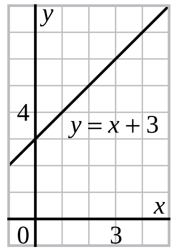
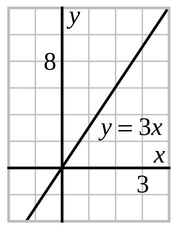
Selected Solutions — Chapter 5

Investigate, page 247

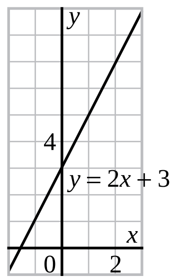
Rule 1. **b)**



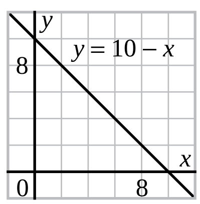
Rule 2. **b)**



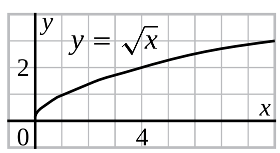
Rule 3. **b)**



Rule 4. **b)**



Rule 5. **b)**



5.1 Exercises, page 250

2. Explanations may vary.

a) False; a function, such as a straight line, may have an infinite number of input numbers. It is impossible to show them all in a table of values.

b) False, some functions are discrete; this means that only certain numbers, usually integers or natural numbers, are valid. In this case, the plotted points are not joined.

3. Explanations may vary.

a) An input number is entered, and only one output number is shown on the display.

Selected Solutions — Chapter 5

b) $y = \sqrt{x}$: x cannot be negative, since the square root of a negative number is not defined.

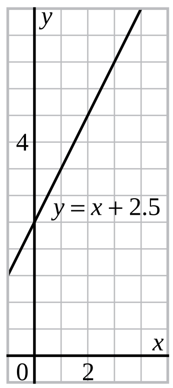
$y = \frac{1}{x}$: x cannot be 0, since division by 0 is not defined.

4. Descriptions may vary.

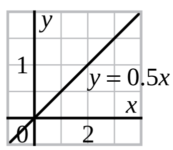
a) All the graphs move up to the right.

b) The points in part i are joined by a straight line. The points in part ii are not joined because you cannot buy part of a ticket. The points in part iii are joined by horizontal line segments. The cost is the same for any time after zero hours to 1 h; then the cost jumps, and is again the same for any time over 1 h up to 2 h, and so on.

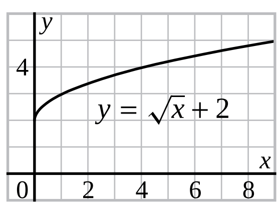
5. b) i)



ii)

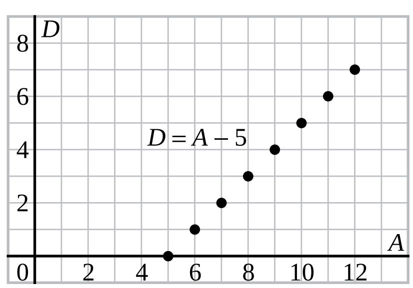


iii)



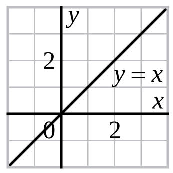
d) Answers may vary. For part iii: Since the square root of a negative number is undefined, negative input numbers cannot be used for $y = \sqrt{x} + 2$.

7. c)

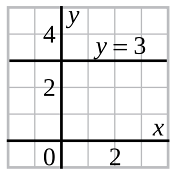


Selected Solutions — Chapter 5

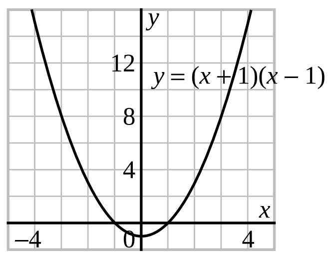
8. a)



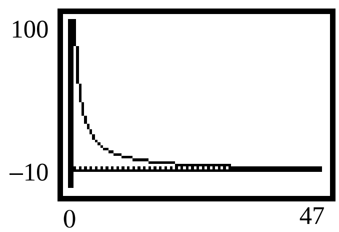
b)



c)



10. a)



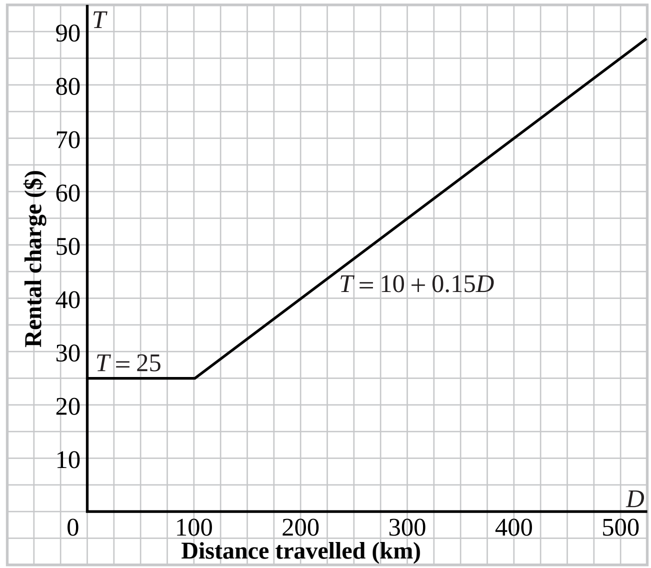
d) The advantages of using the window setting in part a, are that whole number values of X are displayed when tracing, and the values are displayed below the X-axis and do not interfere with the graph.

e) Doubling the protection factor has the greatest effect when *s* is small. If you double the protection factor from 1 to 2, the percent penetration goes from 100 to 50, a decrease of 50. If you double the protection factor from 10 to 20, the percent penetration goes from 10 to 5, a decrease of 5.

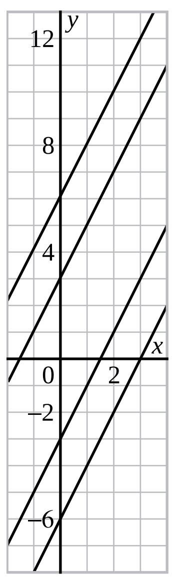
f) The percent penetration decreases as the protection factor increases.

Selected Solutions — Chapter 5

11. b)



12. a)



Rule 2: $y = 2(x + 3)$

Rule 1: $y = 2x + 3$

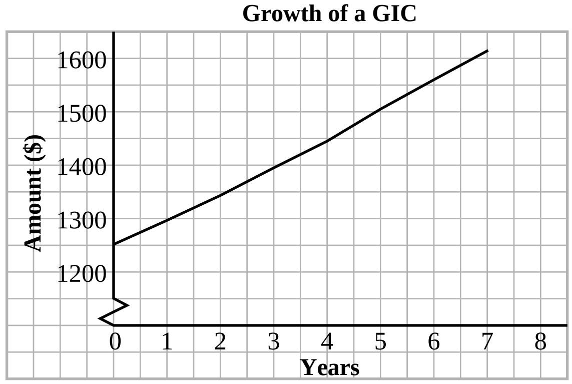
Rule 3: $y = 2x - 3$

Rule 4: $y = 2(x - 3)$

b) All the graphs are parallel because they have the same slope, but their y-intercepts are different.

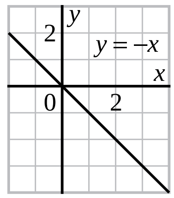
13. a)

Growth of a GIC

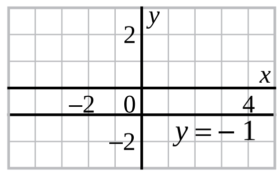


Selected Solutions — Chapter 5

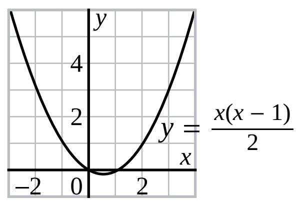
14. a) i)



ii)

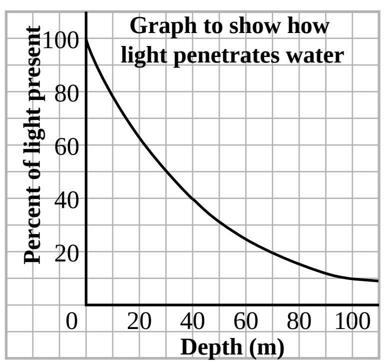


iii)

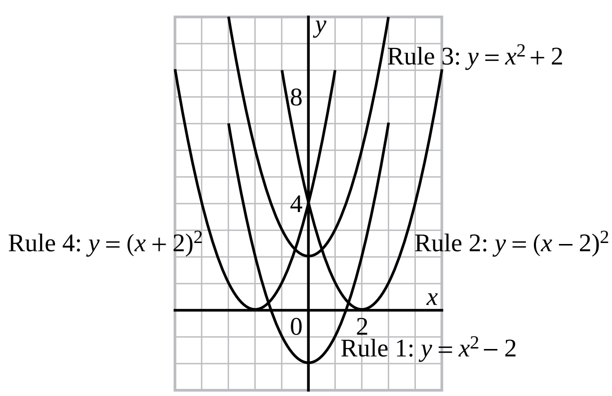


b) Answers may vary. For part iii: I made a table of values for x from –3 to 4. The ordered pairs were (–3, 6), (–2, 3), (–1, 1), (0, 0), (1, 0), (2, 1), (3, 3), and (4, 6). I drew a smooth curve through the points. To find where the curve turns, I found the y-coordinate for $x = 0.5$, to get the point (0.5, –0.125).

15. a)



16. a)



b) All the graphs are congruent; that is, they have the same shape and size but have different x- and y-intercepts. They are congruent because the equations have identical x^2 terms.

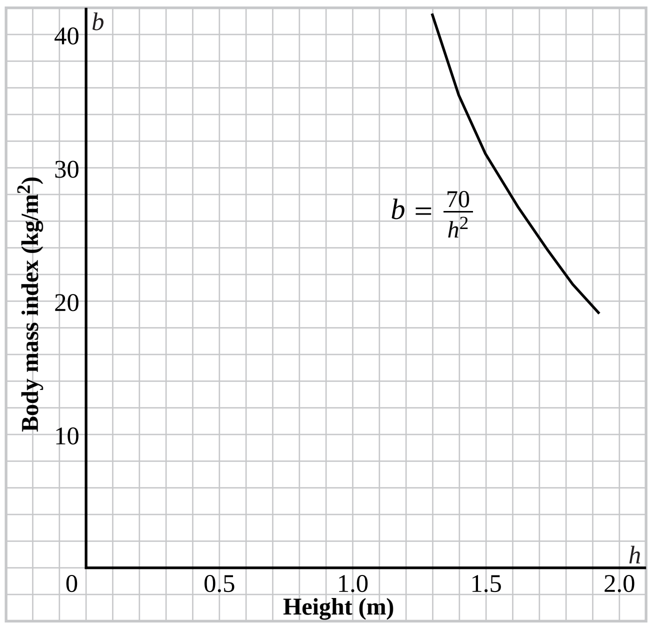
Selected Solutions — Chapter 5

18. a) $\text{BMI} = \frac{70}{(\text{height (m)})^2}$

b) Make a table of values.

Height (m)	BMI (kg/m²)
1.3	41.4
1.4	35.7
1.5	31.1
1.6	27.3
1.7	24.2

Draw the graph.

