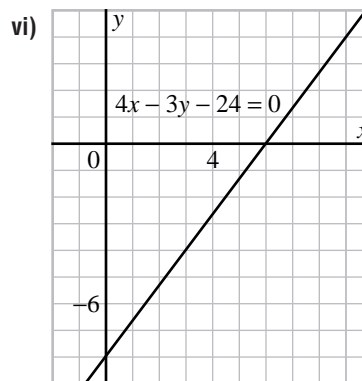
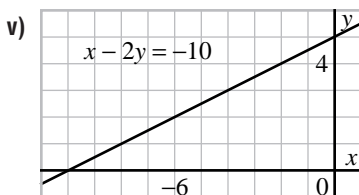
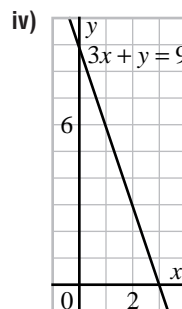
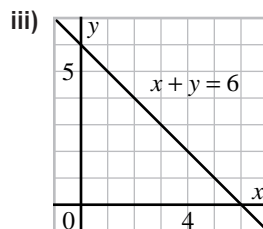
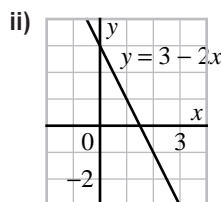
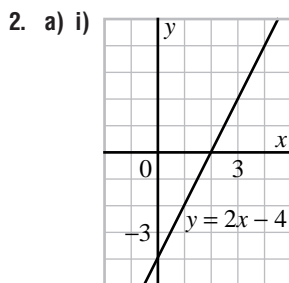


## Selected Solutions — Chapter 4

## 4.1 Exercises, page 202

1. b) Answers may vary. For part iii: I substituted  $x = 0$  in the equation  $4x + y = 12$  to get  $y = 12$ . Then I substituted  $y = 0$  to get  $4x = 12$  and  $x = 3$ . I chose 2 more values for  $x$  and substituted. When  $x = 1$ ,  $4 + y = 12$ , and  $y = 8$ . When  $x = 2$ ,  $8 + y = 12$ , and  $y = 4$ .



- b) Answers may vary. For part v: From the graph, another point on the line is  $(-9, 0.5)$ . Substitute  $-9$  for  $x$  and  $0.5$  for  $y$  in both sides of the equation  $x - 2y = -10$ .

$$\begin{aligned} \text{L.S.} &= -9 - 2(0.5) & \text{R.S.} &= -10 \\ &= -9 - 1 \\ &= -10 \end{aligned}$$

The coordinates  $(-9, 0.5)$  satisfy the equation. This verifies the first part of the property.

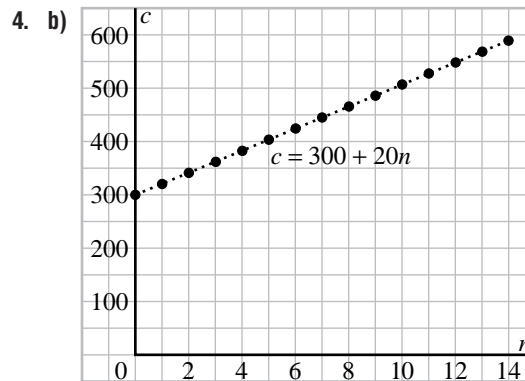
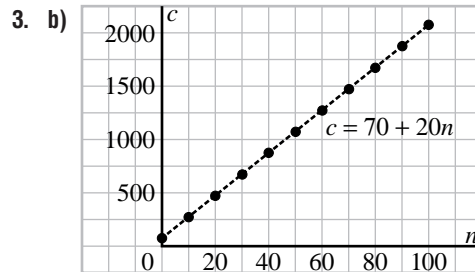
Find another point whose coordinates satisfy the equation.

Substitute  $x = -7$  into  $x - 2y = -10$ .

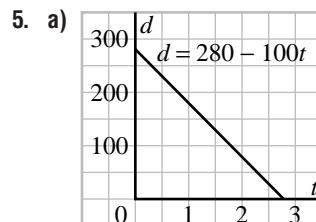
## Selected Solutions — Chapter 4

$$\begin{aligned} -7 - 2y &= -10 \\ -2y &= -3 \\ y &= 1.5 \end{aligned}$$

Another point whose coordinates satisfy the equation is  $(-7, 1.5)$ . This point lies on the graph. This verifies the second part of the property.

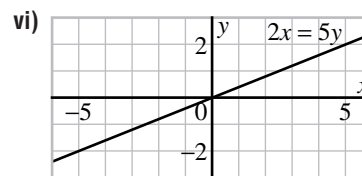
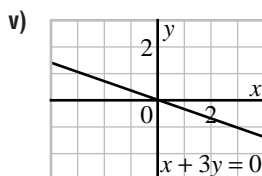
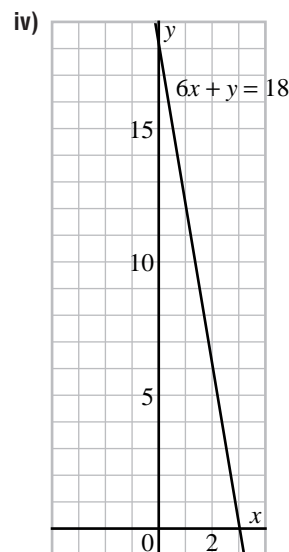
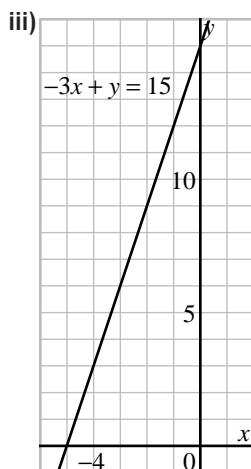
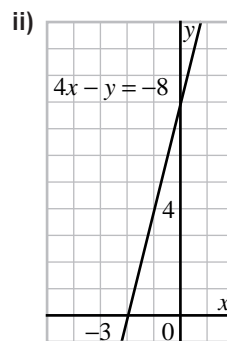
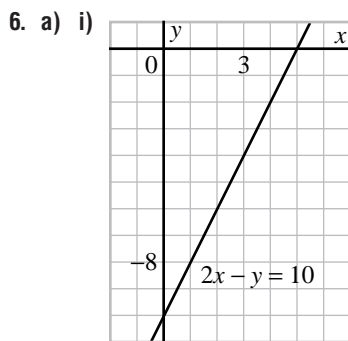
**Modelling the Cost to Play in a Tournament**

The model consists of only the points, since there cannot be a fractional numbers of players. There can only be whole numbers of players, so it is only reasonable to plot points whose  $n$ -coordinates are whole numbers. Also, the maximum value of  $n$  is the maximum number of players who could be on the team. That is,  $n$  would not be as large as 100.



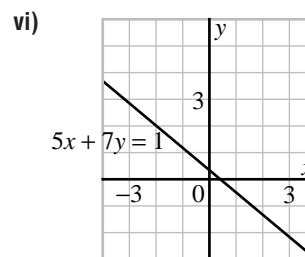
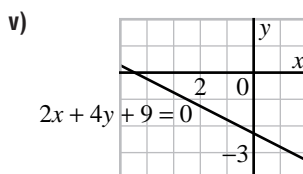
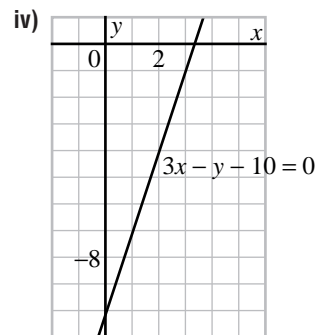
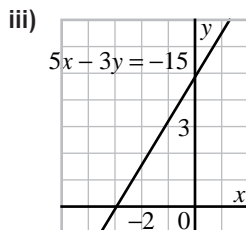
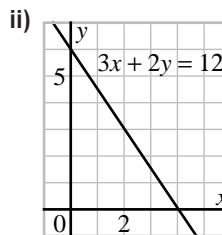
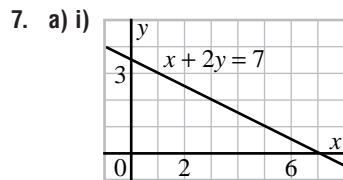
- b) The distance between the two cities is shown when  $t = 0$ ; this is 280 km.
- c) The total travelling time is shown when  $d = 0$ ; this is 2.8 h, or 2 h 48 min.
- e) The distance from Calgary at any time  $t$  hours is 280 km minus the distance travelled in time  $t$  hours at 100 km/h. This distance is represented by the term  $100t$ .

## Selected Solutions — Chapter 4



- b) Answers may vary. For part vi: To draw the graph of  $2x = 5y$ , I substituted  $x = 0$  and found that  $y = 0$ . The graph passes through the origin. I then chose 2 values of  $x$  that would give me integer values of  $y$ . I substituted  $x = 5$  to get  $y = 2$ , and  $x = -5$  to get  $y = -2$ . I then plotted the points  $(-5, -2)$ ,  $(0, 0)$ ,  $(5, 2)$  on a grid, and then drew a straight line through them.

Selected Solutions — Chapter 4



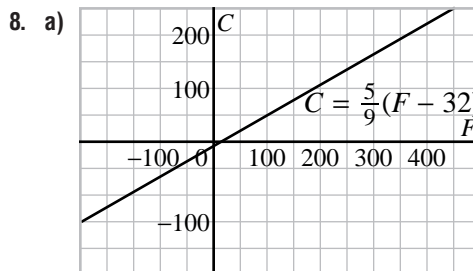
b) Answers may vary. For part i: From the graph, another point on the line is (5, 1). Substitute 5 for  $x$  and 1 for  $y$  into both sides of the equation  $x + 2y = 7$ .

$$\begin{aligned} \text{L.S.} &= 5 + 2(1) & \text{R.S.} &= 7 \\ &= 5 + 2 \\ &= 7 \end{aligned}$$

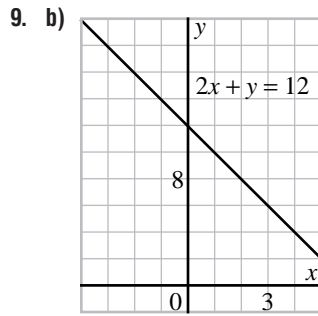
The coordinates (5, 1) satisfy the equation. This verifies the first part of the property. Find another point whose coordinates satisfy the equation. Substitute  $x = 1$  into  $x + 2y = 7$ .

$$\begin{aligned} 1 + 2y &= 7 \\ 2y &= 6 \\ y &= 3 \end{aligned}$$

Another point whose coordinates satisfy the equation is (1, 3). This point lies on the graph. This verifies the second part of the property.

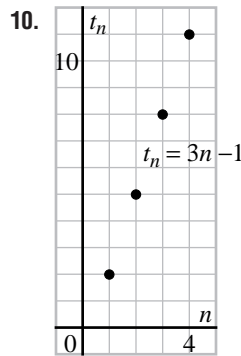


Selected Solutions — Chapter 4



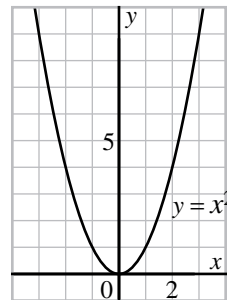
c) Answers may vary. The point  $(-2, 16)$  lies on the graph. If I double  $-2$ , I get  $-4$ , then I add 16, I get 12. The two numbers obey the rule.

d) Answers may vary. The point  $(1, 4)$  does not lie on the graph. If I double 1, I get 2, then I add 4, I get 6. The two numbers do not obey the rule.



