The common denominator is $(x - 3)(x - 6)$. The first expression has the common denominator as its denominator. Multiply the denominator of the second expression by $(x - 3)$ to match the first denominator. Multiply the numerator of the second expression by $x - 3$, so we do not change the value of the second expression.

$\frac{4x}{(x - 3)(x - 6)} + \frac{(2x - 1)(x - 3)}{(x - 6)(x - 3)}$

Expand the numerator of the second expression.

$\frac{4x}{(x - 3)(x - 6)} + \frac{2x^2 - 7x + 3}{(x - 6)(x - 3)}$

Since the denominators are the same, combine the numerators over the same denominator.

$\frac{4x + 2x^2 - 7x + 3}{(x - 3)(x - 6)}$

Collect like terms in the numerator.

$\frac{2x^2 - 3x + 3}{(x - 3)(x - 6)}$

18. Answers may vary.

a) $\frac{2x - 4}{x(x - 4)}$

Write $2x$ as $x + x$.

$\frac{x + x - 4}{x(x - 4)}$

Separate the numerator into 2 parts: x and $x - 4$. Write each part over the denominator.

$\frac{x}{x(x - 4)} + \frac{x - 4}{x(x - 4)}$

Divide the numerator and denominator of each rational expression by their common factors.

$\frac{1}{x - 4} + \frac{1}{x}$

b) $\frac{2x}{x^2 - y^2}$

Factor the denominator.

$\frac{2x}{(x - y)(x + y)}$

Add $y - y$ to the numerator: $2x + y - y$. Then write the numerator as $x + y + x - y$.

$\frac{x + y + x - y}{(x - y)(x + y)}$

Separate the numerator into 2 parts: $x + y$ and $x - y$. Write each part over the denominator.

$$\frac{x+y}{(x-y)(x+y)} + \frac{x-y}{(x-y)(x+y)}$$

Divide the numerators and denominators by their common factors.

$$\frac{1}{x-y} + \frac{1}{x+y}$$

c) $\frac{3x+11}{(x+3)(x+4)}$

Write $3x + 11$ as $x + 3 + 2x + 8$.

$$\frac{x+3+2x+8}{(x+3)(x+4)}$$

Separate the numerator into 2 parts: $x + 3$ and $2x + 8$. Write each part over the denominator.

$$\frac{x+3}{(x+3)(x+4)} + \frac{2x+8}{(x+3)(x+4)}$$

Factor the second denominator.

$$\frac{x+3}{(x+3)(x+4)} + \frac{2(x+4)}{(x+3)(x+4)}$$

Divide the numerators and denominators by their common factors.

$$\frac{1}{x+4} + \frac{2}{x+3}$$

19. a)

$$\begin{aligned}\frac{3x^2+6xy}{3x} - \frac{4y^2-2xy}{2y} &= \frac{3x(x+2y)}{3x} - \frac{2y(2y-x)}{2y}\\ &= x + 2y - (2y - x)\\ &= 2x\end{aligned}$$

b)

$$\begin{aligned}\frac{x^2-5xy+6y^2}{x-3y} - \frac{x^2-xy-12y^2}{x-4y} &= \frac{(x-2y)(x-3y)}{(x-3y)} - \frac{(x-4y)(x+3y)}{(x-4y)}\\ &= x - 2y - (x + 3y)\\ &= -5y\end{aligned}$$

c)

$$\begin{aligned}\frac{x^2-4xy-21y^2}{3x-21y} + \frac{x^2+2xy-24y^2}{2x+2y} &= \frac{(x-7y)(x+3y)}{3(x-7y)} + \frac{(x+6y)(x-4y)}{2(x+y)}\\ &= \frac{x+3y}{3} + \frac{(x+6y)(x-4y)}{2(x+y)}\\ &= \frac{(x+3y)(2)(x+y)}{3(2)(x+y)} + \frac{3(x+6y)(x-4y)}{3(2)(x+y)}\\ &= \frac{2(x+3y)(x+y)+3(x+6y)(x-4y)}{6(x+y)}\\ &= \frac{2(x^2+4xy+3y^2)+3(x^2+2xy-24y^2)}{6(x+y)}\\ &= \frac{2x^2+8xy+6y^2+3x^2+6xy-72y^2}{6(x+y)}\\ &= \frac{5x^2+14xy-66y^2}{6(x+y)}\end{aligned}$$

d)

$$\begin{aligned}\frac{a-b}{a^2+2ab-3b^2} + \frac{a+b}{a^2-2ab-3b^2} &= \frac{a-b}{(a+3b)(a-b)} + \frac{a+b}{(a-3b)(a+b)}\\ &= \frac{1}{a+3b} + \frac{1}{a-3b}\\ &= \frac{a-3b+a+3b}{(a+3b)(a-3b)}\\ &= \frac{2a}{(a+3b)(a-3b)}\end{aligned}$$