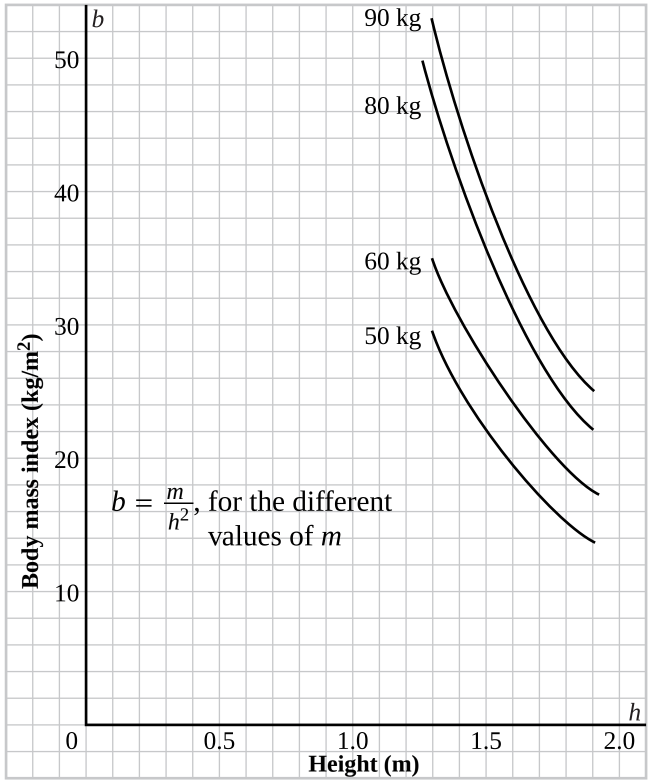


c) The BMI decreases as height increases.

d) The graphs would look similar, but the BMIs would be less for masses less than 70 kg, and greater for masses greater than 70 kg.

Selected Solutions — Chapter 5

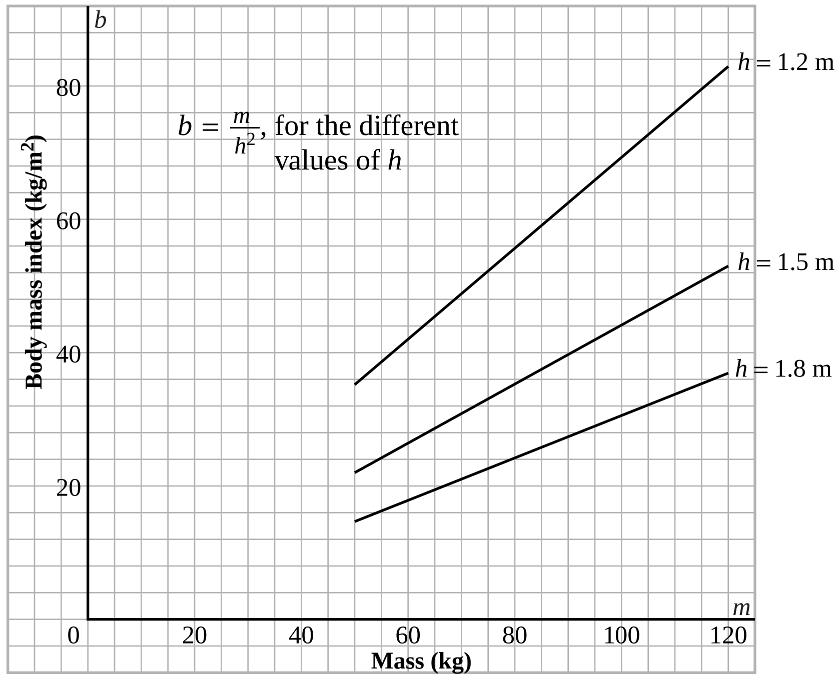
e)



Modelling Physical Fitness

When the height is 1.5 m, the equation of the function is $b = \frac{m}{2.25}$, which is a straight line, with b intercept 0, and slope $\frac{1}{2.25}$. This is different from the functions in exercise 18 because the latter were not represented by straight lines; their equations had the form $b = \frac{k}{h^2}$, where k is the constant mass.

Selected Solutions — Chapter 5



19. a) i) There would be h-values for the half-hours, not just the whole hours. $h = 0.5, 1.5, 2.5, 3.5,...$

ii) There would be h-values for the quarter-hours, not just the whole hours. $h = 0.25, 0.50, 0.75, 1.25, 1.50,...$

b) If the employees are paid for any part of an hour, the graph can be drawn as a straight line. In theory, the employees would be paid for every second they worked.

5.2 Exercises, page 258

1. b; as the person rides up the hill, the speed decreases, reaches zero for an instant at the top of the hill, then increases again as the person rides down the hill.

3. b; the team must sell at least 6 doughnuts to make any profit. The team does not sell parts of doughnuts, so the graph cannot be a straight line.

5. Explanations may vary. For part b: The domain is the possible x-values. I read these from the graph, from the dot at the left side to the dot at the right side. The domain is $-8 \le x \le 8$. The range is the possible y-values. I read these from the graph, from the lower dot to the upper dot. The range is $2 \le y \le 6$.

6. a) ii; the speed is constant, then increases, and then slows down again to the original speed.

b) Explanations may vary. For part i: The speed increases, then slows down, but not back to the original speed. A car speeds up to pass a car, then slows down to a speed faster than the original speed. For part ii: A car cruises at constant speed, accelerates instantaneously, then continues to increase its speed.

Selected Solutions — Chapter 5

7. c) She walks 10 min away from home, then stops for 5 min, then walks for 10 min away from home, then stops for 10 min, then walks for 5 min towards home, then stops for 5 min, then walks for 10 min towards home, then stops for 10 min, then walks for 5 min after which time she is home.

8. Answers may vary.

a) A person starts 5 blocks away from the school, and walks to 7 blocks away from the school. The person walks at a constant speed without stopping. The *x*-axis represents time, and the *y*-axis represents distance from the school. The *y*-intercept represents the person's starting point. The range is from 5 blocks to 7 blocks. The domain is from 0 min to 10 min.

b) A driver brakes and slows to a constant speed. The *x*-axis represents time, and the *y*-axis speed. The *y*-intercept is the speed the car was travelling when the driver began to brake. The range is 50 km/h to 100 km/h. The domain is 0 min to 1 min.

c) A car accelerates continuously from a certain speed. The *x*-axis represents time, and the *y*-axis distance. The *y*-intercept is the distance the car had travelled before measurements of time began. The range is 100 m to 1000 m. The domain is 0 s to 10 s.

d) Jane is at the grocery store for a time, and then walks home. After being at home for a while, she realizes that she forgot to buy milk, and returns to the grocery store. The *x*-axis represents time, and the *y*-axis distance from home. The *y*-intercept is the distance the grocery store is from home. The first *x*-intercept is the time between entering the grocery store and getting home. The second *x*-intercept is the time between entering the grocery store and leaving home. The range is 0 m to 100 m. The domain is 0 min to 4 min.

e) A pendulum swings back and forth. The *x*-axis represents time, and the *y*-axis distance from the side of the clock. The pendulum hangs from the clock at a certain distance from the side of the clock. The lowest points of the graph represent the closest distance to the side of the clock and the highest points represent the farthest distance measured from the same side of the clock. The *y*-intercept is the closest distance to the side of the clock. The range is 2 cm to 6 cm. The domain is 0 s to 4.5 s.

f) A car is parked in a lot. The *x*-axis represents time, and the *y*-axis parking fees. The parking lot charges $1 for each hour or part of an hour. The *y*-intercept is the minimum parking charge of $1. The range is $1 to $3. The domain is 0 h up to 3 h, not including 3 h.

9. e) There is a flat rate of $150 plus 15¢ per copy for the first 500 copies, and 10¢ per copy for any copies after 500.

Selected Solutions — Chapter 5

10. **b)** The amount of fuel decreases because the helicopter is flying, and using up the fuel. At 1.5 h, the helicopter refuels.

c) The helicopter is not flying from 1.5 h to 2 h, then flies from 2 h to about 2.75 h, then stops flying again.

d) The slope is negative because the amount of fuel is decreasing.

11. **a)** As time passes, more pages are printed, so the graph slants up to the right.

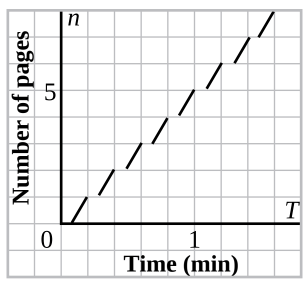
c) The slope is the rate at which the pages are printed; that is, the number of pages per minute.

d) There is no n intercept. The T-intercept represents the time the printer takes to warm up; the time before the first page is printed.

Modelling the Printing of Pages

Answers may vary.

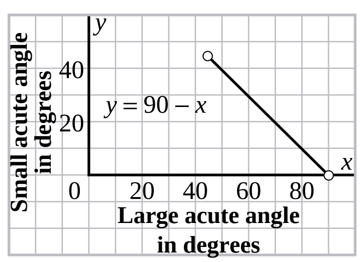
The action of a printer



a) The graph would be a broken-line graph, with line segments having different slopes and lengths. Each segment would span 1 square vertically, to represent 1 printed page, but the segment would have a lesser slope and, hence, a longer length the longer the page took to print.

b) The graph would be a series of non-intersecting line segments, similar in length and slope to the segments in the graph in part a.

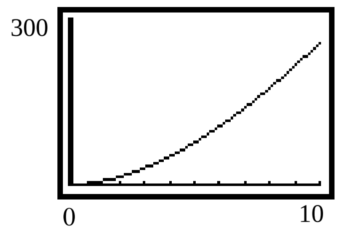
13. **a)**



c) As x decreases, y increases. On the graph, as you move left along the line, you also move up. In the equation, as smaller numbers are subtracted from 90, the differences are greater.

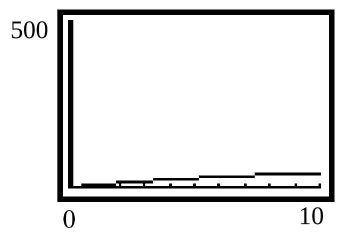
Selected Solutions — Chapter 5

15. a)



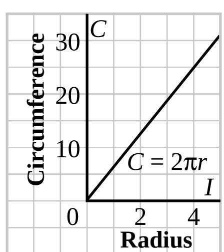
b) Part ii represents velocity as a function of time. At $t = 0$, $v = 0$. As t increases, v increases, but the car eventually reaches its maximum speed and stays there. Part i represents acceleration. The car starts accelerating at $t = 0$ with an acceleration that decreases with time. As time passes, the car continues to accelerate, but eventually stops accelerating as it reaches its maximum speed.

c)

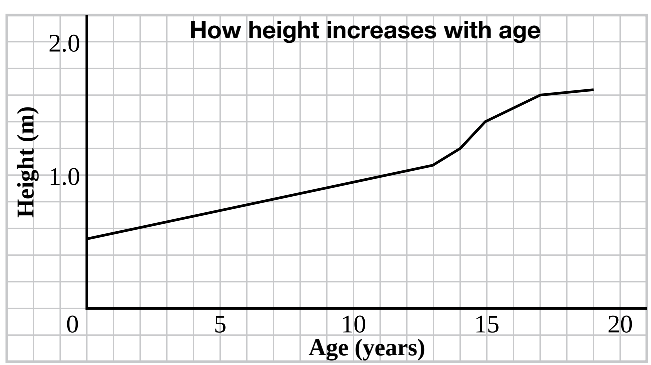


16. Graphs may vary.

a)

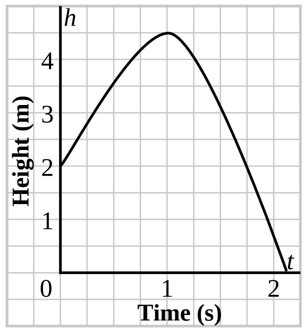


b)

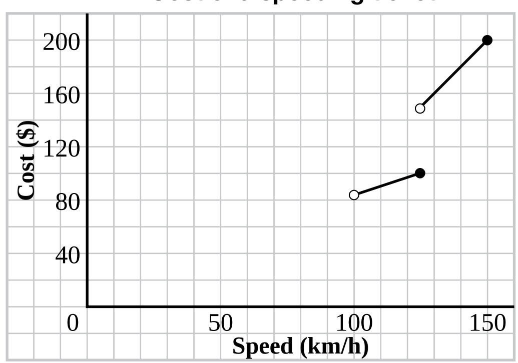


Selected Solutions — Chapter 5

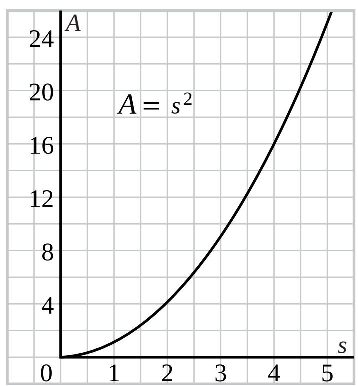
c) **Height of a basketball during a free throw**