11. Answers may vary.  $y = x^2$

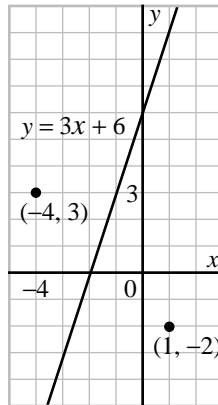
$x$	$y$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



12. a) The Equation of a Line Property says that the coordinates of every point on the line satisfy the equation of the line, and every point whose coordinates satisfy the equation of the line is on the line. Assume that the coordinates of a point not on the line satisfy the equation of the line. The second part of the property says that this point is on the line. This is a contradiction. The coordinates of a point not on the line cannot satisfy the equation of the line.

## Selected Solutions — Chapter 4

b) I drew the line  $y = 3x + 6$ .



I chose a point below the line,  $(1, -2)$ , and substituted its coordinates in the equation  $y = 3x + 6$ .

$$\begin{aligned} \text{L.S.} &= -2 & \text{R.S.} &= 3(1) + 6 \\ & & &= 9 \end{aligned}$$

The L.S. is less than the R.S.

I chose a point above the line,  $(-4, 3)$ , and substituted its coordinates in the equation  $y = 3x + 6$ .

$$\begin{aligned} \text{L.S.} &= 3 & \text{R.S.} &= 3(-4) + 6 \\ & & &= -6 \end{aligned}$$

The L.S. is greater than the R.S.

I deduced that for all points above the line,  $y > 3x + 6$ ; and for all points below the line,  $y < 3x + 6$ .

**Linking Ideas: Mathematics and Technology**

**Investigating Hall Rental Costs, page 206**

4. The formula  $C = 125 + 6.85n$  is better for someone renting a hall for a function that has 83 people or fewer attending.

Substitute  $n = 83$  in each equation to check.

$$\begin{aligned} C &= 125 + 6.85(83) & C &= 425 + 3.25(83) \\ &= 125 + 568.55 & &= 425 + 269.75 \\ &= 693.55 & &= 694.75 \end{aligned}$$

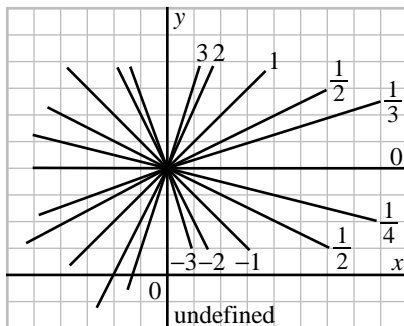
If the function has 84 or more people at the hall, then the formula  $C = 425 + 3.25n$  is better.

$$\begin{aligned} C &= 125 + 6.85(84) & C &= 425 + 3.25(84) \\ &= 125 + 575.4 & &= 425 + 273 \\ &= 700.40 & &= 698.00 \end{aligned}$$

# Selected Solutions — Chapter 4

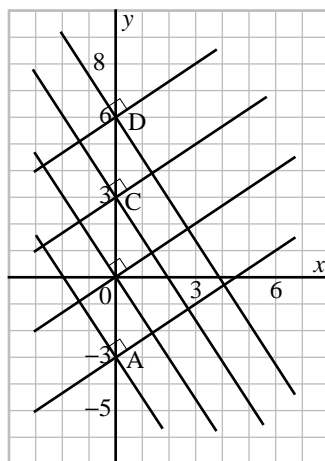
## 4.2 Exercises, page 210

2. a)

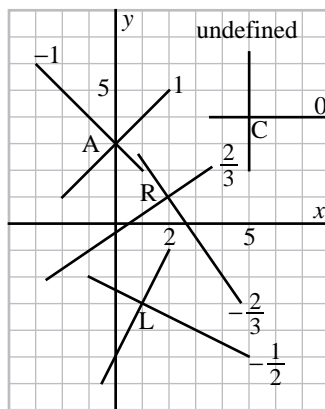


b) Answers may vary. For part v: The slope is  $\frac{1}{3}$ ; that is, the rise is 1 and the run is 3. I started at  $(0, 4)$ , moved 1 up and 3 right to reach the point  $(3, 5)$ ; then I moved 1 up and 3 right again to reach the point  $(6, 6)$ .

3.

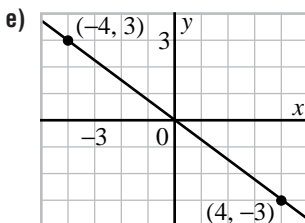
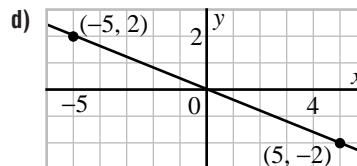
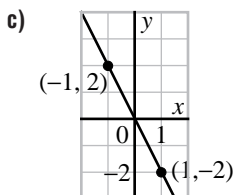
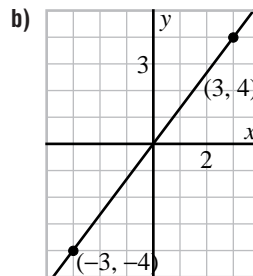
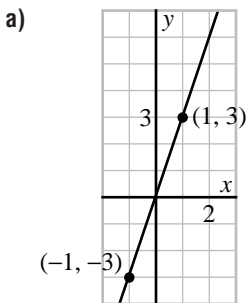


4.

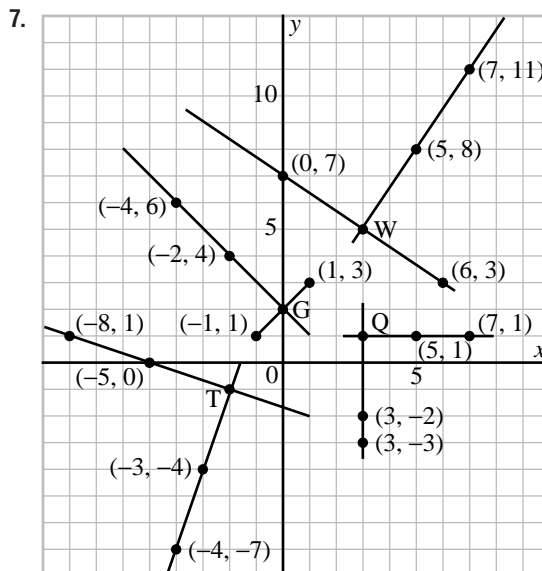


# Selected Solutions — Chapter 4

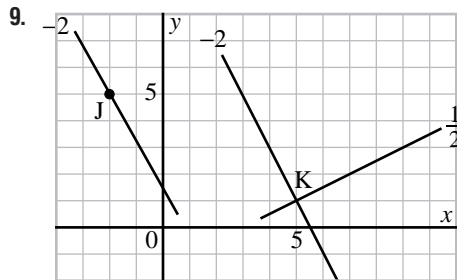
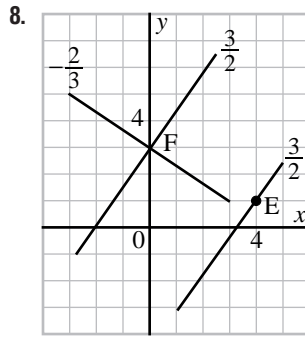
5. Lines may vary.



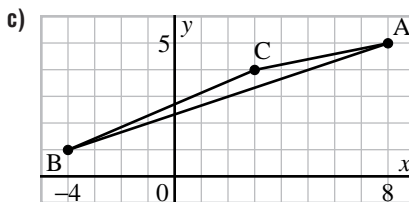
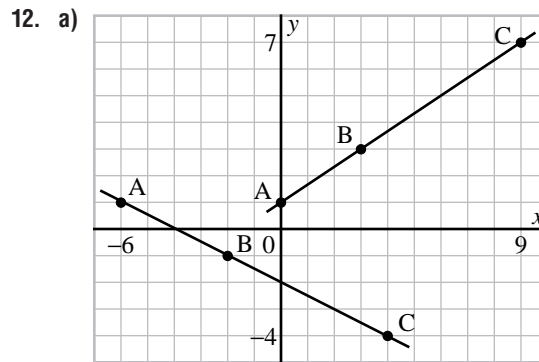
6. It is unlikely that two students will draw the same line and, hence, choose the same pair of points. There are an infinite number of lines with the same slope on a grid.



Selected Solutions — Chapter 4

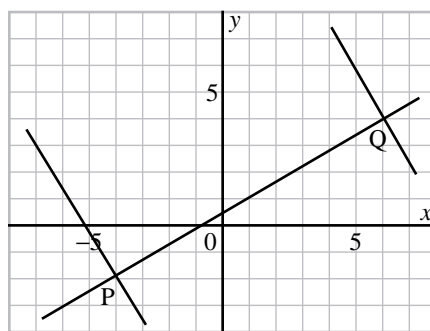


10. b) I used the formula for slope  $= \frac{y_2 - y_1}{x_2 - x_1}$ , where  $(x_1, y_1)$  and  $(x_2, y_2)$  are the coordinates of two points on the line. I know the slope is 4, so I substituted  $y_2 = k$ ,  $y_1 = 8$ ,  $x_2 = 2$ , and  $x_1 = 3$  to get  $4 = \frac{k - 8}{2 - 3}$ . This simplifies to  $4 = \frac{k - 8}{-1}$ , or  $4 = 8 - k$ . I solved this equation to get  $k = 4$ .



## Selected Solutions — Chapter 4

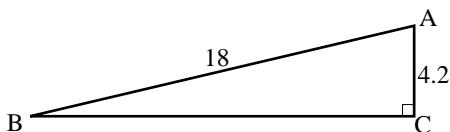
17. a)



18. Draw a diagram to represent how far the space shuttle travels in 1 min.

$$\begin{aligned}
 \text{Distance} &= \text{Speed} \times \text{Time} \\
 &= 1080 \text{ km/h} \times \frac{1}{60} \text{ h} \\
 &= \frac{1080 \text{ km}}{60} \\
 &= 18 \text{ km}
 \end{aligned}$$

During this time, the shuttle drops 4200 m, or 4.2 km.



Use the Pythagorean Theorem to determine the run, BC.

$$\begin{aligned}
 BC &= \sqrt{18^2 - 4.2^2} \\
 &= \sqrt{306.36} \\
 &\doteq 17.5031
 \end{aligned}$$

$$\begin{aligned}
 \text{Slope} &= \frac{\text{rise}}{\text{run}} \\
 &\doteq \frac{4.2}{17.5031} \\
 &\doteq 0.2399 \\
 &\doteq 0.24
 \end{aligned}$$

The slope of the reentry path was 0.24.

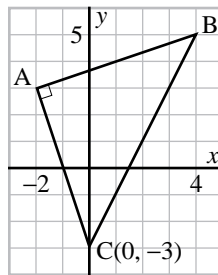
### Modelling the Space Shuttle's Reentry Path

The force of gravity may cause the path of the shuttle to curve. The straight line path may not take the shuttle to the desired landing site. The slope would approach 0 as the shuttle approached the ground.