Selected Solutions — Chapter 7

20. Predictions may vary.

a) Subtract 1 from each denominator.

$$\frac{1}{x-1}+\frac{1}{x}=\frac{x+x-1}{x(x-1)}$$
$$=\frac{2x-1}{x(x-1)}$$
$$\frac{1}{x-2}+\frac{1}{x-1}=\frac{x-1+x-2}{(x-2)(x-1)}$$
$$=\frac{2x-3}{(x-2)(x-1)}$$
$$\frac{1}{x-3}+\frac{1}{x-2}=\frac{x-2+x-3}{(x-3)(x-2)}$$
$$=\frac{2x-5}{(x-3)(x-2)}$$

b) Subtract 1 from each numerator and each denominator.

$$\frac{0}{x}+\frac{1}{x+1}=\frac{1}{x+1}$$
$$\frac{-1}{x-1}+\frac{0}{x}=\frac{-1}{x-1}$$
$$\frac{-2}{x-2}-\frac{1}{x-1}=\frac{-2x+2-x+2}{(x-2)(x-1)}$$
$$=\frac{-3x+4}{(x-2)(x-1)}$$

c) Add 1 to the second factor in the first denominator.
Subtract 1 from the second factor in the second denominator.

$$\frac{1}{x(x)}+\frac{1}{x(x)}=\frac{2}{x^2}$$
$$\frac{1}{x(x+1)}+\frac{1}{x(x-1)}=\frac{x-1+x+1}{x(x^2-1)}$$
$$=\frac{2x}{x(x^2-1)}$$
$$=\frac{2}{x^2-1}$$
$$\frac{1}{x(x+2)}+\frac{1}{x(x-2)}=\frac{x-2+x+2}{x(x^2-4)}$$
$$=\frac{2x}{x(x^2-4)}$$
$$=\frac{2}{x^2-4}$$

7.6 Exercises, page 434

3. Answers may vary. For part h: For the equation $\frac{x}{5}=-\frac{2}{5}+\frac{3}{x}$, the equation is undefined when $x=0$ because a denominator cannot be zero.
To solve the equation, I multiplied each side of the equation by the common denominator $5x$.

$$5x\left(\frac{x}{5}\right)=5x\left(-\frac{2}{5}\right)+5x\left(\frac{3}{x}\right)$$

I then divided numerator and denominator by their common factor, in each term.

$x^2=-2x+15$

I collected terms on the left side of the equation.

$x^2+2x-15=0$

I factored the left side.

$(x+5)(x-3)=0$

I equated each factor to zero, to find the solution.

$x=-5$ or $x=3$

Selected Solutions — Chapter 7

9. Answers may vary. For part a: The equation is undefined when a denominator is equal to 0. This occurs when $x - 2 = 0,\ x = 2$; or $2x + 1 = 0,\ x = -\frac{1}{2}$.

$$\frac{3}{x-2} = \frac{5}{2x+1}$$

I used the shortcut method.

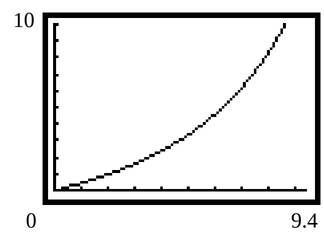
$$3(2x + 1) = 5(x - 2)$$
$$6x + 3 = 5x - 10$$
$$x = -13$$

7.7 Exercises, page 439

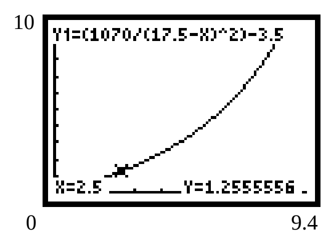
3. The formula is unchanged, but the rational expression $\frac{500}{165 - x}$ now represents the flight out time and $\frac{500}{165 + x}$ represents the return trip time.