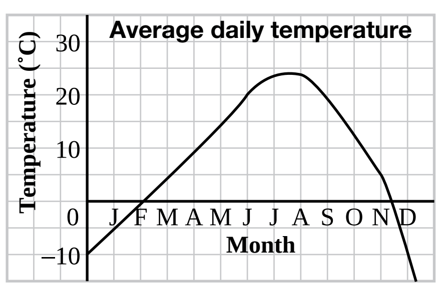
d) **Cost of a speeding ticket**



e)

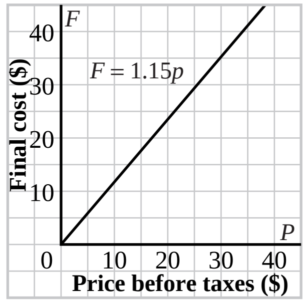


f)

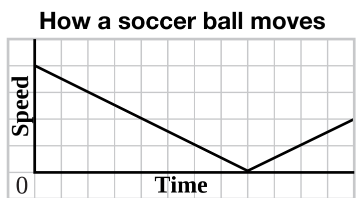


Selected Solutions — Chapter 5

g)



h)



17. a) From the graph, after 1 year, the value has decreased from $3500 to $1500.

b) From the graph, after 2 years, the value is $1000.

c) The y-intercept is the original value of the computer.

d) It is uncertain, although possible, that the line will intersect the x-axis. If it does, then the computer will be worthless.

e) For the segment from 0 years to 1 year, the slope is

$$\frac{1500 - 3500}{1 - 0} = \frac{-2000}{1} = -2000$$

The computer's value is reduced by $2000 per year for the first year.

For the segment from 1 year to 3 years, the slope is

$$\frac{500 - 1500}{3 - 1} = \frac{-1000}{2} = -500$$

The computer's value is reduced by $500 per year for the 2nd and 3rd years.

For the segment from 3 years onward, the slope is approximately

$$\frac{300 - 500}{3.5 - 3} = \frac{-200}{0.5} = -400$$

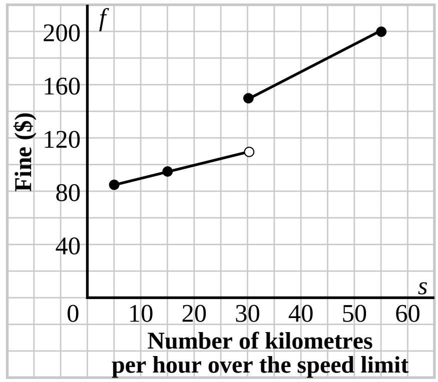
The computer's value is reduced by $400 per year for the 4th year.

The computer's depreciation rate slows down after 1 year and 3 years.

Selected Solutions — Chapter 5

Investigate, page 266

1.



4. Answers may vary.

Table C

Number of kilometres over the speed limit	Fine ($)
60	220
65	240
70	260

5.3 Exercises, page 269

2. Answers may vary.

a) I notice that the x-values increase by 1 as do the y-values, and each y-value is 5 more than the corresponding x-value. The equation is $y = x + 5$.

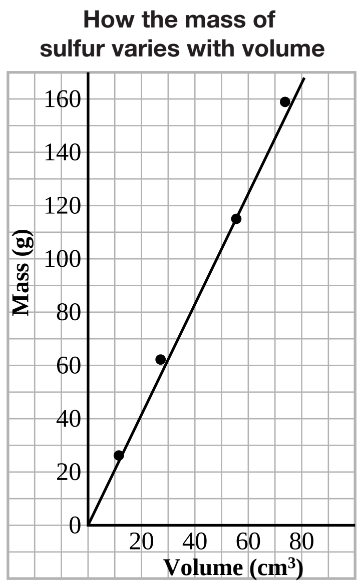
b) I notice that the x-values increase by 1 while the y-values increase by 2. So, I know the equation contains $2x$. I also notice that each y-value is 1 less than 2 times the corresponding x-value. The equation is $y = 2x - 1$.

c) I notice that the x-values increase by 1 and the y-values increase by 3, by 5, by 7, and by 9. So, I know the graph is not a straight line. I check for x^2-values. Each y-value is 2 more than the square of the corresponding x-value. The equation is $y = x^2 + 2$.

5. Answers may vary. For part b: The x-values increase by 4, 3, 3, and 2. The y-values increase by –2, –1.5, –1.5, and –1. So, I know the equation contains $\frac{-1}{2}x$. Each y-value is $-\frac{1}{2}$ times the corresponding x-value. The equation is $y = \frac{-1}{2}x$.

Selected Solutions — Chapter 5

7. a)

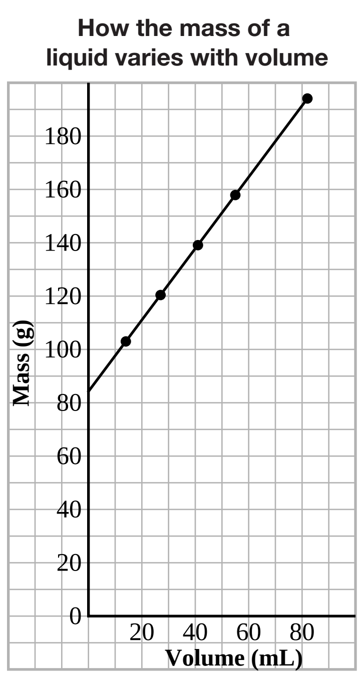


e) Errors could have been made in the measurements of volumes and masses.

9. Answers may vary. For part f: The x-values increase by –21, –1, –2, and –3. The y-values increase by 3.5, 2, –18, and +9. There is no pattern in these numbers. So, the graph is not a straight line. The y-values are not squares or related to squares. So, the equation does not involve x^2. I notice that if I multiply corresponding x- and y-values, I always get 12. The equation is $xy = 12$, or $y = \frac{12}{x}$.

11. e) Both equations would become $y = 4x + 4$.

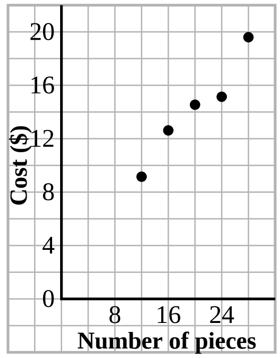
12. a)



d) Density $= \frac{\text{mass}}{\text{volume}}$; so, determine the mass and volume of an amount of the liquid, then divide mass by volume.

Selected Solutions — Chapter 5

13. a) Cost of different pizzas



b) The cost increases as the number of pieces increases.

15. a) $x = 1,\ y = 2(1) + 0.1(1 - 1)(1 - 2)(1 - 3)(1 - 4)(1 - 5)$
$= 2 + 0.1(0)(-1)(-2)(-3)(-4)$
$= 2 + 0$
$= 2$

$x = 2,\ y = 2(2) + 0.1(2 - 1)(2 - 2)(2 - 3)(2 - 4)(2 - 5)$
$= 4 + 0.1(1)(0)(-1)(-2)(-3)$
$= 4 + 0$
$= 4$

$x = 3,\ y = 2(3) + 0.1(3 - 1)(3 - 2)(3 - 3)(3 - 4)(3 - 5)$
$= 6 + 0.1(2)(1)(0)(-1)(-2)$
$= 6 + 0$
$= 6$

$x = 4,\ y = 2(4) + 0.1(4 - 1)(4 - 2)(4 - 3)(4 - 4)(4 - 5)$
$= 8 + 0.1(3)(2)(1)(0)(-1)$
$= 8 + 0$
$= 8$

$x = 5,\ y = 2(5) + 0.1(5 - 1)(5 - 2)(5 - 3)(5 - 4)(5 - 5)$
$= 10 + 0.1(4)(3)(2)(1)(0)$
$= 10 + 0$
$= 10$

$x = 6,\ y = 2(6) + 0.1(6 - 1)(6 - 2)(6 - 3)(6 - 4)(6 - 5)$
$= 12 + 0.1(5)(4)(3)(2)(1)$
$= 12 + 0.1(120)$
$= 24$

x	*y*
1	2
2	4
3	6
4	8
5	10
6	24

b) The tables are the same up to $x = 5,\ y = 10$. In exercise 3a, the next entry would be (6, 12).

Selected Solutions — Chapter 5

c) No, the rules for the two tables are not the same because all the corresponding ordered pairs are not the same.

16. a) The pattern looks like it comes from the equation $y = 5x$.

b) The pattern does not come from the equation $y = 5x$.

c) The equation must be similar to the one in exercise 15.
$y = 5x + a(x - 1)(x - 2)(x - 3)(x - 4)(x - 5)(x - 6)$
Substitute $x = 7$ and $y = 755$.

$$755 = 35 + a(7 - 1)(7 - 2)(7 - 3)(7 - 4)(7 - 5)(7 - 6)$$
$$720 = a(6)(5)(4)(3)(2)(1)$$
$$720 = 720a$$
$$a = 1$$

The equation is
$y = 5x + (x - 1)(x - 2)(x - 3)(x - 4)(x - 5)(x - 6)$.

d) Substitute $x = 7$ and $y = 100$ in the equation in part c.

$$100 = 35 + a(7 - 1)(7 - 2)(7 - 3)(7 - 4)(7 - 5)(7 - 6)$$
$$65 = 720a$$
$$a = \frac{65}{720}$$
$$= \frac{13}{144}$$

The equation is
$y = 5x + \frac{13}{144}(x - 1)(x - 2)(x - 3)(x - 4)(x - 5)(x - 6)$.

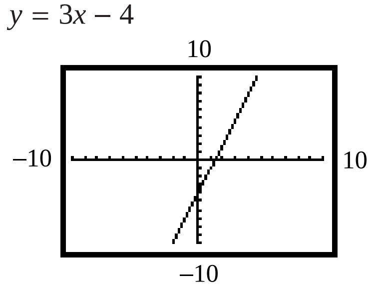
e) Use the equation in part c. Substitute $x = 7$ and y equal to any number, then determine a. For example, $x = 7$ and $y = 50$.

$$50 = 35 + a(6)(5)(4)(3)(2)(1)$$
$$15 = 720a$$
$$a = \frac{1}{48}$$

The equation is
$y = 5x + \frac{1}{48}(x - 1)(x - 2)(x - 3)(x - 4)(x - 5)(x - 6)$.