19. a) Plot the points on a grid. Join AB. From the grid, the slope of AB is  $\frac{1}{3}$ . Angle A =  $90^\circ$ , so the slope of AC is  $-3$ .

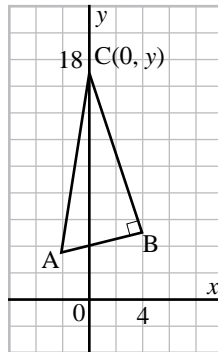
From A, draw a line with slope  $-3$ ; that is, move 3 down and 1 right, to reach  $(-1, 0)$ , then draw a line through A and this point. Extend the line until it meets the y-axis at  $(-3, 0)$ . This is point C.

## Selected Solutions — Chapter 4

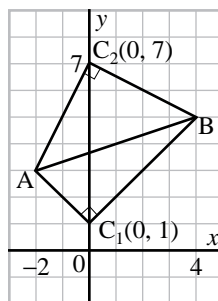


- b) Plot the points on a grid. Join AB. As in part a, the slope of AB is  $\frac{1}{3}$ . Angle B =  $90^\circ$ , so the slope of BC is  $-3$ .

From B, move 3 up and 1 left to reach (3, 8), then draw a line through B and this point. Extend the line until it meets the y-axis at (0, 17). This is point C.



- c) Plot the points on a grid. Join AB. The right angle is at point C. With a set square, or the corner of a piece of paper, place the right angle on the y-axis. Move it up and down, and rotate it until the arms of the right angle pass through A and B. This occurs twice: at (0, 7) with C above AB; and at (0, 1) with C below AB.



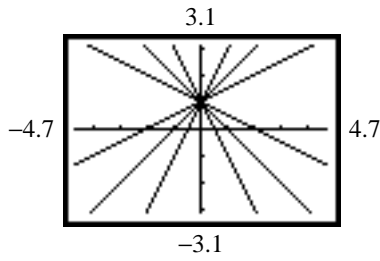
Count squares to calculate the rise and run for AC and CB in each case, to check that the segments are perpendicular.

**Exploring with a Graphing Calculator: Investigating  $y = mx + b$ , page 213**

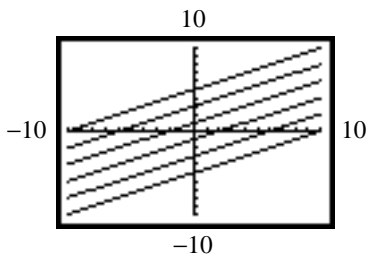
1. b) Answers may vary. The first screen shows more of the graph, but the scales on the axes are different. The second screen has equal scales on the axes, but does not show as much of the graph.

Selected Solutions — Chapter 4

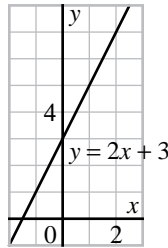
2. a)



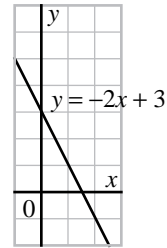
3. a)



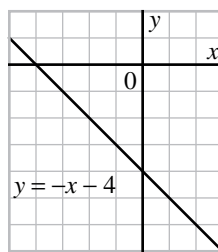
5. a)



b)

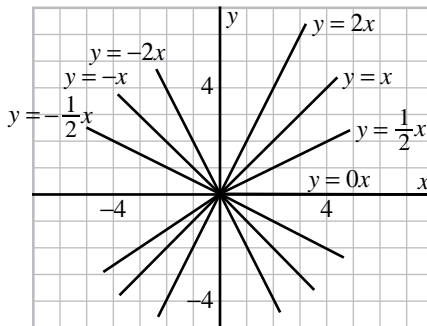


c)

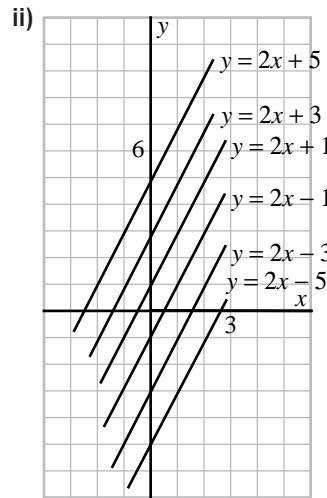
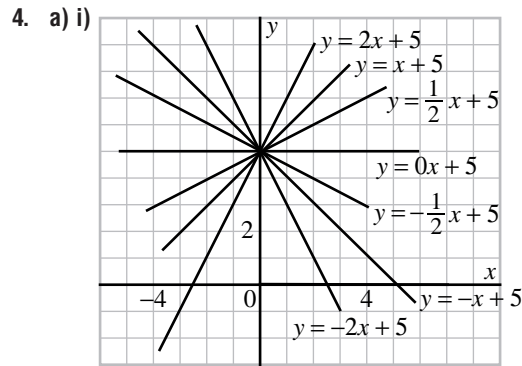


Investigate, page 214

1. a)

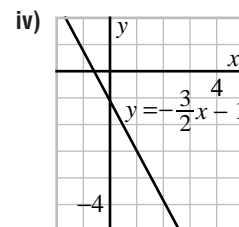
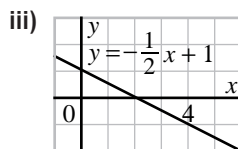
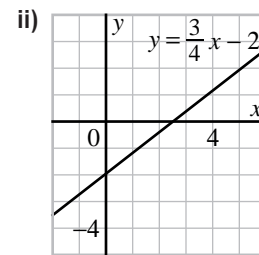
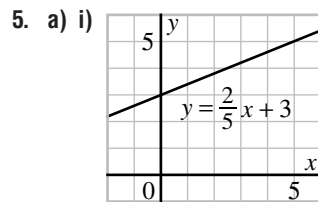


Selected Solutions — Chapter 4

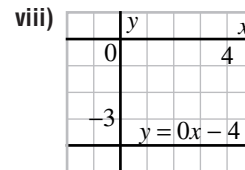
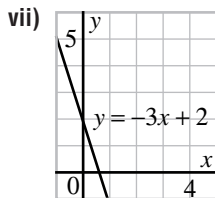
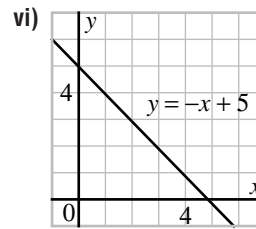
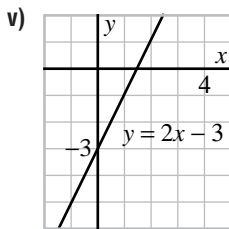


4.3 Exercises, page 217

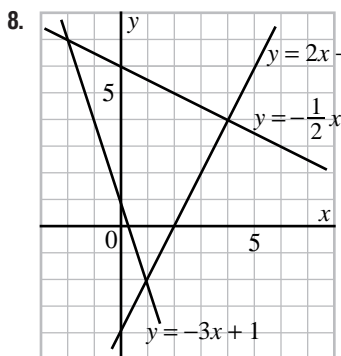
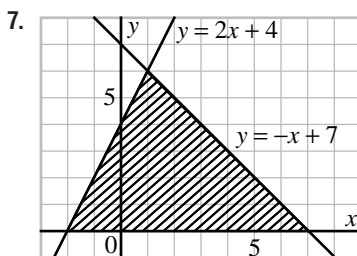
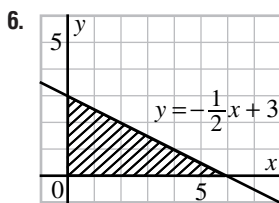
4. b) Answers may vary. For part b: From the graph, I counted squares to find the rise and run, then calculated the slope as  $-\frac{3}{2}$ . The y-intercept is  $-3$ . I substituted  $m = -\frac{3}{2}$  and  $b = -3$  in the equation  $y = mx + b$  to get  $y = -\frac{3}{2}x - 3$ .



Selected Solutions — Chapter 4



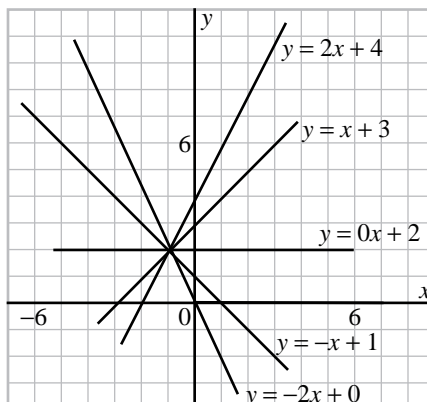
- b) Answers may vary. For part i: From the equation, the slope is  $\frac{2}{5}$ . The y-intercept is 3. The corresponding point has coordinates (0, 3). I drew x, y axes on a grid. From the point (0, 3), I moved 2 up and 5 right to reach the point (5, 5). I drew a line through (0, 3) and (5, 5), and extended it in both directions.



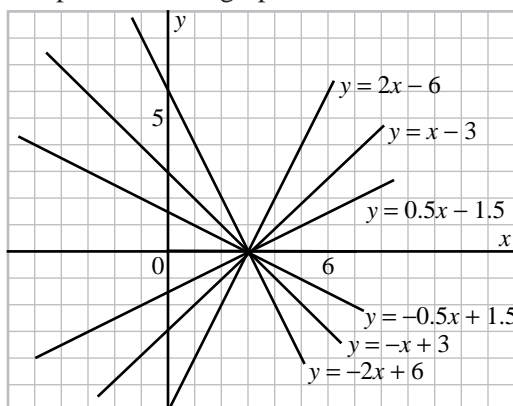
9. a) i) The pattern in the equations is that the y-intercept is 2 greater than the slope.  
 ii) The pattern in the equations is that the y-intercept is the opposite of 3 times the slope.

## Selected Solutions — Chapter 4

- b) i) The pattern in the graphs is that all the lines intersect at  $(-1, 2)$ .



- ii) The pattern in the graphs is that all the lines intersect at  $(3, 0)$ .



- c) i) Let  $y = mx + (m + 2)$  represent any one of the equations. Verify that  $(-1, 2)$  satisfies this equation, no matter what the value of  $m$  is. When  $x = -1$  and  $y = 2$ , the left side becomes 2 and the right side becomes  $m(-1) + m + 2$ , or 2. Hence, the equation is satisfied. This means that every line whose  $y$ -intercept is 2 greater than the slope passes through  $(-1, 2)$ .
- ii) Let  $y = mx - 3m$  represent any one of the equations. Verify that  $(3, 0)$  satisfies this equation. When  $x = 3$  and  $y = 0$ , the left side becomes 0 and the right side becomes  $3m - 3m$ , or 0. Hence, the equation is satisfied. This means that every line whose  $y$ -intercept is the opposite of 3 times the slope passes through  $(3, 0)$ .

10. b) Answers may vary. For part iii: Since the line passes through  $A(3, -2)$ , its coordinates satisfy the equation. I substituted  $x = 3$  and  $y = -2$ , then solved the equation for  $b$ .