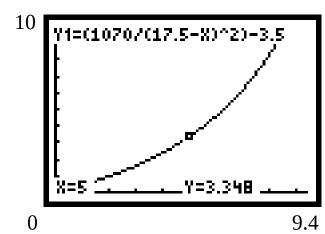
$$t = \frac{500}{165 - x} + \frac{500}{165 + x}$$

8. **a)** $y = \dfrac{1070}{(17.5 - x)^2} - 3.5$



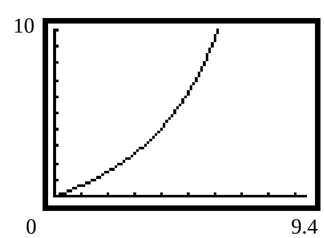
b)





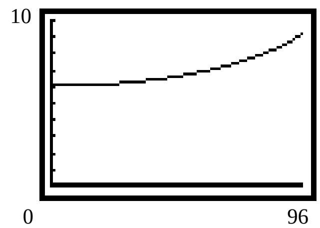
c) Predictions may vary. The graph is similar, but gets steeper more quickly since the function is undefined at $x = 12.4$ instead of at $x = 17.5$.

$$y = \frac{535}{(12.4 - x)^2} - 3.48$$

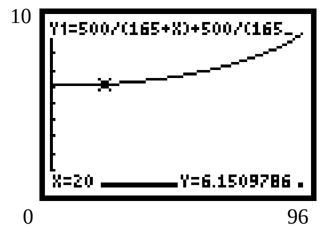


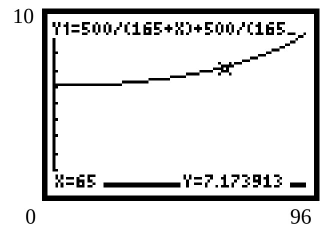
Selected Solutions — Chapter 7

12. a) $y = \frac{500}{165 + x} + \frac{500}{165 - x}$



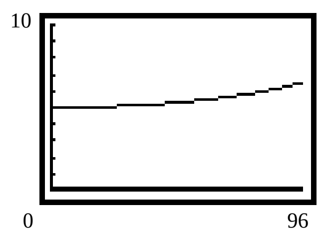
b)





c) Predictions may vary. The *y*-intercept is smaller and the graph does not rise as quickly.

$y = \frac{500}{200 + x} + \frac{500}{200 - x}$



Problem Solving:
Exploring Averages, page 442

1. Total distance is 24 km.
Total time is $\frac{24 \text{ km}}{12 \text{ km/h}}$, or 2 h
But time to go up the hill is 2 h.

2. Let x represent the downhill speed in kilometres per hour.

	Distance (km)	Speed (km/h)	Time (h)
Uphill	12	6	2
Downhill	12	x	$\frac{12}{x}$

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$12 = \frac{24}{2 + \frac{12}{x}}$$

Use the shortcut method.

$$12\left(2 + \frac{12}{x}\right) = 24$$

Divide by 12.

Selected Solutions — Chapter 7

$$2 + \frac{12}{x} = 2$$
$$\frac{12}{x} = 0$$

This has no solution.

There is no value of x that makes $\frac{12}{x} = 0$.

Problems, page 443

1. a) Mark had 7 tests each with an average of 60%, and 3 tests each with an average of 80%.

$$\begin{aligned}\text{Average of 10 tests} &= \frac{7 \times 60\% + 3 \times 80\%}{10}\\ &= \frac{420\% + 240\%}{10}\\ &= \frac{660\%}{10}\\ &= 66\%\end{aligned}$$

b) Since each average is for a different number of tests, the average for 10 tests is not the mean of the two averages.

c) Yes; to average 70% on all ten tests, the total marks would have to be 70% × 10 = 700%.
The marks for the first seven tests are
60% × 7 = 420%.
So, for the last three tests, the total marks would have to be
700% − 420% = 280%.
To get 280%, Mark would need an average of $\frac{280\%}{3}$, or approximately 94% on each test.

2. a) The time for the trip to Vancouver is $\frac{1800}{737}$ h. The time for the return trip to Winnipeg is $\frac{1800}{937}$ h. The total time for the trip is $\left(\frac{1800}{737} + \frac{1800}{937}\right)$ h. The average speed is the total distance, 3600 km, divided by the total time.

$$\begin{aligned}\text{Average speed} &= \frac{3600}{\frac{1800}{737} + \frac{1800}{937}}\\ &= \frac{2}{\frac{1}{737} + \frac{1}{937}}\end{aligned}$$

Multiply numerator and denominator by their common denominator (937)(737).

$$= \frac{2(937)(737)}{(937 + 737)} \doteq 825.05$$

The average speed is about 825 km/h.

b) Since the time for each trip is different, you can’t calculate the mean of the two speeds to determine the average speed for the trip.

c) The total time is now $\left(\frac{1800}{737} + \frac{1800}{937} + 2\right)$ h.

$$\begin{aligned}\text{Average speed} &= \frac{3600}{\frac{1800}{737} + \frac{1800}{937} + 2}\\ &\doteq 565.7\end{aligned}$$

The average speed is about 566 km/h.