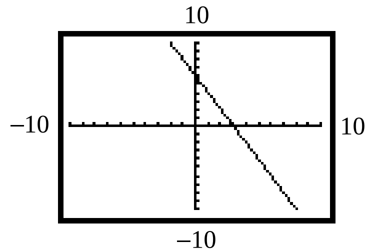
Investigate, page 274

1. $y = 3x - 4$

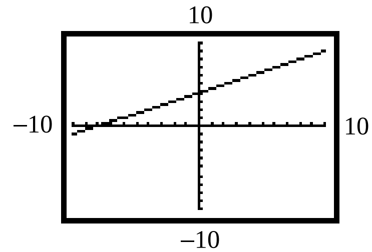


Selected Solutions — Chapter 5

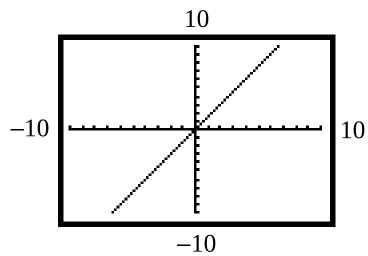
2. $2x + y = 6$



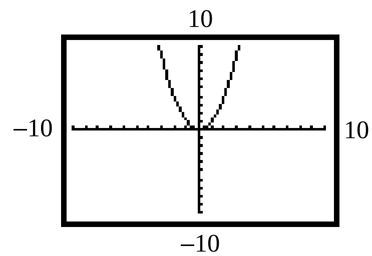
$x - 2y + 8 = 0$



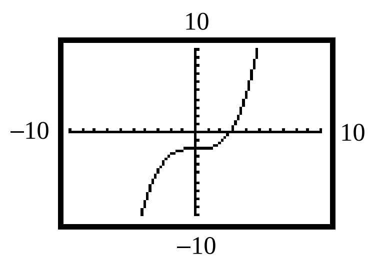
$3x = 2y$



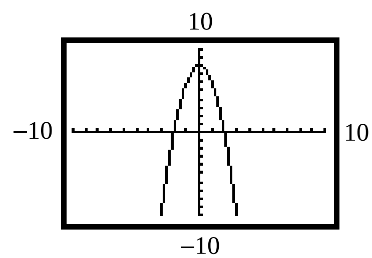
$y = x^2$



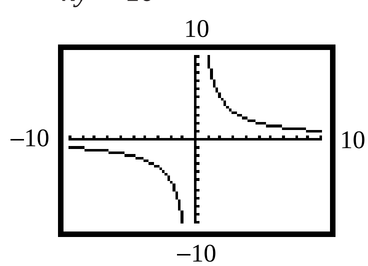
$y = 0.1x^3 - 2$



$2x^2 + y = 8$



$xy = 10$



Modelling the Dependence of Boiling Point of Water on Altitude, page 279

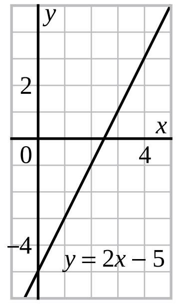
Answers may vary. The boiling point of water depends on air pressure. As altitude increases, pressure decreases, and water boils at a lower temperature.

Answers may vary. Those depths are within Earth, and should not count as depths below sea level.

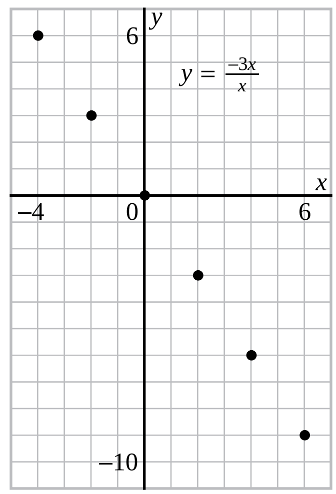
Selected Solutions — Chapter 5

5.4 Exercises, page 279

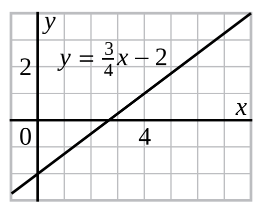
2. a)



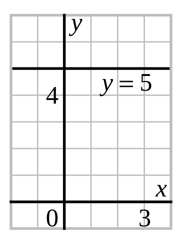
b)



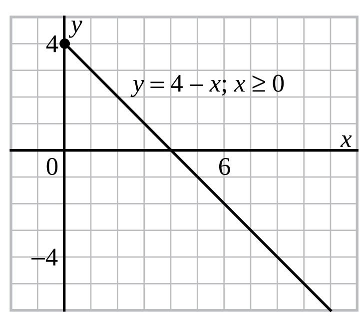
c)



d)

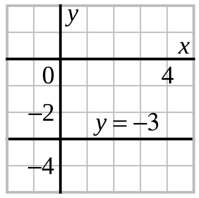


e)

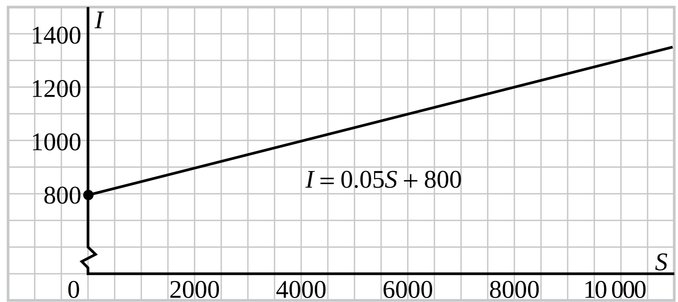


Selected Solutions — Chapter 5

f)



3. a)

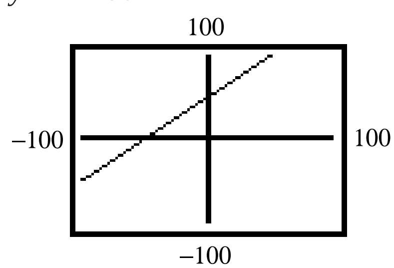


b) The slope represents 5% commission on sales.

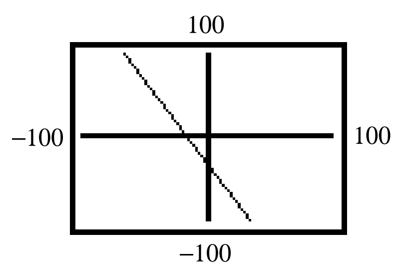
c) The *I*-intercept represents Franc's base salary of $800 per month. There is no *S*-intercept.

e) Franc's monthly income is equal to $800 plus 5% of his monthly sales.

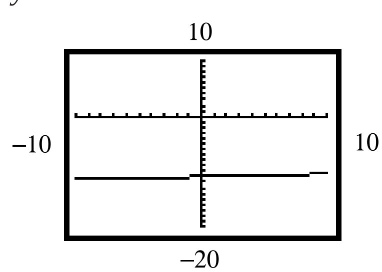
4. a) $y = x + 50$



b) $y = -2(x + 16)$



c) $y = 0.05x - 11$



Make sure that the scales of the axes are such that the graph is

Selected Solutions — Chapter 5

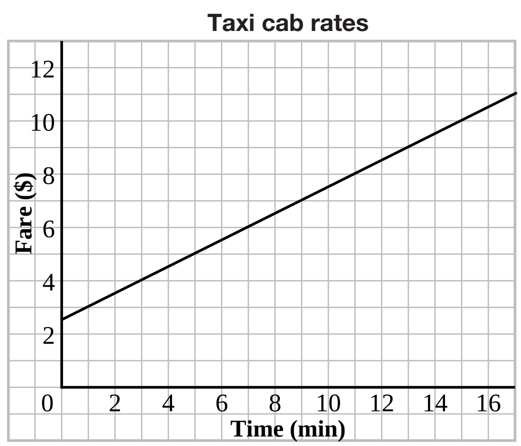
displayed.

5. b) Answers may vary. For part i: From the graph, $x \le 0$ and $y \le 2$. The domain is $x \le 0$ and the range is $y \le 2$. The slope is 2 and the y-intercept is 2. Use the general equation $y = mx + b$, where m is the slope and b the y-intercept. The equation is $y = 2x + 2$.

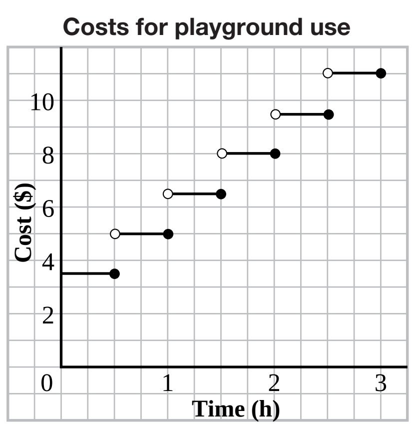
8. h) The car continues at the same speed.

9. Note that, since the fare is constant for each 12 s of time, the graph is really a step function — for each minute, there are 6 steps. Because the steps are so small, the graph is drawn as a straight line.

c)



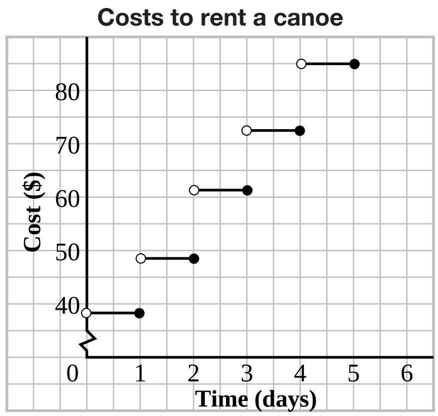
10. a)



11. Subtract $2 from the total fee, then divide by the number of half-hours the park was used. This will be the fee per half-hour.

Selected Solutions — Chapter 5

12. a)



c) Subtract the cost for 3 days from the cost for 5 days to calculate the daily cost for 2 days.

$85 – $61 = $24

Divide by 2 to find the daily rate.

The daily rate is $12.

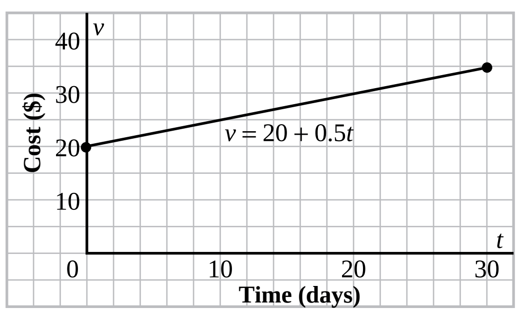
The daily cost for 5 days is $5 \times \$12 = \$60$.

Subtract this cost from $85 to calculate the fixed cost.

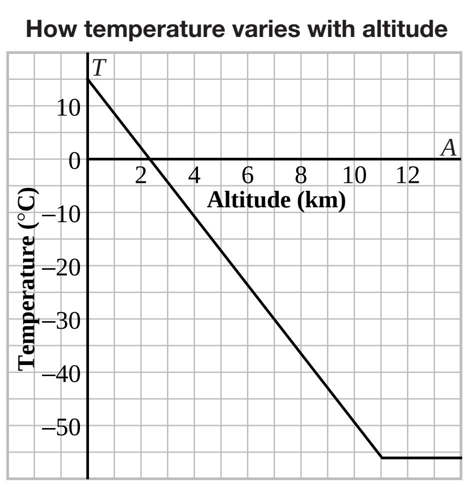
$85 – $60 = $25

The fixed cost is $25.

13. a)



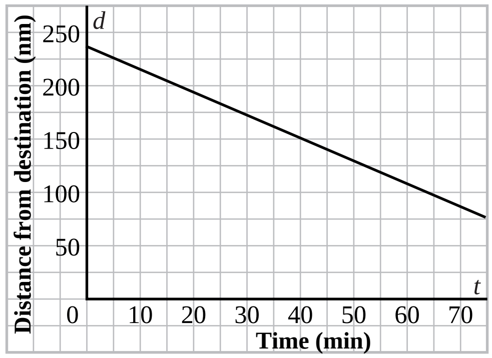
14. a), d)



Selected Solutions — Chapter 5

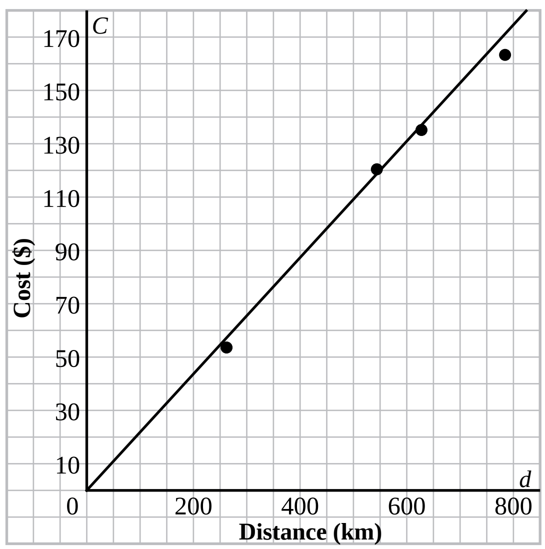
15. a)

Helicopter's Journey



16. a)

Cost of a round trip

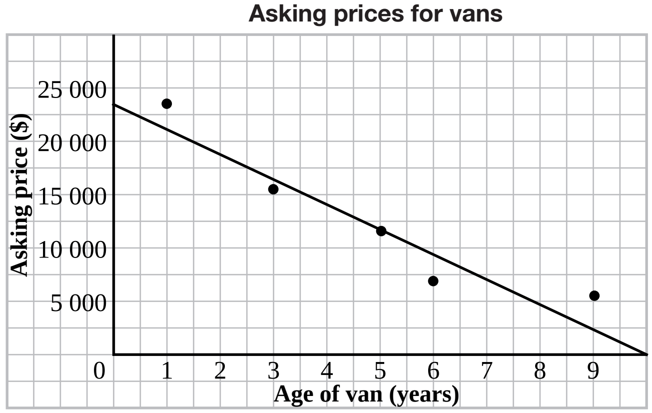


b) The points cannot be joined by a straight line.

g) Answers may vary. The equation of the straight line depends on its slope and *y*-intercept. The slope and *y*-intercept depend on how the line was drawn through the points. The equation in the student text is calculated using a graphing calculator; see pages 286, 287.