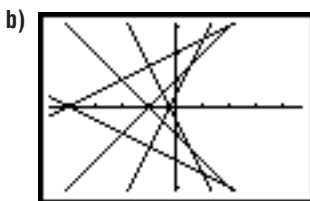
$$-2 = 3(3) + b$$

$$-2 = 9 + b$$

$$b = -11$$

12. a) The pattern in the equations is that the slope and  $y$ -intercept are reciprocals.

## Selected Solutions — Chapter 4

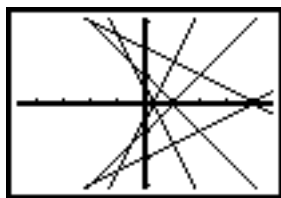


The graphs appear to form a curve. Pairs of graphs are symmetrical about the  $x$ -axis.

- c) Visualize what happens if the  $y$ -intercept starts out very small and positive, and begins to increase. If the  $y$ -intercept is very small, the slope is very large (for example, when the  $y$ -intercept is 0.1, the slope is 10). As the  $y$ -intercept increases, the slope decreases. Hence, the lines form the upper part of the curve that can be seen in the first quadrant. The situation is similar if the  $y$ -intercept and slope are negative, and everything is reflected in the  $x$ -axis. Hence, the pattern is symmetrical about the  $x$ -axis.

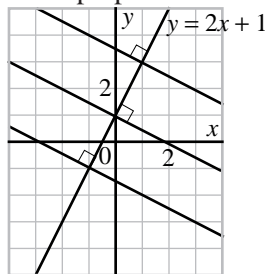
13. Answers may vary. The equations could be:

$$\begin{array}{ll} y = x - 1 & y = -x + 1 \\ y = 2x - 0.5 & y = -2x + 0.5 \\ y = 0.5x - 2 & y = -0.5x + 2 \end{array}$$



The graphs form a curve, similar to that in exercise 12, but the graphs have been reflected in the  $y$ -axis.

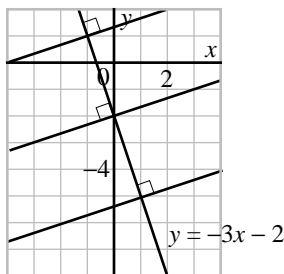
14. Lines perpendicular to  $y = 2x + 1$  may vary.



15. c) There are an infinite number of lines perpendicular to a given line, but they all have the same slope.

## Selected Solutions — Chapter 4

16. a) Lines perpendicular to  $y = -3x - 2$  may vary.

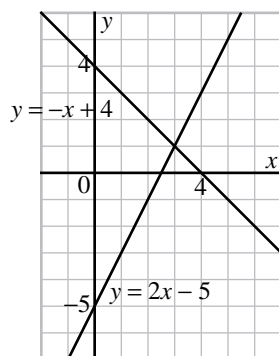


18. Predictions may vary.

- a) All the lines have  $y$ -intercept 3. The slope of each line is 10 times as great as the slope of the preceding line, so the lines become more vertical.
- b) All the lines have  $x$ -intercept  $-3$ . The slope of each line is 10 times as great as the slope of the preceding line, and the  $y$ -intercept is 10 times as great as that of the preceding line. The lines move farther up the  $y$ -axis and become more vertical.

22. Draw the line  $y = 2x - 5$ . Its slope is 2 and its  $y$ -intercept is  $-5$ . Draw the line  $y = -x + 4$ . Its slope is  $-1$  and its  $y$ -intercept is 4.

From the graph, the lines intersect at  $(3, 1)$ .



- a) The line that passes through  $(3, 1)$  and is parallel to  $y = \frac{2}{3}x + 4$  has slope  $\frac{2}{3}$ .

On the graph, from the point  $(3, 1)$ , move 2 down and 3 left to reach the point  $(0, -1)$ . These are the coordinates of the  $y$ -intercept.

The equation of the line is  $y = \frac{2}{3}x - 1$ .

- b) The line that passes through  $(3, 1)$  and is perpendicular to  $y = \frac{3}{4}x - 1$  has slope  $-\frac{4}{3}$ .

On the graph, from the point  $(3, 1)$ , move 4 up and 3 left to reach the point  $(0, 5)$ . These are the coordinates of the  $y$ -intercept.

The equation of the line is  $y = -\frac{4}{3}x + 5$ .

## Selected Solutions — Chapter 4

*Linking Ideas: Mathematics and Technology**Patterns in Equations and Lines, page 221*

3. b) Given the pattern generated from exercises 1 and 2, other lines are:

$$y = x + 1$$

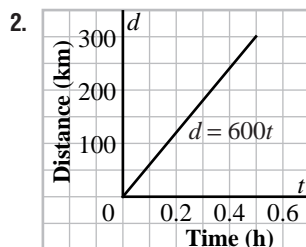
$$y = 2x + 4$$

$$y = 3x + 9$$

$$y = 4x + 16$$

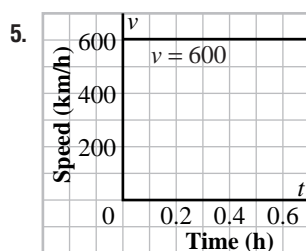
In general, the line is defined by  $y = nx + n^2$ , where  $n$  is any number.

4. The line with the least slope is plotted first. As the other lines are plotted, the  $y$ -intercept moves down the  $y$ -axis. None of the lines enters the white space at the bottom of the plot bounded by the curve made by the lines. The  $y$ -intercept reaches the origin. The line is horizontal. The following lines have a positive slope. The  $y$ -intercept moves up the axis. None of the positive slope lines enters the white space bounded by the curve made by the lines.
5. The space above the white space fills up with lines until it is densely shaded.

*Linking Ideas: Mathematics and Science**How Slope Applies to Speed and Acceleration, page 222*

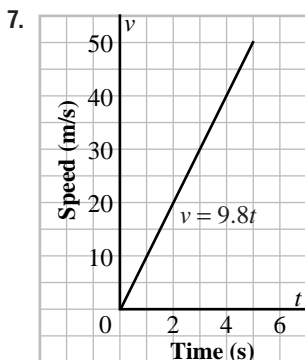
- c) The line passes through the origin, and has slope 600. I used the equation  $y = mx + b$ , and substituted  $y = d$ ,  $m = 600$ ,  $x = t$ , and  $b = 0$ , to get  $d = 600t$ .

4. The plane is travelling at a constant speed of 600 km/h, so the acceleration is 0.



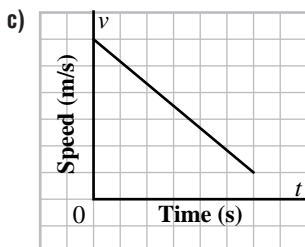
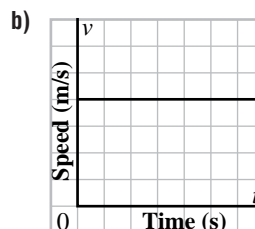
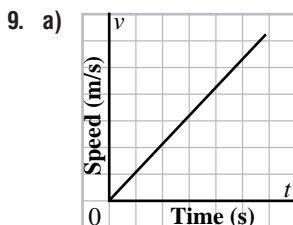
- c) The line is horizontal and has  $v$ -intercept 600. I used the equation  $y = mx + b$ , and substituted  $y = v$ ,  $m = 0$ ,  $x = t$ , and  $b = 600$ , to get  $v = 600$ .

Selected Solutions — Chapter 4

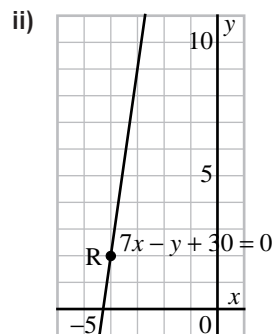
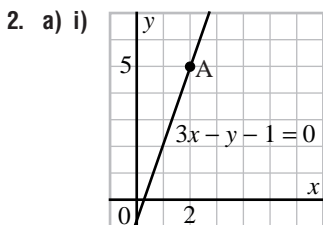


c) The line passes through the origin, and has slope 9.8. I used the equation  $y = mx + b$ , and substituted  $y = v$ ,  $m = 9.8$ ,  $x = t$ , and  $b = 0$ , to get  $v = 9.8t$ .

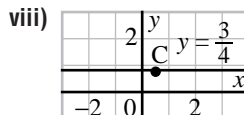
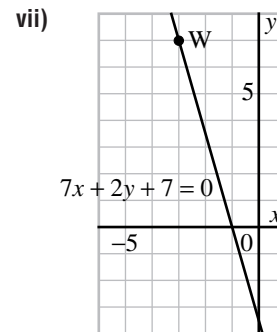
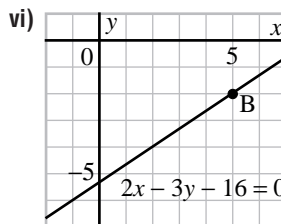
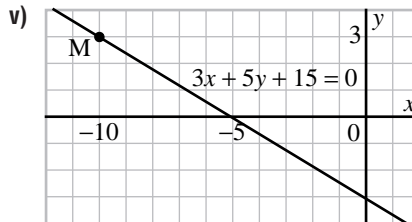
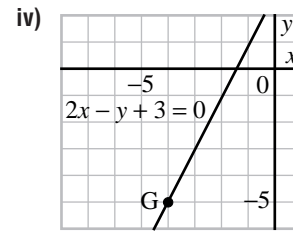
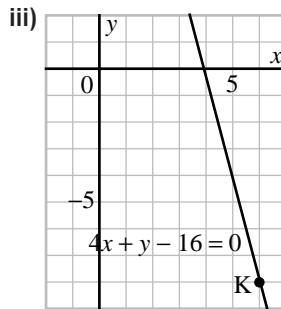
8. The slope of a distance-time graph is speed. The slope of a speed-time graph is acceleration.



4.4 Exercises, page 226



Selected Solutions — Chapter 4



b) Answers may vary. For part v: The line passes through  $M(-10, 3)$ , and has slope  $-\frac{3}{5}$ . I let  $P(x, y)$  be any point on the line. Then, slope of  $MP$  is  $-\frac{3}{5} = \frac{y-3}{x+10}$ .

I simplified the equation using the shortcut method.

$$\begin{aligned} -3(x + 10) &= 5(y - 3) \\ -3x - 30 &= 5y - 15 \end{aligned}$$

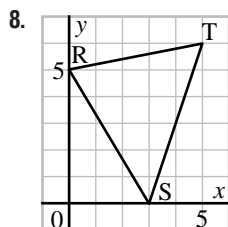
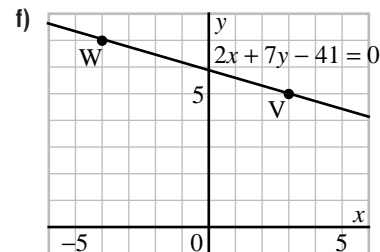
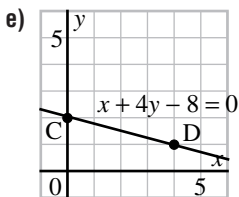
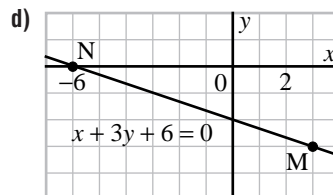
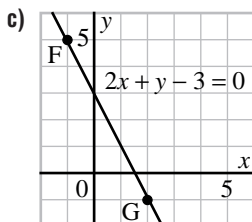
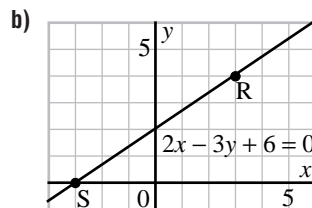
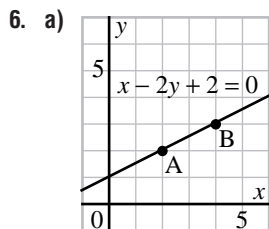
I collected all the terms on the right side  $0 = 3x + 5y + 15$ . This is the equation of the line.