Selected Solutions — Chapter 7

3. a) Time $= \frac{\text{distance}}{\text{speed}}$

$= \frac{12}{15}$

$= 0.8$

It would take 0.8 h, or 0.8 × 60 min, which is 48 min to cycle down the hill.

b) Cycling up the hill took 2 h, and cycling down took 48 min. The total time for the trip is 2 h 48 min, which is $2\frac{48}{60}$ h, or 2.8 h.

c) Average speed $= \frac{\text{distance}}{\text{time}}$

$= \frac{24}{2.8}$

$\doteq 8.57$

The average speed is 8.6 km/h.

4. a) Time $= \frac{\text{distance}}{\text{speed}}$

$= \frac{12}{x}$

It takes $\frac{12}{x}$ hours to go down the hill.

b) Cycling up the hill took 2 h, and cycling down took $\frac{12}{x}$ h. The total time for the trip is $\left(2 + \frac{12}{x}\right)$ hours.

c) Average speed $= \frac{\text{distance}}{\text{time}}$

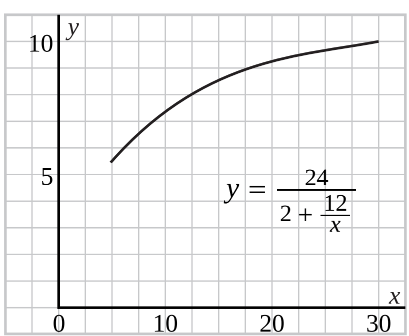
$= \frac{24}{2 + \frac{12}{x}}$

The average speed is $\left(\frac{24}{2 + \frac{12}{x}}\right)$ kilometres per hour.

5. a) Tables may vary.

x	Average speed (km/h)
0	0
5	5.5
10	7.5
15	8.6
20	9.2
25	9.7
30	10

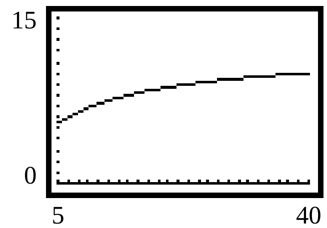
b)



Selected Solutions — Chapter 7

6. Both graphs are the same.

$$y = \frac{24}{2 + \frac{12}{x}}$$



Exploring with a Graphing Calculator: Speed-Time Graphs, page 444

1. b) When $x = 0$, $y = \frac{500}{165}$ for both graphs.

c) As x increases, for the top curve, $y = \frac{500}{165 - x}$ increases because the denominator 165 - x decreases as x increases.
As x increases, for the bottom curve, $y = \frac{500}{165 + x}$ decreases because the denominator $165 + x$ increases as x increases.

d) As x increases, $\frac{500}{165 - x}$ increases faster than $\frac{500}{165 + x}$ decreases.

2. b) When $x = 0$, $y = \frac{1000}{165}$ for this graph, which is double the y-intercept (or the sum of the y-intercepts) for the other two graphs.

c) Its y-values are the sum of the y-values of the other two graphs. Hence, its y-values are always greater than the corresponding y-values of each of the other graphs.

4. The graphs would be similar except the starting points would be (0, 2.5) and (0, 5).

7 Review, page 445

5. Answers may vary. For part d: To simplify $\frac{3x^2y(2x - y)}{15xy} \div \frac{2xy^2(2x - y)}{5x^2y^3}$,
I first wrote the expression as a product:
$$\frac{3x^2y(2x - y)}{15xy} \times \frac{5x^2y^3}{2xy^2(2x - y)}$$
I then divided numerator and denominator by the common factors 3, x^2, y^3, 5, and $2x - y$ to get: $\frac{x^2y}{2}$

9. Answers may vary. For part i: To simplify $\frac{2a + 3}{10a^2} - \frac{7a - 4}{15a^2}$, I found the smallest monomial that $10a^2$ and $15a^2$ divide into; this is $30a^2$. I multiplied the first expression by $\frac{3}{3}$ and the second expression by $\frac{2}{2}$ to get the denominators the same.

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$\frac{2a+3}{10a^2} \times \frac{3}{3} - \frac{7a-4}{15a^2} \times \frac{2}{2}$

I multiplied numerators and multiplied denominators.

$\frac{6a+9}{30a^2} - \frac{14a-8}{30a^2}$

I combined the numerators (remembering to change the sign of the terms in the second numerator) and wrote them over a common denominator.

$\frac{6a+9-14a+8}{30a^2}$

I then collected like terms in the numerator to get:

$\frac{17-8a}{30a^2}$

16. Answers may vary. For part c: To solve $\frac{6}{x-2} = \frac{21}{x^2-4} + 1$, I first identified the common denominator. The denominator $x^2 - 4$ factors as $(x-2)(x+2)$, and this is the common denominator. I also identified nonpermissible values for x; that is, $x - 2 \neq 0$, $x \neq 2$ and $x + 2 \neq 0$, $x \neq -2$. I multiplied each term of the equation by the common denominator.