17. a), b)

Asking prices for vans

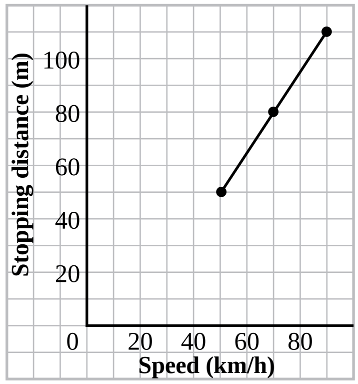


c) The equation in part b was calculated using a graphing calculator. Since only one point out of 5 lies on the line, a linear function does not describe the data well.

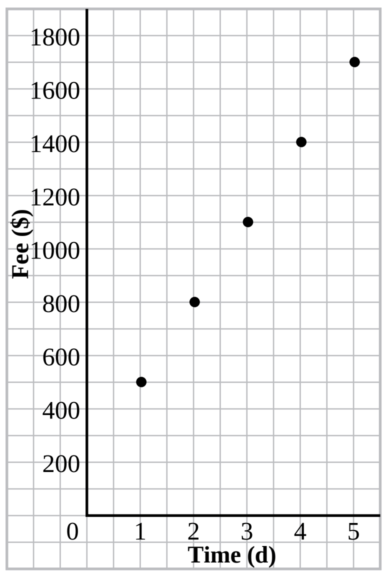
Selected Solutions — Chapter 5

d) According to the line of best fit, a 6-year-old van should have an asking price of about \$9000. An asking price of \$6000 is low.

20. a) Stopping distance for a truck



21. a) Fees charged by a computer programmer



d) I wrote an expression for the general term of an arithmetic sequence where first term $a = 500$ and common difference $d = 300$.

$t_n = a + (n - 1)d$
$\quad = 500 + (n - 1)300$
$\quad = 200 + 300n$

I substituted 5000 for t_n.

$5000 = 200 + 300n$
$4800 = 300n$
$\quad n = 16$

The evaluator would have to work for 16 days to earn \$5000.

22. a) The area of a trapezoid is given by $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is the perpendicular distance between them.

Selected Solutions — Chapter 5

For APQD, AP = x
and DQ = y

For PBCQ, PB = $10 - x$
QC = $6 - y$

For each trapezoid, let the perpendicular distance between the parallel sides be h.
Area APQD = area PBCQ
Substitute the known lengths.

$$\frac{1}{2}(x + y)h = \frac{1}{2}(10 - x + 6 - y)h$$

Divide each side by $\frac{1}{2}h$.

$$x + y = 16 - x - y$$
$$2x + 2y = 16$$

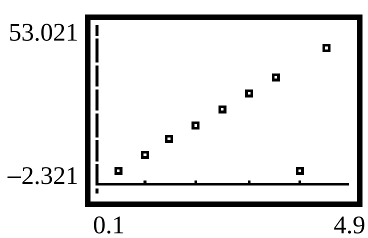
Divide each term by 2.

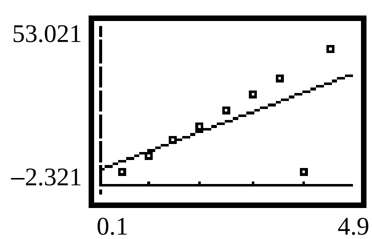
$$x + y = 8$$

b) The sum of x and y must be 8.
Since AB = 10 cm, AP = x must be less than 10.
But x cannot be greater than 8, so $x < 8$.
Since DC = 6 cm, DQ = y must be less than 6.
Therefore, x must be greater than $8 - 6$, or 2.
Hence, x must be between 2 cm and 8 cm for the trapezoids to have equal areas.

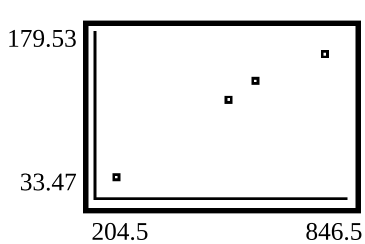
Exploring with a Graphing Calculator: The Line of Best Fit, page 287

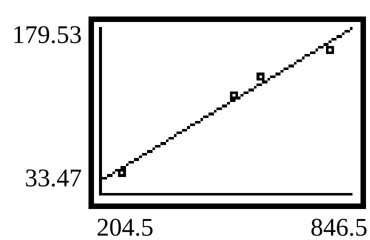
1. a), c) Notice the one point (4.0, 4.7) that probably represents an error in measurement. If this point is ignored, the line of best fit would pass through the remaining points and the origin. This illustrates that care must be taken when drawing a line of best fit. It would have been better to ignore this point when drawing the line of best fit.





2.a), b)





c) Answers may vary. The equation determined by the calculator is probably close to the equation determined by drawing the line of best fit by eye.

Selected Solutions — Chapter 5

d) Answers may vary. The equations from the graphing calculator should be the same.

Problem Solving: Use a Graph, page 289

1. Solve the equation on page 288.

$$\frac{9 - 0.061x}{130 - x} = 0.083$$
$$9 - 0.061x = 0.083(130 - x)$$
$$9 - 0.061x = 10.79 - 0.083x$$
$$0.022x = 1.79$$
$$x = \frac{1.79}{0.022}$$
$$x = 81.\overline{36}$$

The amount to be invested at 6.1% is

$81.\overline{36} \times \$1000 \doteq \$81\ 364$

The amount to be invested at 8.3% is

$130 000 – $81364 = $48 636

2. Answers may vary. Perhaps investing it all at 8.3% would result in her having to pay much higher income taxes.

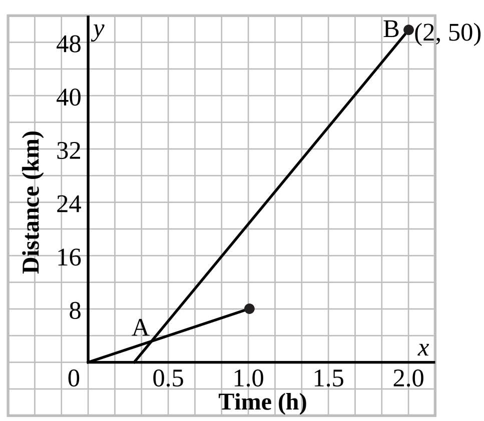
3. **a)** No; most investments are in whole numbers (nearest dollar).

b) Answers may vary. She would be satisfied with an income that is close to $9000.

c) Answers may vary. She could invest $50 000 at 8.3% and $80 000 at 6.1%. Her annual income, before taxes, would be $9030.

4. Sketch a distance-time graph.

Draw a line from (0, 0) to (1, 8) to represent a speed of 8 km/h for jogging. Plot the point B(2, 50). This represents the end of the exercise. Draw a line left from this point, with slope 30 km/h to represent cycling.



Selected Solutions — Chapter 5

Let the point of intersection have coordinates $A(x, 8x)$.

Slope of AB:

$$\frac{50 - 8x}{2 - x} = 30$$

$$50 - 8x = 60 - 30x$$

$$22x = 10$$

$$x = \frac{10}{22}$$

$$\doteq 0.\overline{45}$$

Multiply by 60 to calculate the time in minutes:

$0.\overline{45} \times 60 \doteq 27$

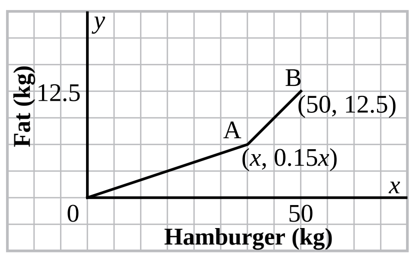
Brenda jogs for about 27 min.

5. Sketch a graph of fat content against hamburger.

For hamburger: $y = 0.15x$ represents the amount of fat in x kilograms of lean beef, and $y = 1x$ represents the amount of fat in x kilograms of fat trim.

The hamburger is to have 25% fat. So, in 50 kg of hamburger, the fat content is 0.25×50 kg $= 12.5$ kg.

Sketch the line $y = 0.15x$ to represent the lean beef. Stop at point A(x, 0.15x). Join A to B(50, 12.5). AB has slope 1, since the fat trim is 100% fat.



Slope of AB:

$$\frac{12.5 - 0.15x}{50 - x} = 1$$

$$12.5 - 0.15x = 50 - x$$

$$37.5 = 0.85x$$

$$x = \frac{37.5}{0.85}$$

$$x \doteq 44.1$$

This represents the mass of lean beef.

To calculate the mass of fat trim, subtract x from 50.

$50 - x \doteq 5.9$

The butcher needs 44.1 kg of lean beef and 5.9 kg of fat trim.

6. The car uses 12.5 L for 100 km of city driving.

Hence, for 1 km of driving, the car uses

$\frac{12.5 \text{ L}}{100} = 0.125$ L

The car uses 7.5 L for 100 km of city driving.

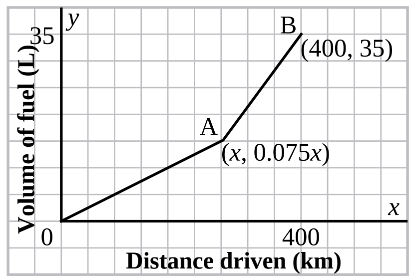
Selected Solutions — Chapter 5

Hence, for 1 km of driving, the car uses

$\frac{7.5 \text{ L}}{100} = 0.075 \text{ L}$

Let y litres represent the volume of gas used when driving x kilometres.

Sketch the line $y = 0.075x$ to represent highway driving. Stop at point $A(x, 0.075x)$. Join A to $B(400, 35)$. AB has slope 0.125 to represent 12.5 L/100 km.



Slope of AB:

$$\frac{35 - 0.075x}{400 - x} = 0.125$$

$$35 - 0.075x = 50 - 0.125x$$

$$15 = 0.05x$$

$$x = \frac{15}{0.05}$$

$$x = 300$$

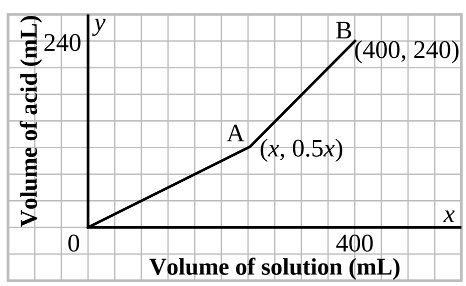
The distance travelled in highway driving is 300 km.

7. Draw a graph of acid content against solution.

For the 50% solution, $y = 0.5x$ represents the volume of 50% acid in x litres of solution.

For the 100% solution, $y = x$ represents the volume of 100% acid in x litres of solution.

The final solution has $0.6 \times 400 \text{ mL} = 240 \text{ mL}$ of acid.



Sketch the line $y = 0.5x$ to represent the 50% solution. Stop at point A(x, 0.5x). Join A to B(400, 240). AB has slope 1 to represent the 100% solution.