4. b) Answers may vary. For part iii: If the line  $3x - y + 14 = 0$  passes through  $(-4, 2)$ , then the coordinates of the point satisfy the equation. I substituted  $x = -4$  and  $y = 2$ .

$$\begin{aligned} \text{L.S.} &= 3x - y + 14 & \text{R.S.} &= 0 \\ &= 3(-4) - 2 + 14 \\ &= -12 - 2 + 14 \\ &= 0 \end{aligned}$$

Since  $\text{L.S.} = \text{R.S.} = 0$ , the line passes through the point.

Selected Solutions — Chapter 4



9. b) Answers may vary. For part vii: The  $x$ -intercept has coordinates  $R(-\frac{2}{3}, 0)$ . The slope is  $\frac{5}{6}$ . I let  $P(x, y)$  be any point on the line. Then the slope of  $RP$  is  $\frac{5}{6} = \frac{y - 0}{x - (-\frac{2}{3})}$ .

I used the shortcut method to simplify the equation.

$$5(x + \frac{2}{3}) = 6y$$

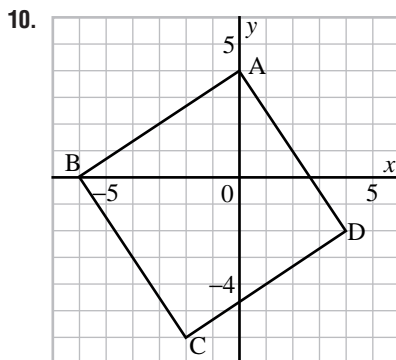
$$5x + \frac{10}{3} = 6y$$

I collected all the terms on the left side.

$$5x - 6y + \frac{10}{3} = 0$$

This is the equation of the line. I multiplied every term by 3 to get rid of the fraction:  $15x - 18y + 10 = 0$ .

## Selected Solutions — Chapter 4



11. b) Answers may vary. For part iii: Since the line  $3x + 2y + k$  passes through the point  $C(5, 2)$ , its coordinates must satisfy the equation of the line. I substituted  $x = 5$  and  $y = 2$  in the equation, then solved for  $k$ .

$$\begin{aligned} 3(5) + 2(2) + k &= 0 \\ 15 + 4 + k &= 0 \\ k &= -19 \end{aligned}$$

14. a) Refer to the diagram on page 228 of the student text.

- b) For the line AB: From the graph, the coordinates are  $A(4, 8)$  and  $B(1, 2)$ , and the slope of AB is  $\frac{6}{3}$ , or 2. Let  $P(x, y)$  be a point on

line AB. Then the slope of PB is  $2 = \frac{y-2}{x-1}$

$$2(x-1) = y-2$$

$$2x-2 = y-2$$

$$2x-y=0$$

The equation of AB is  $2x - y = 0$ .

For the line CD: From the graph, the coordinates are  $C(8, 4)$  and  $D(2, 1)$ , and the slope of CD is  $\frac{3}{6}$ , or  $\frac{1}{2}$ .

Let  $P(x, y)$  be a point on line CD.

Then, the slope of PC is  $\frac{1}{2} = \frac{y-4}{x-8}$

$$x-8 = 2y-8$$

$$x-2y=0$$

The equation of CD is  $x - 2y = 0$ .

For the line EF: From the graph, the coordinates are  $E(-2, 6)$  and  $F(-1, 3)$ , and the slope of EF is  $\frac{-3}{1}$ , or  $-3$ .

Let  $P(x, y)$  be a point on line EF.

Then, the slope of PE is  $-3 = \frac{y-6}{x+2}$

$$-3x-6 = y-6$$

$$3x+y=0$$

The equation of EF is  $3x + y = 0$ .

For the line GH: From the graph, the coordinates are  $G(6, -2)$  and  $H(3, -1)$ , and the slope of GH is  $-\frac{1}{3}$ .

## Selected Solutions — Chapter 4

Let  $P(x, y)$  be a point on line GH.

Then, slope of PG is  $-\frac{1}{3} = \frac{y+2}{x-6}$

$$-x + 6 = 3y + 6$$

$$x + 3y = 0$$

The equation of GH is  $x + 3y = 0$ .

All the lines pass through the origin.

- c) For the line AI: From the graph, the coordinates are  $A(4, 8)$  and  $I(10, 11)$ , and the slope of AI is  $\frac{3}{6}$ , or  $\frac{1}{2}$ .

Let  $P(x, y)$  be a point on line AI.

Then, slope of PA is  $\frac{1}{2} = \frac{y-8}{x-4}$

$$x - 4 = 2y - 16$$

$$x - 2y + 12 = 0$$

The equation of AI is  $x - 2y + 12 = 0$ .

For the line CJ: From the graph, the coordinates are  $C(8, 4)$  and  $J(11, 10)$ , and the slope of CJ is  $\frac{6}{3}$ , or 2.

Let  $P(x, y)$  be a point on line CJ.

Then, slope of PC is  $2 = \frac{y-4}{x-8}$

$$2x - 16 = y - 4$$

$$2x - y - 12 = 0$$

The equation of CJ is  $2x - y - 12 = 0$ .

For the line KL: From the graph, the coordinates are  $K(6, 14)$  and  $L(9, 13)$ , and the slope of KL is  $-\frac{1}{3}$ .

Let  $P(x, y)$  be a point on line KL.

Then, slope of PK is  $-\frac{1}{3} = \frac{y-14}{x-6}$

$$-x + 6 = 3y - 42$$

$$x + 3y - 48 = 0$$

The equation of KL is  $x + 3y - 48 = 0$ .

For the line MN: From the graph, the coordinates are  $M(14, 6)$  and  $N(13, 9)$ , and the slope of MN is  $-\frac{3}{1}$ , or  $-3$ .

Let  $P(x, y)$  be a point on line MN.

Then, slope of PM is  $-3 = \frac{y-6}{x-14}$

$$-3x + 42 = y - 6$$

$$3x + y - 48 = 0$$

The equation of MN is  $3x + y - 48 = 0$ .

Extend each of the 4 lines above on the grid. All of them appear to pass through the point  $(12, 12)$ .

To verify, substitute the coordinates  $(12, 12)$  in each equation in turn.

## Selected Solutions — Chapter 4

For the line AI:  $x - 2y + 12 = 0$

$$\begin{aligned} \text{L.S.} &= 12 - 2(12) + 12 & \text{R.S.} &= 0 \\ &= 12 - 24 + 12 \\ &= 0 \end{aligned}$$

For the line CJ:  $2x - y - 12 = 0$

$$\begin{aligned} \text{L.S.} &= 2(12) - 12 - 12 & \text{R.S.} &= 0 \\ &= 24 - 24 \\ &= 0 \end{aligned}$$

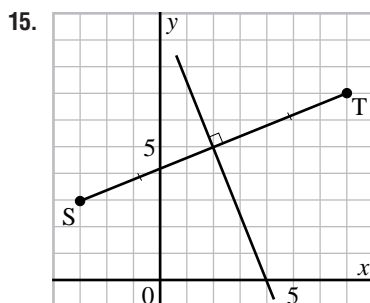
For the line KL:  $x + 3y - 48 = 0$

$$\begin{aligned} \text{L.S.} &= 12 + 3(12) - 48 & \text{R.S.} &= 0 \\ &= 12 + 36 - 48 \\ &= 0 \end{aligned}$$

For the line MN:  $3x + y - 48 = 0$

$$\begin{aligned} \text{L.S.} &= 3(12) + 12 - 48 & \text{R.S.} &= 0 \\ &= 36 - 36 \\ &= 0 \end{aligned}$$

For each line, L.S. = R.S., so the point lies on each line.



b) The midpoint of ST has coordinates  $M(\frac{-3+7}{2}, \frac{3+7}{2})$ , or  $M(2, 5)$ .

From the graph, the slope of ST is  $\frac{4}{10}$ , or  $\frac{2}{5}$ .

So, the slope of the perpendicular to ST is  $-\frac{5}{2}$ .

Draw the line through  $M(2, 5)$  with slope  $-\frac{5}{2}$ .

c) Let  $P(x, y)$  be any point on the perpendicular bisector.

$$\begin{aligned} \text{Then, slope of PM is } -\frac{5}{2} &= \frac{y-5}{x-2} \\ -5x + 10 &= 2y - 10 \\ 5x + 2y - 20 &= 0 \end{aligned}$$

The equation of the perpendicular bisector is

$$5x + 2y - 20 = 0.$$

16. a) i) The midpoint of BC is  $M(\frac{5+9}{2}, \frac{-1+9}{2})$ , or  $M(7, 4)$ . Draw the line through AM.

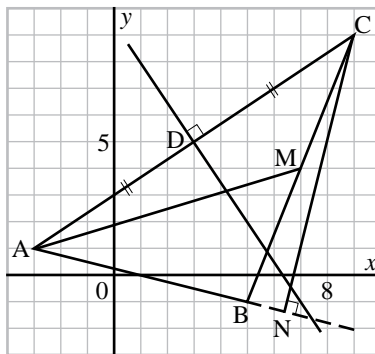
ii) From the graph, the slope of AB is  $-\frac{2}{8}$ , or  $-\frac{1}{4}$ .

So, the slope of the altitude is 4. Draw the line through C with slope 4 to meet AB extended to N.

## Selected Solutions — Chapter 4

iii) The midpoint of AC is  $D(\frac{-3+9}{2}, \frac{1+9}{2})$ , or  $D(3, 5)$ .