$\frac{6}{(x-2)} \times (x-2)(x+2)$

$= \frac{21}{(x-2)(x+2)} \times (x-2)(x+2) + 1(x-2)(x+2)$

I then divided each numerator and denominator by their common factors.

$6(x+2) = 21 + (x-2)(x+2)$

I then expanded to remove brackets.

$6x + 12 = 21 + x^2 - 4$

I collected all terms on the right side, so x^2 is positive.

$0 = x^2 - 6x + 5$

I factored the right side.

$0 = (x-5)(x-1)$

I equated each factor to zero.

$x - 5 = 0$ or $x - 1 = 0$

$x = 5$ $\quad$ $x = 1$

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2. b) Answers may vary. The sequences have common difference 1, and their first terms are 0, 1, 2, which is an arithmetic sequence. The general terms are $n - 1$, n, and $n + 1$, which is an arithmetic sequence.

3. b) Answers may vary. For part i: I found the common difference, $d = 7 - 3$, or 4. I used the expression for the general term $t_n = a + (n-1)d$. I substituted $t_n = 71$, $a = 3$, and $d = 4$.

$71 = 3 + (n-1)4$

I solved this equation for n.

$71 = 3 + 4n - 4$

$71 = -1 + 4n$

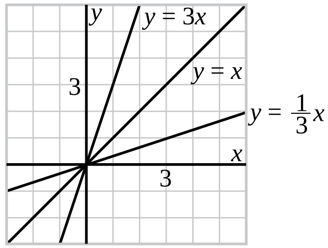
$72 = 4n$

$n = 18$

71 is the 18th term in the sequence.

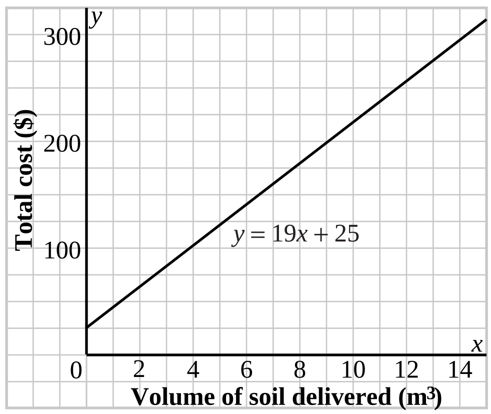
Selected Solutions — Chapter 7

14.

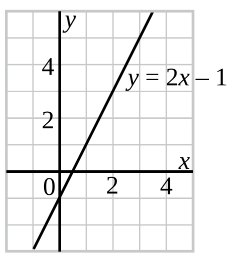


15.

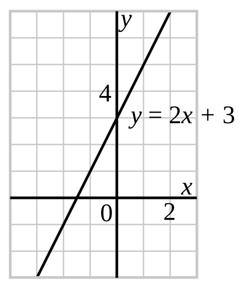
Costs for soil delivery



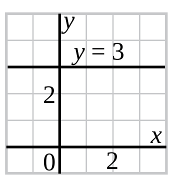
16. a)



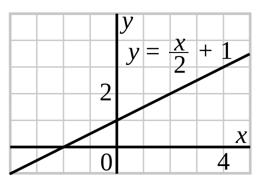
b)



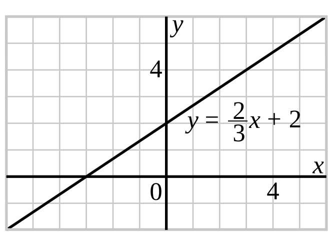
c)



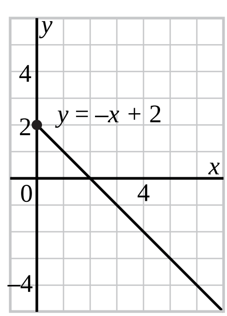
d)



e)



f)



Selected Solutions — Chapter 7

19. **e)** For part b: If the radius is doubled, then the circumference is doubled, and the area is four times as great.
For part d: If the radius is doubled, then the surface area is four times as great and the volume is eight times as great.
These results are valid for all circles and spheres.
If the radius of the circle is multiplied by a factor of x, then the circumference is multiplied by a factor of x, and the area is multiplied by a factor of x^2.
If the radius of a sphere is multiplied by a factor of x, then the surface area is multiplied by a factor of x^2, and the volume is multiplied by a factor of x^3.