Selected Solutions — Chapter 5

Slope of AB:

$$\frac{240 - 0.5x}{400 - x} = 1$$

$$240 - 0.5x = 400 - x$$

$$160 = 0.5x$$

$$x = \frac{160}{0.5}$$

$$x = 320$$

This represents the volume of 50% solution. To calculate the volume of 100% solution, subtract x from 400.

$400 - x = 80$

The technician needs 320 mL of the 50% solution, and 80 mL of the 100% solution.

8. Tap A takes 5 min to drain the tank. Tap B takes 10 min to drain the tank. Let the tank have volume x litres.

The draining speed of tap A is $\frac{x}{5}$ litres/min.

The draining speed of tap B is $\frac{x}{10}$ litres/min.

Their speed together is $(\frac{x}{5} + \frac{x}{10})$litres/min.

Let t minutes be the time it takes to drain the tank.

Then $\frac{x}{5} + \frac{x}{10} = \frac{x}{t}$

Divide both sides by x.

$$\frac{1}{5} + \frac{1}{10} = \frac{1}{t}$$

$$\frac{2}{10} + \frac{1}{10} = \frac{1}{t}$$

$$\frac{3}{10} = \frac{1}{t}$$

$$\frac{10}{3} = t$$

$$t = 3.\overline{3}$$

It takes about 3.3 min to drain the tank using both taps.

An alternative solution: Place a ruler vertically on the graph at the vertical axis. Move the ruler to the right until the sum of the vertical distance to the graph for tap A and the vertical distance to the graph for tap B is 100%.

For example, at 2 min, the sum of the distances is $20 + 40 = 60$.

At 3 min, the sum of the distances is $30 + 60 = 90$.

At 4 min, the sum of the distances is $40 + 80 = 120$.

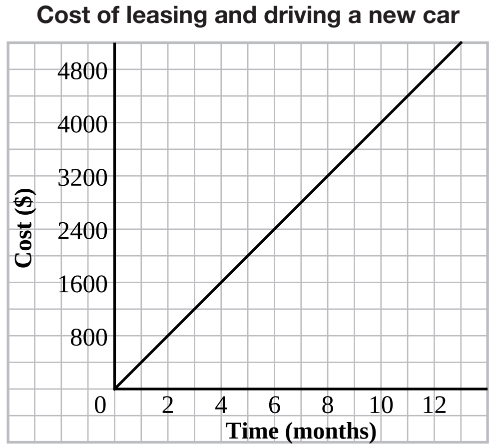
At about 3.3 min, the sum of the distances is approximately $33 + 67 = 100$.

At this time, tap B has emptied 33% of the tank and tap B has emptied 67% of the tank.

Selected Solutions — Chapter 5

Investigate, page 290

2.



3. b) Find an entry in the month column, and the entry that is double that entry. Look at the corresponding cost entries.
Find a point on the Time axis, and another point that represents twice that time. Draw vertical lines from these points to the graph. Draw horizontal lines from the graph to intersect the Cost axis. The corresponding costs will be in the ratio 2 : 1.
Repeat the procedure for a time and triple that time.

Modelling the Distance to a Thunderstorm, page 293

Because light travels so much faster than sound, we assume that the lightning struck when we saw the flash.
The thunder then travels to us at a speed of 350 m/s.

Convert this to kilometres per second by dividing by 1000; that is, 0.35 km/s.

We know that distance = speed × time.

The distance travelled, d kilometres, in time, t seconds, at a speed of 0.35 km/s is given by $d = 0.35t$.

A person may need to know how far she is from a thunderstorm so she can take cover, to avoid being struck by lightning, or to avoid getting wet.

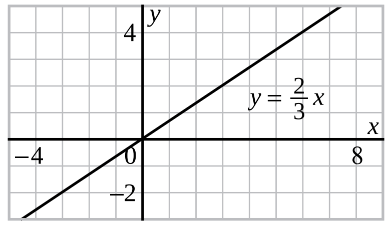
5.5 Exercises, page 295

3. We know that when two quantities vary directly, the graph of their equation passes through the origin (0, 0). Draw axes, label the vertical axis D, the horizontal axis T, and draw a line through (0, 0) and (4, 28).

4. b) Answers may vary. For part iv: The area, A, of a square depends on the square of the length of a side, s. The equation has the form $A = s^2$, which is not a linear equation. Hence, the situation does not represent a direct variation.

Selected Solutions — Chapter 5

6. c) i)



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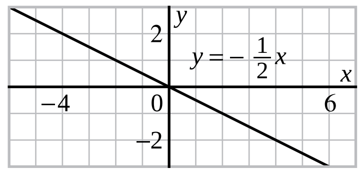
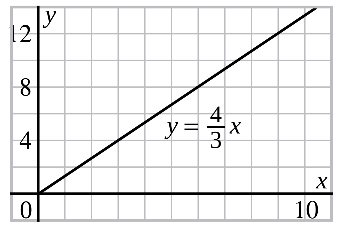


iii)



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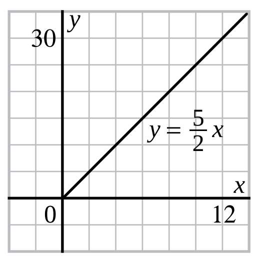
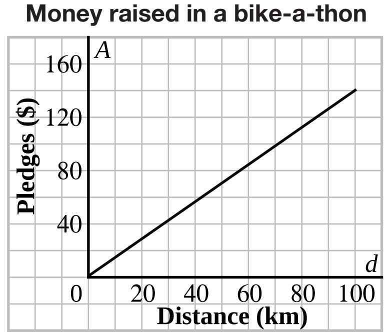
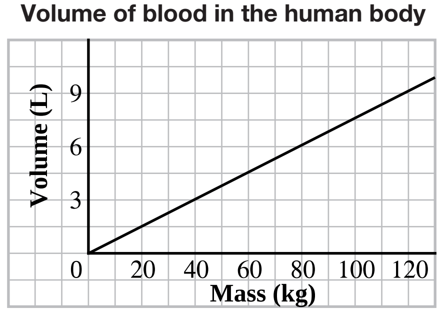


7. d) **Money raised in a bike-a-thon**



9. c) **Volume of blood in the human body**



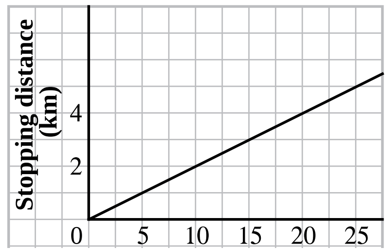
10. c) Explanations may vary. The greater the reaction time, the greater the distance travelled before the brakes are applied.

Selected Solutions — Chapter 5

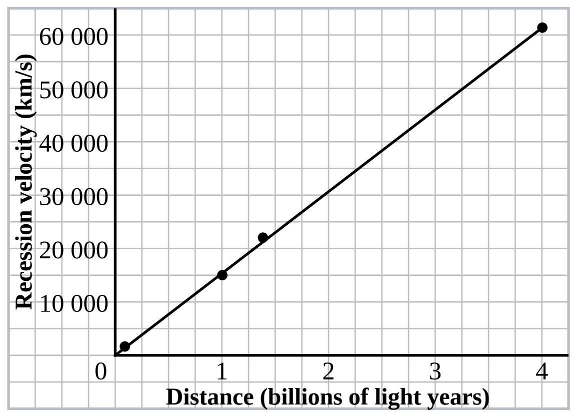
11. b) This is an example of direct variation if time begins at 0 h. That is, every hour from 0 h, 240 acres are lost.

c) The sequence is arithmetic because each term is determined by adding 240 to the preceding term.

12. c) Stopping distance for a super tank



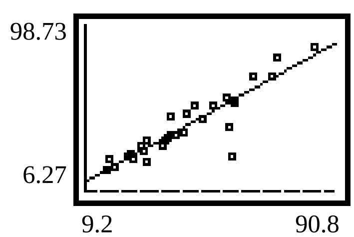
14. a)



b) The graph passes through the origin since the recession velocities vary directly as the distances.

15. a) No, a husband may be older than, younger than, or the same age as his wife.

b), c) The relationship is not linear. The points are scattered over the plot. There is a general trend up and to the right.



d) Answers may vary. A line of best fit is a straight line. If it passes through the origin, it represents direct variation. The line in part b does not pass through the origin, so it does not represent direct variation.

f) The slope of the line represents the husband's age divided by the wife's age. Since a husband and wife are approximately the same age, the slope is approximately 1.

Selected Solutions — Chapter 5

16. a) The area of the inner circle is πx^2. The area of the outer circle is πy^2. The area of the shaded region is $\pi y^2 - \pi x^2$, or $\pi(y^2 - x^2)$. Since this shaded area is equal to the area of the inner circle, write an equation.

$\pi x^2 = \pi(y^2 - x^2)$

Divide each term by π.

$x^2 = y^2 - x^2$

$2x^2 = y^2$

Take the square root of each side.

$y = \sqrt{2}x$

b) y varies directly as x, because the equation relating y and x has the form $y = mx$, where m is a constant.

17. By symmetry, r is the hypotenuse of a right triangle with legs $\frac{1}{2}x$ and x. Apply the Pythagorean Theorem.

$x^2 + (\frac{1}{2}x)^2 = r^2$

$x^2 + \frac{1}{4}x^2 = r^2$

$\frac{5}{4}x^2 = r^2$

Take the square root of each side.

$r = \frac{\sqrt{5}}{2}x$

Thus, r varies directly as x, with the constant of proportionality $\frac{\sqrt{5}}{2}$.

18. a) Let d metres represent distance, and t seconds time.

Since d varies as t^2, write an equation, with k as the constant of proportionality.

$d = kt^2$

Substitute $d = 490$ and $t = 10$.

$490 = 100k$

$k = 4.9$

Thus, the equation is $d = 4.9t^2$.

b) $d = 4.9t^2$

Substitute $t = 5$.

$d = 4.9(5)^2$

$= 122.5$

The object will travel 122.5 m.

c) $d = 4.9t^2$

Substitute $t = 15$.

$= 4.9(15)^2$

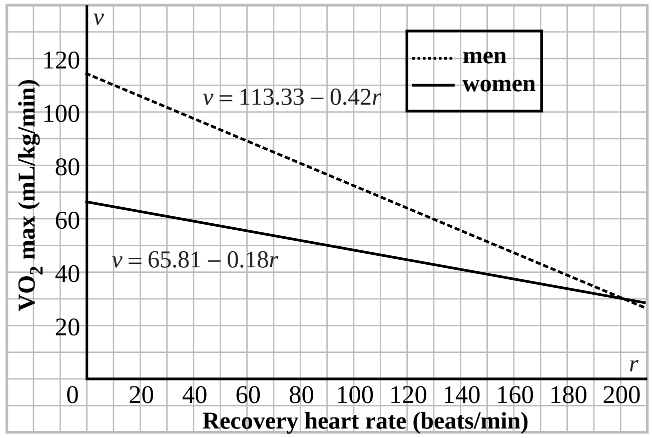
$= 1102.5$

The object will travel 1102.5 m.

Selected Solutions — Chapter 5

Mathematical Modelling: Pushing Your Physical Limits, page 303

1. a)



b) Answers may vary. For a person at rest, the heart rate is about 70 beats per minute. This would be the lowest value of r. For a person exercising, the heart rate can be as high as 200 beats per minute. This would be the greatest value of r.

For men: when $r = 70$, $v \doteq 84$

when $r = 200$, $v \doteq 29$

For women: when $r = 70$, $v \doteq 53$

when $r = 200$; $v \doteq 30$

2. a) $v = 65.81 - 0.18r$

Substitute $r = 145$ in the equation.

$$\begin{aligned} v &= 65.81 - 0.18(145) \\ &= 65.81 - 26.10 \\ &= 39.7 \end{aligned}$$

The VO_2 max is approximately 40 mL/kg/min.

b) $v = 65.81 - 0.18r$

Substitute $r = 130$ in the equation.

$$\begin{aligned} v &= 65.81 - 0.18(130) \\ &= 65.81 - 23.40 \\ &= 42.4 \end{aligned}$$

The VO_2 max is approximately 42 mL/kg/min.