From the graph, the slope of AC is  $\frac{8}{12}$ , or  $\frac{2}{3}$ . So, the slope of the perpendicular bisector is  $-\frac{3}{2}$ . Draw the line through D with slope  $-\frac{3}{2}$ .



b) i) For line AM: From the graph, the coordinates of M are (7, 4) and the slope of AM is  $\frac{3}{10}$ . Let  $P(x, y)$  be a point on AM.

$$\text{Then, slope of PM is } \frac{3}{10} = \frac{y-4}{x-7}$$

$$3x - 21 = 10y - 40$$

$$3x - 10y + 19 = 0$$

The median from A to BC has equation  $3x - 10y + 19 = 0$ .

ii) For line CN: The coordinates of C are (9, 9). From part a, the slope of CN is 4. Let  $P(x, y)$  be a point on CN.

$$\text{Then, slope of PC is } 4 = \frac{y-9}{x-9}$$

$$4x - 36 = y - 9$$

$$4x - y - 27 = 0$$

The altitude from C to AB has equation  $4x - y - 27 = 0$ .

iii) For the perpendicular bisector of AC: From part a, D has coordinates (3, 5), and the perpendicular bisector has slope  $-\frac{3}{2}$ . Let  $P(x, y)$  be a point on the perpendicular bisector.

$$\text{Then slope of PD is } -\frac{3}{2} = \frac{y-5}{x-3}$$

$$-3x + 9 = 2y - 10$$

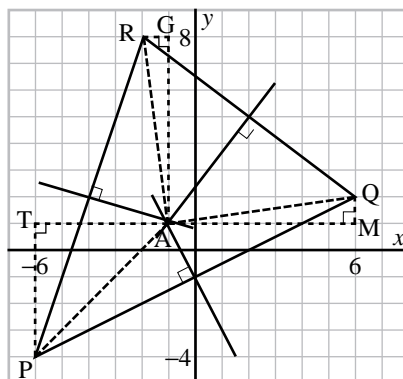
$$3x + 2y - 19 = 0$$

The perpendicular bisector of AC has equation

$$3x + 2y - 19 = 0.$$

## Selected Solutions — Chapter 4

17.



a) For side RQ: The points are  $R(-2, 8)$  and  $Q(6, 2)$ . From the graph, the slope of RQ is  $-\frac{6}{8}$ , or  $-\frac{3}{4}$ . Let  $B(x, y)$  be a point on RQ.

$$\begin{aligned} \text{Then, the slope of BR is } -\frac{3}{4} &= \frac{y-8}{x+2} \\ -3x - 6 &= 4y - 32 \\ 3x + 4y - 26 &= 0 \end{aligned}$$

The equation of RQ is  $3x + 4y - 26 = 0$ .

For side RP: The points are  $R(-2, 8)$  and  $P(-6, -4)$ . From the graph, the slope of RP is  $\frac{12}{4}$ , or 3. Let  $C(x, y)$  be a point on RP.

$$\begin{aligned} \text{Then, slope of CR is } 3 &= \frac{y-8}{x+2} \\ 3x + 6 &= y - 8 \\ 3x - y + 14 &= 0 \end{aligned}$$

The equation of RP is  $3x - y + 14 = 0$ .

For side QP: The points are  $Q(6, 2)$  and  $P(-6, -4)$ . From the graph, the slope of QP is  $\frac{6}{12}$ , or  $\frac{1}{2}$ . Let  $D(x, y)$  be a point on QP.

$$\begin{aligned} \text{Then, slope of DQ is } \frac{1}{2} &= \frac{y-2}{x-6} \\ x - 6 &= 2y - 4 \\ x - 2y - 2 &= 0 \end{aligned}$$

The equation of QP is  $x - 2y - 2 = 0$ .

b) For side RQ: The midpoint has coordinates  $(\frac{-2+6}{2}, \frac{8+2}{2})$ , or  $(2, 5)$ . The slope of the perpendicular bisector is  $\frac{4}{3}$ . Let  $B(x, y)$  be a point on the perpendicular bisector.

$$\begin{aligned} \text{Then, } \frac{4}{3} &= \frac{y-5}{x-2} \\ 4x - 8 &= 3y - 15 \\ 4x - 3y + 7 &= 0 \end{aligned}$$

The equation of the perpendicular bisector of RQ is  $4x - 3y + 7 = 0$ .

For side RP: The midpoint has coordinates  $(\frac{-2-6}{2}, \frac{-4+8}{2})$ , or  $(-4, 2)$ . The slope of the perpendicular bisector is  $-\frac{1}{3}$ .

## Selected Solutions — Chapter 4

Let  $C(x, y)$  be a point on the perpendicular bisector.

$$\text{Then, } -\frac{1}{3} = \frac{y-2}{x+4}$$

$$-x - 4 = 3y - 6$$

$$x + 3y - 2 = 0$$

The equation of the perpendicular bisector of RP is

$$x + 3y - 2 = 0.$$

For side QP: The midpoint has coordinates

$$\left(\frac{-6+6}{2}, \frac{2-4}{2}\right), \text{ or } (0, -1).$$

The slope of the perpendicular bisector is  $-2$ . Let  $D(x, y)$  be a point on the perpendicular bisector.

$$\text{Then, } -2 = \frac{y+1}{x-0}$$

$$-2x = y + 1$$

$$2x + y + 1 = 0$$

The equation of the perpendicular bisector of QP is

$$2x + y + 1 = 0.$$

c) From the graph, the perpendicular bisectors intersect at  $A(-1, 1)$ .

d) For the length of AP, consider right  $\triangle APT$ .

Count squares to determine  $AT = 5$  and  $PT = 5$ .

$$\begin{aligned} \text{Use the Pythagorean Theorem: } AP &= \sqrt{5^2 + 5^2} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \end{aligned}$$

For the length of AQ, consider right  $\triangle AMQ$ .

Count squares to determine  $AM = 7$  and  $MQ = 1$ .

$$\begin{aligned} \text{Use the Pythagorean Theorem: } AQ &= \sqrt{7^2 + 1^2} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \end{aligned}$$

For the length of AR, consider right  $\triangle ARG$ .

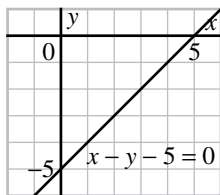
Count squares to determine  $RG = 1$  and  $AG = 7$ .

$$\begin{aligned} \text{Use the Pythagorean Theorem: } AR &= \sqrt{1^2 + 7^2} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \end{aligned}$$

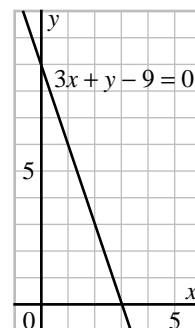
## Selected Solutions — Chapter 4

## 4.5 Exercises, page 233

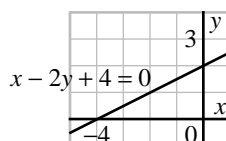
4. a) i)



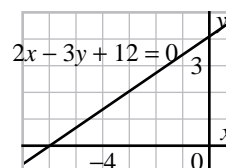
ii)



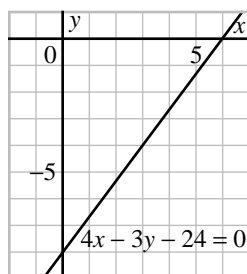
iii)



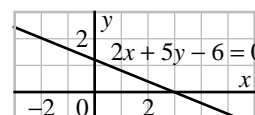
iv)



v)



vi)



b) Answers may vary. For part iv: For  $2x - 3y + 12 = 0$ ; to determine the  $x$ -intercept, I substituted  $y = 0$ , then solved for  $x$ .

$$\begin{aligned} 2x - 3(0) + 12 &= 0 \\ 2x &= -12 \\ x &= -6 \end{aligned}$$

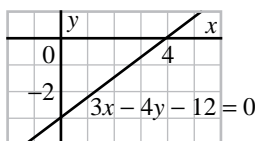
The  $x$ -intercept is  $-6$ .

To determine the  $y$ -intercept, I substituted  $x = 0$ , then solved for  $y$ .

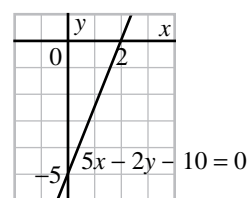
$$\begin{aligned} 2(0) - 3y + 12 &= 0 \\ 3y &= 12 \\ y &= 4 \end{aligned}$$

The  $y$ -intercept is  $4$ .

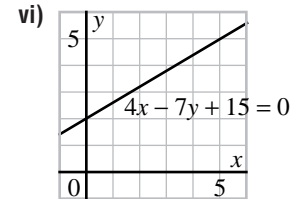
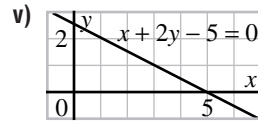
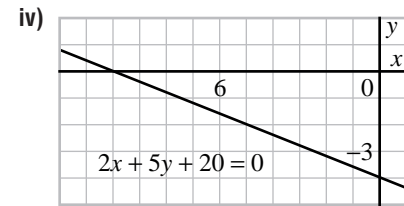
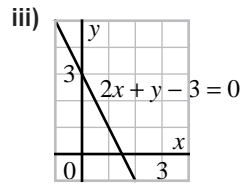
6. a) i)



ii)

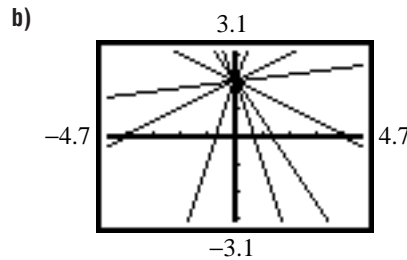


Selected Solutions — Chapter 4



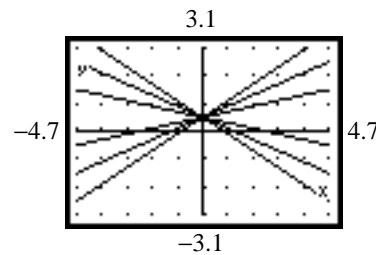
b) Answers may vary. For part iv:  $2x + 5y + 20 = 0$ ; I solved the equation for  $y$ .  $5y = -2x - 20$ , then divided each term by 5 to get  $y = -\frac{2}{5}x - 4$ . From the equation in this form, the slope of the line is  $-\frac{2}{5}$ , and the  $y$ -intercept is  $-4$ .

7. a) The  $y$ -coefficients are the same, the constant terms are the same, and the  $x$ -coefficients are different.

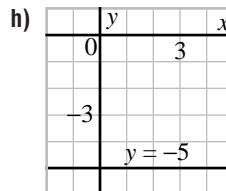
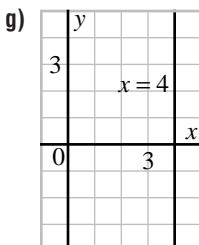
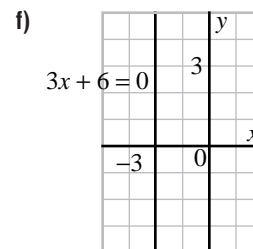
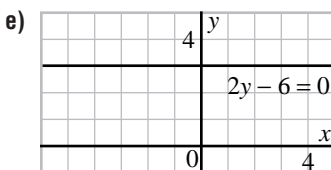
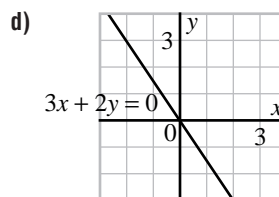
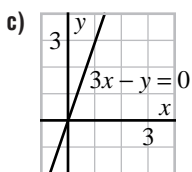
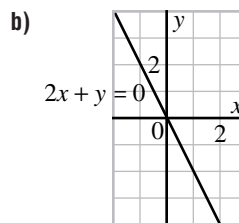
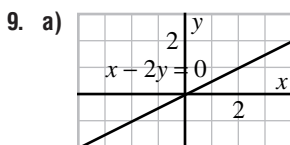
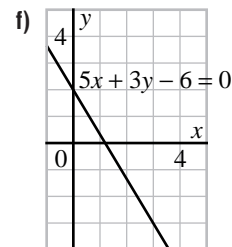
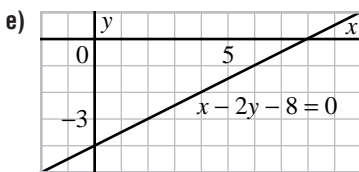
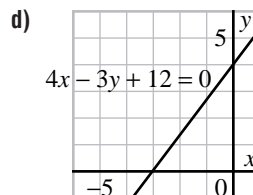
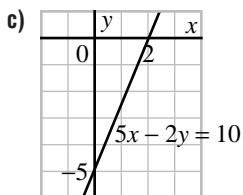
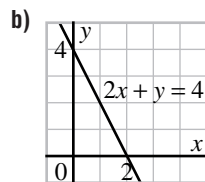
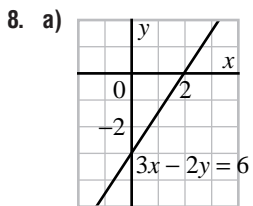


All the lines have the same  $y$ -intercept. The  $y$ -intercept is the opposite of the constant term divided by the coefficient of  $y$ . Since the  $y$ -coefficients are the same and the constant terms are the same, the  $y$ -intercepts will be the same.

c) Answers may vary.  $2x + 10y - 5 = 0$ ;  $4x + 10y - 5 = 0$ ,  $6x + 10y - 5 = 0$ ,  $-2x + 10y - 5 = 0$ ,  $-4x + 10y - 5 = 0$ ,  $-6x + 10y - 5 = 0$ . All the lines have the same  $y$ -intercept, which is 0.5.