Answers may vary. Silken Laumann is a professional athlete and would typically have a higher VO_2 max than a moderate exerciser; perhaps 50 mL/kg/min.

3. a) $v = 113.33 - 0.42r$

Substitute $v = 88$ in the equation.

$$\begin{aligned} 88 &= 113.33 - 0.42r \\ -25.33 &= -0.42r \\ r &= 60.3 \end{aligned}$$

The recovery heart rate is about 60 beats/min.

b) $v = 113.33 - 0.42r$

Substitute $v = 55$ in the equation.

Selected Solutions — Chapter 5

$$55 = 113.33 - 0.42r$$
$$-58.33 = -0.42r$$
$$= 138.9$$

The recovery heart rate is about 139 beats/min.

4. As recovery heart rate increases, VO_2 max decreases. An exceptionally fit person would have a VO_2 max and a recovery heart rate that tend to oppose this trend. This person would have a relatively high VO_2 max and a relatively low recovery heart rate. To see what this means, consider an exceptionally fit person and a person who is less fit. If their recovery heart rates are the same, the fit person's heart would deliver more oxygen to the muscles. Another way of looking at it is to say that if both their hearts deliver the same amount of oxygen to the muscles, the fit person's heart would not be working as hard.

5. Men and women are physiologically different.

6. Answers may vary.

7. Answers may vary. Using a high percent of available oxygen indicates that you cannot transfer the oxygen quickly enough to your muscle tissue (low VO_2 max). If your body can transfer oxygen quickly to your muscle tissue (high VO_2 max), then you are not using a high percent of your available oxygen, and will not be winded when finishing a race.

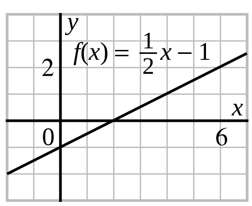
5.6 Exercises, page 307

3. b) Answers may vary. For part ii: To evaluate $f(\sqrt{2})$ for $f(x) = 2x^3 + 5x^2 + 3x - 4$, I substituted $\sqrt{2}$ for x, then simplified.

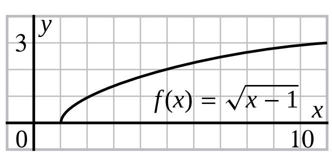
$$f(\sqrt{2}) = 2(\sqrt{2})^3 + 5(\sqrt{2})^2 + 3\sqrt{2} - 4$$
$$= 4\sqrt{2} + 10 + 3\sqrt{2} - 4$$

Then I collected like terms to get $7\sqrt{2} + 6$.

4. a) i)

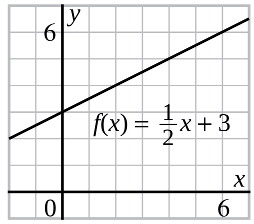


ii)

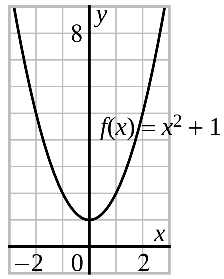


Selected Solutions — Chapter 5

10. a) i)



ii)



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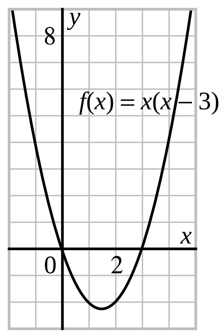


b) Answers may vary. For part iii: For the function $f(x) = x(x - 3)$, I chose values for x and found corresponding values of $f(x)$, or y. For example,

for $x = -2$, $y = -2(-2 - 3)$
$= -2(-5)$
$= 10$

x	***y***
–2	10
–1	4
0	0
1	–2
2	–2
3	0
4	4
5	10

I plotted the points on a grid. To find the lowest point of the graph, I calculated the y-value corresponding to $x = 1.5$.

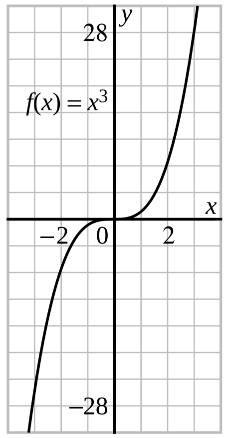
$y = 1.5(1.5 - 3)$
$= 1.5(-1.5)$
$= -2.25$

The domain is all possible x-values, which is all real numbers.

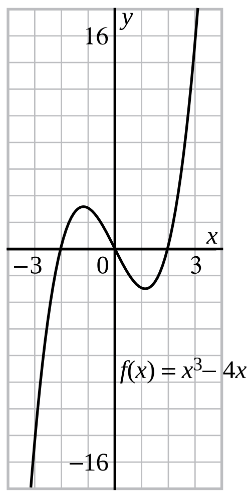
Selected Solutions — Chapter 5

The range is all possible y-values which, from the graph, are values so that $y \geq -2.25$.

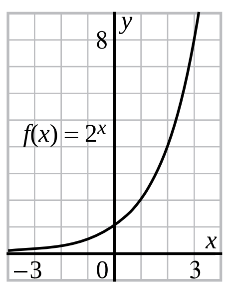
11. a)



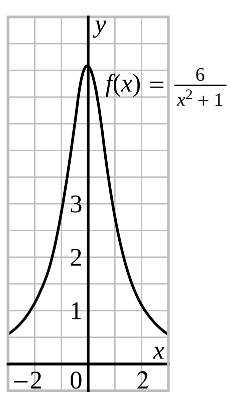
b)



c)



d)



Selected Solutions — Chapter 5

13. **d)** Answers may vary. For part c: Since $x = \sqrt{3}$, I chose a function that contains x^2, so the value of x will be squared.

If $f(x) = x^2$, then $f(\sqrt{3}) = (\sqrt{3})^2 = 3$

But I need $f(\sqrt{3})$ to equal 1. So, I subtracted 2 from 3 to get 1.

This means I must subtract 2 from $f(x)$ as well.

I ended up with $f(x) = x^2 - 2$.

18. **a)** **i)** To get $f(2)$, substitute $x = 2$ in $f(x) = \frac{x}{1+x}$.

To get $f(\frac{1}{2})$, substitute $x = \frac{1}{2}$ in $f(x) = \frac{x}{1+x}$.

$$\begin{aligned} f(2) + f(\tfrac{1}{2}) &= \frac{2}{1+2} + \frac{\frac{1}{2}}{1+\frac{1}{2}} \\ &= \frac{2}{3} + \frac{\frac{1}{2}}{\frac{3}{2}} \\ &= \frac{2}{3} + \frac{1}{3} \\ &= 1 \end{aligned}$$

ii) Substitute $x = 3$, then $x = \frac{1}{3}$ in $f(x)$.

$$\begin{aligned} f(3) + f(\tfrac{1}{3}) &= \frac{3}{1+3} + \frac{\frac{1}{3}}{1+\frac{1}{3}} \\ &= \frac{3}{4} + \frac{\frac{1}{3}}{\frac{4}{3}} \\ &= \frac{3}{4} + \frac{1}{4} \\ &= 1 \end{aligned}$$

b) Since the answer was 1 in both parts of part a, predict $f(n) + f(\frac{1}{n}) = 1$. Substitute to check.

Substitute $x = n$, then $x = \frac{1}{n}$ in $f(x)$.

$$\begin{aligned} f(n) + f(\tfrac{1}{n}) &= \frac{n}{1+n} + \frac{\frac{1}{n}}{1+\frac{1}{n}} \\ &= \frac{n}{1+n} + \frac{\frac{1}{n}}{\frac{n+1}{n}} \\ &= \frac{n}{1+n} + \frac{1}{n+1} \\ &= \frac{n+1}{n+1} \\ &= 1 \end{aligned}$$

c) We cannot divide by 0, so n cannot be zero. n can be any other real number.

Selected Solutions — Chapter 5

20. a) i) $g(x) = 3^x$

Replace x with $2x$.

$$\begin{aligned} g(2x) &= 3^{2x} \\ \text{and } [g(x)]^2 &= (3^x)^2 \\ &= 3^{2x} \end{aligned}$$

Hence, $g(2x) = [g(x)]^2$

ii) $g(x) = 3^x$

Replace x with $3x$.

$$\begin{aligned} g(3x) &= 3^{3x} \\ \text{and } [g(x)]^3 &= (3^x)^3 \\ &= 3^{3x} \end{aligned}$$

Hence, $g(3x) = [g(x)]^3$

b) Following the pattern of part a, $g(nx) = [g(x)]^n$

21. b) 7^4, 7^8, 7^{12},...; All these powers are multiples of four.

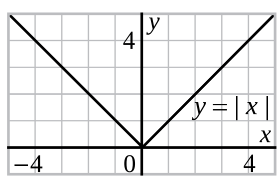
c) Since 7^4, 7^8, 7^{12}, and so on, all have the last digit 1, then any power of 7 that has an exponent that is a multiple of 4 will have the last digit 1.

In general, if n is any natural number, then 7^{4n} will have the last digit 1.

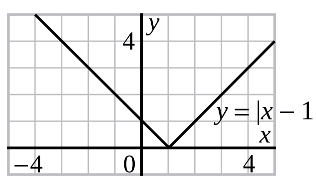
Hence, $x = 4n$

Mathematics File: Absolute Value, page 310

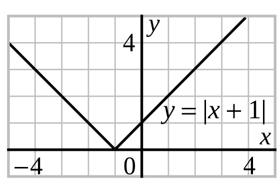
5. a)



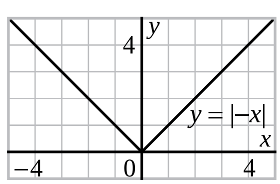
6. a)



b)

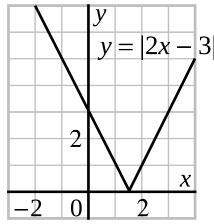


c)



Selected Solutions — Chapter 5

d)



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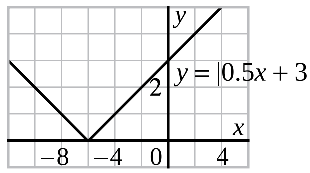
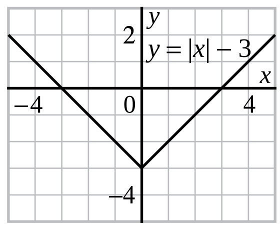


f)



7. c) Graph 1: $y = x - 2$ represents a linear function. The graph has slope 1 and y-intercept −2. The x-intercept is 2.

Graph 2: $y = |x - 2|$; The negative values of $y = x - 2$ are reflected in the x-axis.

Graph 3: $y = |x| - 2$; The graph $y = |x|$ is translated down 2 units.

Graph 4: $y = ||x| - 2|$; The negative values of $y = |x| - 2$ are reflected in the x-axis.

5.7 Exercises, page 316

1. Yes, a function produces one output number for every valid input number. A relation may produce one or more output numbers for each input number.

2. No, some relations produce more than one output number for every valid input number. Only those relations that produce one output number for each input number are functions.

3. No, for a circle, each input number has two output numbers, so the graph does not represent a function.

4. The same y-value can occur twice in a function; for example, the function $y = x^2$ has y-values of 1 for $x = 1$ and $x = -1$. However, the same y-value occurs twice in the relation $x^2 + y^2 = 25$; that is, $y = 3$ when $x = 4$ and $x = -4$.

7. For part a: The graph does not obey the vertical line test. A vertical line (such as $x = 1$) intersects the graph in 3 points. The relation is not a function.

Selected Solutions — Chapter 5

For part b: The graph does obey the vertical line test. Any vertical line intersects the graph only once. The relation is a function.

For part c: The graph obeys the vertical line test so the relation is a function.

For part d: The graph does not obey the vertical line test. A vertical line (such as $x = 3$) intersects the graph in 2 points. The relation is not a function.

For part e: The graph does not obey the vertical line test. A vertical line (such as $x = 2$) intersects the graph in 2 points. The relation is not a function.

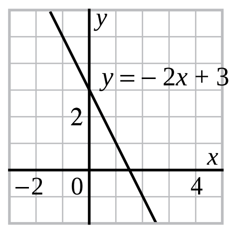
For part f: The graph obeys the vertical line test so the relation is a function.

For part g: The graph obeys the vertical line test so the relation is a function.

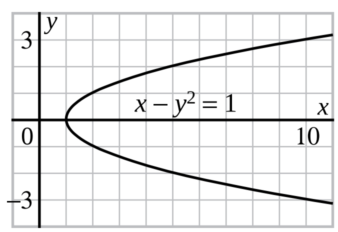
For part h: The graph obeys the vertical line test so the relation is a function.

9. a) The relation is a function.

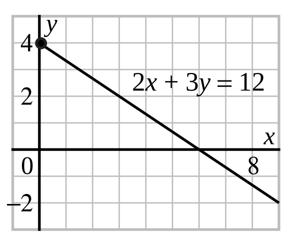
The graph obeys the vertical line test.



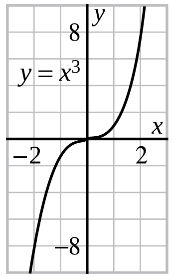
b) The relation is not a function. The graph does not obey the vertical line test.



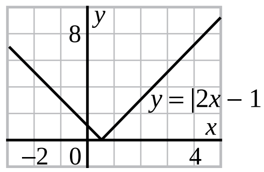
c) The relation is a function. The graph obeys the vertical line test.



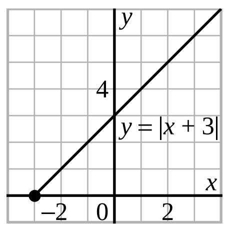
d) The relation is a function. The graph obeys the vertical line test.



e) The relation is a function. The graph obeys the vertical line test.



f) The relation is a function. The graph obeys the vertical line test.



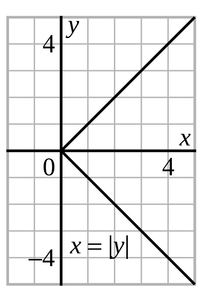
10. **a)** The relation is not a function. Altitude is not a function of temperature. For some temperatures, there is more than one altitude.

b) The temperature is a function of altitude. Draw the graph with temperature on the vertical axis and altitude on the horizontal axis. Altitude is on the vertical axis to make the graph more visual. That is, since altitude is height, it makes sense to represent height on the vertical rather than the horizontal axis.

11. The statement is true. It is the converse of the statement of the vertical line test.

12. **a)** The relation $x = |y|$ is not a function because its graph does not obey the vertical line test. For each x-value, there is more than one y-value.

b)



ADDISON-WESLEY Mathematics 10

Selected Solutions — Chapter 5

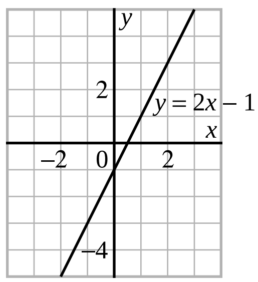
Mathematics File: Classifying Functions, page 319

1. Answers may vary.
   Increasing: page 260, exercise 8a; page 323, exercise 9iii
   Decreasing: page 278, Example 4; page 280, exercise 5ii
   Periodic: page 260, exercise 8e
   Piecewise linear: page 261, exercise 10; page 263, exercise 14
   Step: page 260, exercise 8f; page 323, exercise 9vi
   Discontinuous: page 307, exercise 5b; page 317, exercise 6c

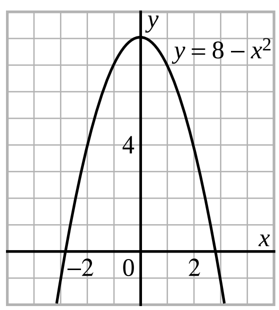
2. Answers may vary. Parabolas: page 253, exercise 17i; page 270, exercise 6ii

5 Review, page 320

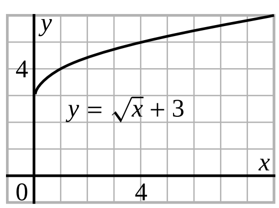
1. b) i)



ii)

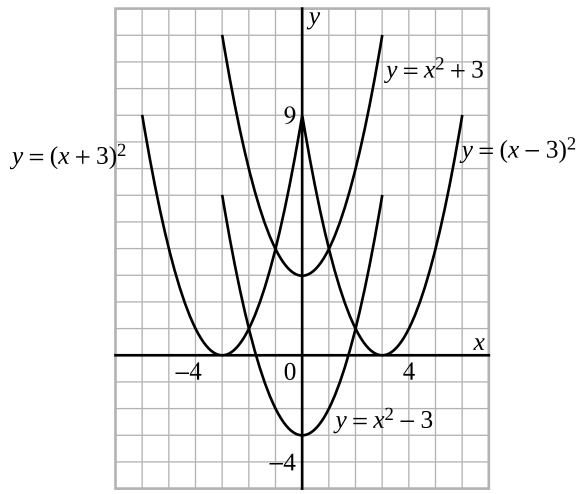


iii)



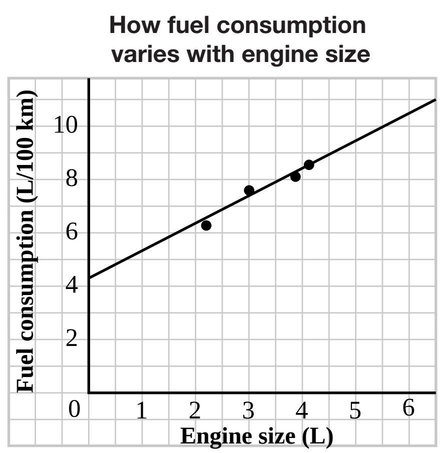
Selected Solutions — Chapter 5

2. a)



b) All the graphs have the same size, shape, and orientation, but they have different intercepts.

3. a) Straight lines may vary.

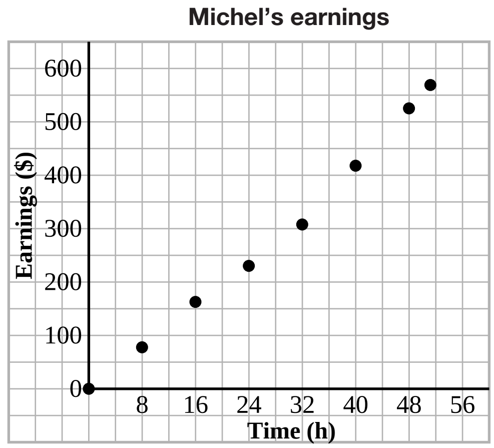


e) The estimate in part e is more likely to be incorrect, since it is an extrapolation. Also, we cannot be sure that the linear relationship is true for engines as large as 6.0 L.

f) The equations are probably different because the equation depends on where the line is drawn. Different people may choose different lines of best fit.

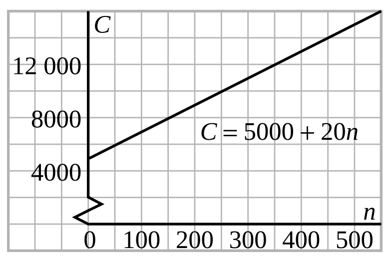
Selected Solutions — Chapter 5

4. b)

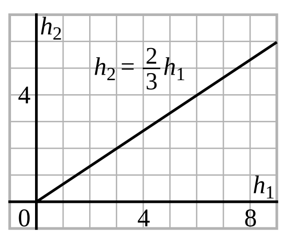


The points should not be joined since we are not told that Michel earns money for parts of hours worked.

5. c)



6. d)



10. Answers may vary. For part a: The relation is not a function since there are two y-values when $x = -1$.

For part b: The relation is a function since its graph obeys the vertical line test.

For part c: The relation is not a function since its graph does not obey the vertical line test.

For part d: The relation is not a function since its graph does not obey the vertical line test.

For part e: The relation is not a function since its graph does not obey the vertical line test.

For part f: The relation is a linear function since it is a non-vertical straight line.

Selected Solutions — Chapter 5

5 Cumulative Review, page 322

2. The pattern is that each sum is 45 more than the preceding sum. Consider the first and second series.

The first series is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

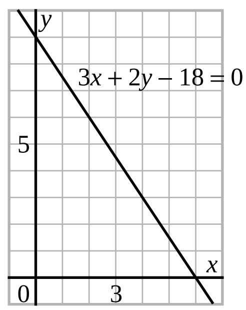
The second series is $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$

The difference between corresponding terms is $0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$

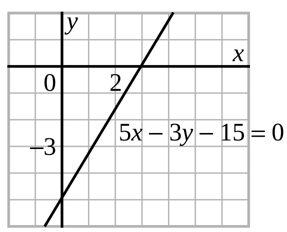
The sum of this series is 45.

The same relationship occurs between any two consecutive series in exercise 1.

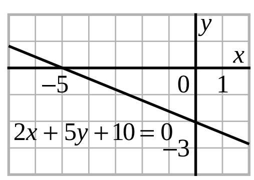
7. a)



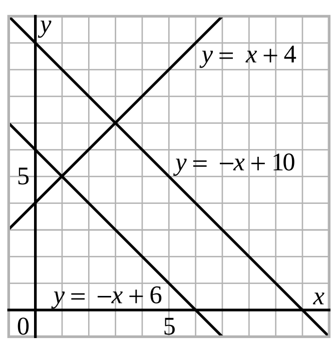
b)



c)



8. a)



Selected Solutions — Chapter 5

10. a) Answers may vary.

For graph i: The *x*-axis represents the time of day and the *y*-axis represents the number of cars. The domain is from 7:00 a.m. to 5:00 p.m. The range is from 0 to 100 cars. The *x*- and *y*-intercepts at the origin represent 0 cars at 7:00 a.m. The minimum point in the middle of the graph represents lunchtime when some cars leave the parking lot. The minimum point at the right of the graph represents the parking lot at 5:00 p.m., when few cars remain.

For graph ii: The *x*-axis represents the person's age in years. The *y*-axis represents the person's mass. The domain is from 0 years to 70 years. The range is from 3 kg to 70 kg. The *x*- and *y*-intercepts at the origin represent the person's mass at birth. The maximum point represents the person's mass at death.

For graph iii: The *x*-axis represents speed and the *y*-axis represents braking distance. The domain is from 0 km/h to 120 km/h. The range is from 0 m to 120 m. The *x*- and *y*-intercepts at the origin represent a braking distance of 0 m at a speed of 0 km/h. The maximum point represents the braking distance at maximum measured speed.

For graph iv: The *x*-axis represents the time and the *y*-axis represents the temperature. The domain is from 0 min to 10 min. The range is from 10˚C to 100˚C. There are no intercepts. The maximum point represents the temperature at the time the watch was started at the highest temperature of the coffee. The minimum point represents the time and temperature at the end of the period for which the temperature was measured.

For graph v: The *x*-axis represents the time and the *y*-axis represents the height of the football. The domain is from 0 s to 5 s. The range is from 0 m to 30 m. The *x*- and *y*-intercepts at the origin represent the position of the ball just as it is kicked (0 m, ground level at 0 s). The *x*-intercept at the right represents the position of the ball when it has landed (0 m, at time $t = 5$ s, assuming the ball was in the air for 5 s). The maximum point represents the maximum height of the ball at time $t = 2.5$ s.

For graph vi: The *x*-axis represents time and the *y*-axis represents cost. The domain is 0 h to 4 h. The range is from $1 to $7. The *y*-intercept represents the minimum cost of $1 to park the car.

b) Answers may vary. For v: The football starts on the ground. It reaches a maximum height, then returns to the ground. As time increases, the height first increases, then decreases. I estimate that the greatest height a football can be kicked is 30 m, and it takes 5 s for the ball to land after it has been kicked.