Selected Solutions — Chapter 4



## Selected Solutions — Chapter 4

13. Answers may vary. For exercise 11: For the line  $2x - y + 8 = 0$ , I solved for  $y$  to get the equation in the  $y = mx + b$  form; that is,  $y = 2x + 8$ . In this form, I identified the slope as 2 and  $y$ -intercept as 8. The lines intersect at  $(0, 8)$ . The slope of the perpendicular line is  $-\frac{1}{2}$ . The equation of the perpendicular line is  $-\frac{1}{2} = \frac{y-8}{x-0}$ , which simplifies to  $-x = 2y - 16$ , or  $x + 2y - 16 = 0$ .

For exercise 12: For the line  $2x + y + 6 = 0$ , I solved for  $y$  to get the equation in the  $y = mx + b$  form; that is,  $y = -2x - 6$ . In this form, I identified the slope as  $-2$ . To find the  $x$ -intercept, I substituted  $y = 0$  to get  $0 = -2x - 6$ , which simplifies to  $x = -3$ . The lines intersect at  $(-3, 0)$ . The slope of the perpendicular line is  $\frac{1}{2}$ . The equation of the perpendicular line is  $\frac{1}{2} = \frac{y-0}{x+3}$ , which simplifies to  $x + 3 = 2y$ , or  $x - 2y + 3 = 0$ .

19. Answers may vary. For exercise 18: I wrote each equation in the form  $y = mx + b$ .

$$x + 2y - 4 = 0 \text{ becomes } 2y = -x + 4 \text{ or } y = -\frac{1}{2}x + 2.$$

The slope is  $-\frac{1}{2}$ , the  $y$ -intercept is 2, and the  $x$ -intercept is the value of  $x$  when  $y = 0$ ; that is,  $x = 4$ .

$$x - y + 2 = 0 \text{ becomes } y = x + 2.$$

The slope is 1, the  $y$ -intercept is 2, and the  $x$ -intercept is  $-2$ .

$$3x + 6y + 5 = 0 \text{ becomes } 6y = -3x - 5 \text{ or } y = -\frac{3}{6}x - \frac{5}{6}.$$

The slope is  $-\frac{1}{2}$ , the  $y$ -intercept is  $-\frac{5}{6}$ , and the  $x$ -intercept is  $-\frac{5}{3}$ .

$$2x - y - 8 = 0 \text{ becomes } y = 2x - 8.$$

The slope is 2, the  $y$ -intercept is  $-8$ , and the  $x$ -intercept is 4.

$$2x + 4y - 9 = 0 \text{ becomes } 4y = -2x + 9 \text{ or } y = -\frac{2}{4}x + \frac{9}{4}.$$

The slope is  $-\frac{1}{2}$ , the  $y$ -intercept is  $\frac{9}{4}$ , and the  $x$ -intercept is  $\frac{9}{2}$ .

$$4x - 3y + 6 = 0 \text{ becomes } 3y = 4x + 6 \text{ or } y = \frac{4}{3}x + \frac{6}{3}.$$

The slope is  $\frac{4}{3}$ , the  $y$ -intercept is 2, and the  $x$ -intercept is  $-\frac{3}{2}$ .

$$5x - 7y - 20 = 0 \text{ becomes } 7y = 5x + 20 \text{ or } y = \frac{5}{7}x + \frac{20}{7}.$$

The slope is  $\frac{5}{7}$ , the  $y$ -intercept is  $\frac{20}{7}$ , and the  $x$ -intercept is 4.

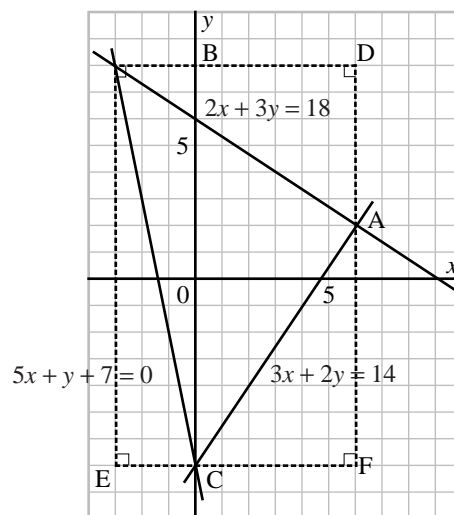
a) The lines  $2x - y - 8 = 0$  and  $5x - 7y - 20 = 0$  have the same  $x$ -intercept as  $x + 2y - 4 = 0$ .

b) The lines  $x - y + 2 = 0$  and  $4x - 3y + 6 = 0$  have the same  $y$ -intercept as  $x + 2y - 4 = 0$ .

c) The lines  $3x + 6y + 5 = 0$  and  $2x + 4y - 9 = 0$  have the same slope as  $x + 2y - 4 = 0$ .

## Selected Solutions — Chapter 4

21.



- a) To graph the line  $2x + 3y = 18$ , the  $x$ -intercept is 9 and the  $y$ -intercept is 6.

To graph the line  $5x + y + 7 = 0$ , the  $x$ -intercept is  $-\frac{7}{5}$  and the  $y$ -intercept is  $-7$ . Since  $-\frac{7}{5}$  is difficult to plot, use the slope,  $-5$ , instead. To graph the line  $3x - 2y = 14$ , the  $y$ -intercept is  $-7$  and the slope is  $\frac{3}{2}$ . From the graph, the vertices are  $A(6, 2)$ ,  $B(-3, 8)$ , and  $C(0, -7)$ .

- b) To calculate the lengths of the sides, consider the right triangle for which each side is a hypotenuse, then use the Pythagorean Theorem to calculate the length of the hypotenuse.

For AC, consider  $\triangle ACF$ .

$$AF = 9, CF = 6, AC = \sqrt{9^2 + 6^2} \\ = \sqrt{117}$$

For BC, consider  $\triangle EBC$ .

$$EC = 3, BE = 15, BC = \sqrt{3^2 + 15^2} \\ = \sqrt{234}$$

For AB, consider  $\triangle ABD$ .

$$BD = 9, AD = 6, AB = \sqrt{9^2 + 6^2} \\ = \sqrt{117}$$

- c) To calculate the slope, use the measures from part b.

The slope of AC is  $\frac{9}{6}$ , or  $\frac{3}{2}$ .

The slope of BC is  $-\frac{15}{3}$ , or  $-5$ .

The slope of AB is  $-\frac{6}{9}$ , or  $-\frac{2}{3}$ .

- d) Since  $AB = AC = \sqrt{117}$ , and slope of AB  $\times$  slope of AC  $= -1$ ,  $\triangle ABC$  is a right isosceles triangle.

## Selected Solutions — Chapter 4

$$\begin{aligned}
 \text{e) The area of the triangle is } & \frac{1}{2}(AB)(AC) \\
 & = \frac{1}{2}(\sqrt{117})(\sqrt{117}) \\
 & = \frac{117}{2} \\
 & = 58.5
 \end{aligned}$$

$$\begin{aligned}
 \text{The perimeter of the triangle is } & AB + AC + BC \\
 & = \sqrt{117} + \sqrt{117} + \sqrt{234} \\
 & = 2\sqrt{117} + \sqrt{234}
 \end{aligned}$$

The area is 58.5 square units and the perimeter is  $(2\sqrt{117} + \sqrt{234})$  units.

**Problem Solving: The Vanishing Square Puzzle, page 236**

1. There are 9 parts of rectangles above the horizontal line, and 9 below it. When the top 2 pieces are interchanged, the 9 top parts align with the 9 bottom parts to make only 9 rectangles.
2. The two trapezoids below the large right triangle in the top diagram do not fit exactly in the positions shown in the second diagram. They overlap the right triangle very slightly, and the little bits of the planes that are in the overlapped part would form an entire plane if they were all put together. The top edges of the two trapezoids in the bottom diagram are 0.077 units above the hypotenuse of the right triangle, and the overlapping part forms a long slender parallelogram with area 1 square unit.

Suppose there are coordinates in the top diagram with the lower left corner of the rectangle at  $(0, 0)$ . Then the coordinates of the endpoints of the hypotenuse of the triangle are  $(0, 6)$  and  $(13, 1)$ . The slope of this segment is  $\frac{6-1}{0-13} = -\frac{5}{13}$ . Now consider the larger trapezoid at the lower right of the top diagram. The vertical side on the right is 1 unit long, and the vertical side on the left looks like it is 4 units long, but it isn't. Let the coordinates of the top vertex on this side be  $(5, y)$ . Then the slope from this point to  $(13, 1)$  is  $\frac{y-1}{5-13}$ , or  $\frac{y-1}{-8}$ . Since its slope is also  $-\frac{5}{13}$ , we can equate the slopes and solve for  $y$  to obtain  $y = 4.077$ . When this trapezoid is placed on the second diagram so that its base just touches the two L-shaped pieces, its left side is 0.077 units too high, and extends above the top left corner of the diagram.

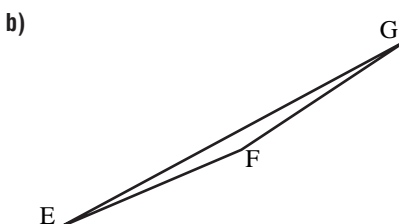
Using a similar analysis, we can show that the right side of this trapezoid is also too high by the same amount. Hence, the upper side of the large trapezoid is 0.077 units above the hypotenuse of the triangle.

The situation with the smaller trapezoid is similar. Its left side in the top diagram is 3 units high and, when it is placed on the second diagram, its top left corner will be 0.077 units too high. We can also show that its right side is too high by the same amount.

## Selected Solutions — Chapter 4

3. a) Both the vanishing plane puzzle and the vanishing square puzzle depend on the fact that the pieces do not fit exactly along the diagonal line. In the vanishing square puzzle, the pieces do not fit along the diagonal in either diagram. In the vanishing plane puzzle, they fit on the first diagram but not on the second.
- b) In both puzzles, one figure (rectangle in the first and plane in the second) can be considered as being divided into little bits and combined with the others. The way they are combined is different. The bits of rectangle are joined to the other rectangles (making them slightly longer), but the little bits of planes are superimposed on top of some of the other planes. In neither case can the figure that disappears be identified. That is, in the vanishing rectangle puzzle, we cannot say which rectangle disappeared and, in the vanishing plane puzzle, we cannot say which plane disappeared.

4. a) 1 square unit is the area of the “missing square.”



- c) For the first rectangle, EG is the diagonal of the rectangle. Use the Pythagorean Theorem.

$$EG^2 = 5^2 + 13^2$$

$$EG = \sqrt{194} \doteq 13.93$$

EF is the hypotenuse of the red triangle.

$$EF^2 = 2^2 + 5^2$$

$$EF = \sqrt{29} \doteq 5.39$$

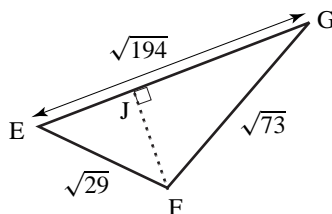
FG is the hypotenuse of the blue triangle.

$$FG^2 = 3^2 + 8^2$$

$$FG = \sqrt{73} \doteq 8.54$$

The perimeter is  $\sqrt{194} + \sqrt{29} + \sqrt{73} \doteq 27.86$  units, where 1 unit is the side of a small square.

- d) Draw the triangle from part b, distorted so the height from EG can be drawn.



Drop the perpendicular from F onto EG at J.

We want to calculate the length JF.

Use the Pythagorean Theorem in  $\triangle JGF$ .

## Selected Solutions — Chapter 4

$$\begin{aligned} JG^2 + JF^2 &= 73 \\ JF^2 &= 73 - JG^2 \end{aligned} \quad \textcircled{1}$$

Use the Pythagorean Theorem in  $\triangle EJF$ .

$$\begin{aligned} EJ^2 + JF^2 &= 29 \\ JF^2 &= 29 - EJ^2 \end{aligned} \quad \textcircled{2}$$

$$\text{From } \textcircled{1} \text{ and } \textcircled{2} \quad 73 - JG^2 = 29 - EJ^2 \quad \textcircled{3}$$

$$\text{But } EJ = \sqrt{194} - JG$$

Substitute for EJ in  $\textcircled{3}$ .

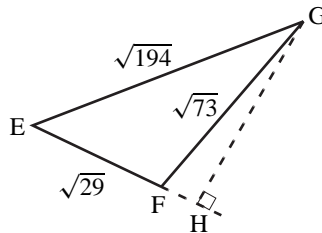
$$\begin{aligned} 73 - JG^2 &= 29 - (\sqrt{194} - JG)^2 \\ 73 - JG^2 &= 29 - (194 - 2\sqrt{194}JG + JG^2) \\ 73 - JG^2 &= 29 - 194 + 2\sqrt{194}JG - JG^2 \\ 73 - 29 + 194 &= 2\sqrt{194}JG \\ JG &= \frac{238}{2\sqrt{194}} \end{aligned}$$

Substitute for JG in  $\textcircled{1}$ .

$$\begin{aligned} JF^2 &= 73 - \left(\frac{238}{2\sqrt{194}}\right)^2 \\ &\doteq 0.0051546 \\ JF &\doteq 0.0718 \end{aligned}$$

The height of the triangle from EG is 0.0718 units, where 1 unit is the side of a small square.

- e) Draw the triangle from part b again, distorted so the height from EF can be drawn.



Drop the perpendicular from G onto EF extended to H.

We want to calculate the length GH.

Use the Pythagorean Theorem in  $\triangle EGH$ .

$$EG^2 = EH^2 + GH^2$$

$$194 = (\sqrt{29} + FH)^2 + GH^2$$

$$GH^2 = 194 - (\sqrt{29} + FH)^2 \quad \textcircled{1}$$

Use the Pythagorean Theorem in  $\triangle FGH$ .

$$FG^2 = FH^2 + GH^2$$

$$73 = FH^2 + GH^2 \quad \textcircled{2}$$

$$GH^2 = 73 - FH^2$$

$$\text{From } \textcircled{1} \text{ and } \textcircled{2}: \quad 194 - (\sqrt{29} + FH)^2 = 73 - FH^2$$

$$194 - 29 - 2\sqrt{29}FH - FH^2 = 73 - FH^2$$

$$92 = 2\sqrt{29}FH$$

$$FH = \frac{92}{2\sqrt{29}}$$

## Selected Solutions — Chapter 4

Substitute for FH in ②.

$$GH^2 = 73 - \left(\frac{92}{2\sqrt{29}}\right)^2$$

$$GH^2 \doteq 0.034\,482\,8$$

$$GH \doteq 0.1857$$

The height of the triangle from EF is 0.1857 units.

6. a) The sum of the angles in a triangle is  $180^\circ$ .  
 b) Answers may vary. Yes, as long as the adjacent sides of the cut-up triangle in the demonstration are collinear.

**Mathematical Modelling: It's All in the Packaging, page 239**

1. The full carton of books is represented by 1.

The thin books fill  $\frac{x}{30}$  of the carton.

The thick books fill  $\frac{y}{20}$  of the carton.

The equation is  $\frac{x}{30} + \frac{y}{20} = 1$ .

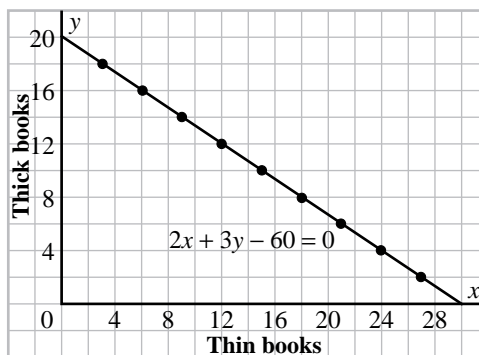
Multiply each side by the common denominator 60.

$$60\left(\frac{x}{30}\right) + 60\left(\frac{y}{20}\right) = 60(1)$$

$$2x + 3y = 60$$

$$\text{or } 2x + 3y - 60 = 0$$

2. a), b)



- c) There is more than one solution because more than one combination of thick and thin books will fill the box. The points representing solutions lie on a straight line because the constant slope property is obeyed. For example, if we have 15 thin books and 10 thick books, the carton is full. If we replace 3 thin books with 2 thick books, we have 12 thin books and 12 thick books, which also fill the carton. On the graph, if we start at the point (15, 10), move 2 up and 3 left, we reach the point (12, 12), which also lies on the line.

## Selected Solutions — Chapter 4

3. From the graph, there is a solution for each point on the line that has whole number coordinates.

$x$	$y$
0	20
3	18
6	16
9	14
12	12
15	10
18	8
21	6
24	4
27	2
30	0

There are 11 solutions.

4.  $2x + 3y = 60$   
 $3y = 60 - 2x$   
 $y = \frac{60}{3} - \frac{2}{3}x$   
 $y = -\frac{2}{3}x + 20$

The  $y$ -intercept is the number of thick books that fit in the box if there are no thin books. Every time you remove 2 thick books, you can add 3 thin books. This is the slope  $-\frac{2}{3}$ .

5.  $a$  is the  $x$ -intercept and  $b$  is the  $y$ -intercept.
6. a) If the carton holds only 18 thick books or 30 thin books, then thick books fill  $\frac{y}{18}$  of the carton, and thin books fill  $\frac{x}{30}$  of the carton.

The equation is  $\frac{x}{30} + \frac{y}{18} = 1$ .

Multiply by the common denominator 90.

$$90\left(\frac{x}{30}\right) + 90\left(\frac{y}{18}\right) = 90(1)$$

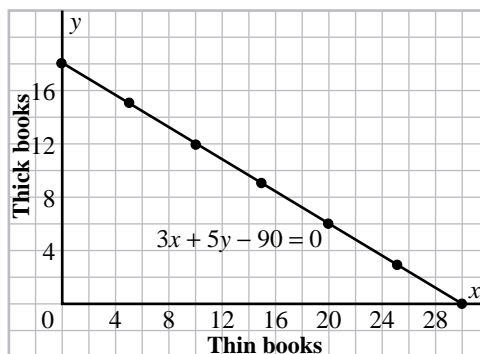
$$3x + 5y = 90$$

or  $3x + 5y - 90 = 0$

Graph this equation.

The solutions are (0, 18), (5, 15), (10, 12), (15, 9), (20, 6), (25, 3), (30, 0).

## Selected Solutions — Chapter 4



- b) If the carton holds 19 thick books, then thick books fill  $\frac{y}{19}$  of the carton.

The equation is  $\frac{x}{30} + \frac{y}{19} = 1$ .

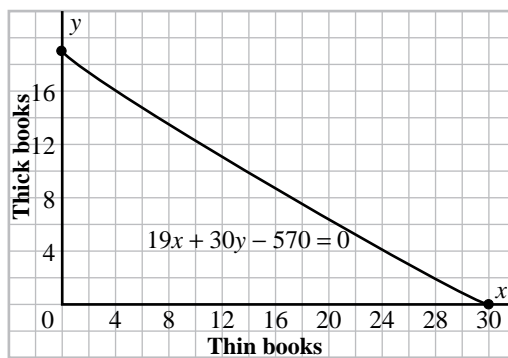
Multiply by the common denominator 570.

$$570\left(\frac{x}{30}\right) + 570\left(\frac{y}{19}\right) = 570(1)$$

$$19x + 30y = 570$$

$$\text{or } 19x + 30y - 570 = 0$$

Graph this equation.



The solutions are (0, 19) and (30, 0), the intercepts. In this case, you could only pack one type of book in the box.

7. a) Tall thin boxes may be difficult to balance and stack. The same volume box would be easier to handle if it were shorter and wider.
- b) Each of the two stacks within the box would follow the equation  $\frac{x}{15} + \frac{y}{10} = 1$ . This allows for more combinations of thin and thick books the box can hold. Each stack can have these combinations: (15, 0), (12, 2), (9, 4), (6, 6), (3, 8), (0, 10).
8. We are using fractions of one unit, so we don't need exact dimensions. The height of the carton is not involved.
9. a) The full carton is represented by 1.  
The thin books fill  $\frac{x}{m}$  of the carton.  
The thick books fill  $\frac{y}{n}$  of the carton.

# Selected Solutions — Chapter 4

The equation is  $\frac{x}{m} + \frac{y}{n} = (mn)(1)$ .

Multiply each side by the common denominator  $mn$ .

$$(mn)\left(\frac{x}{m}\right) + (mn)\left(\frac{y}{n}\right) = (mn)(1)$$

$$nx + my = mn$$

or  $nx + my - mn = 0$

b) The greatest common factor when  $m = 30$  and  $n = 20$  is 10, and there are 11 solutions.

The greatest common factor when  $m = 30$  and  $n = 18$  is 6, and there are 7 solutions.

The greatest common factor when  $m = 30$  and  $n = 19$  is 1, and there are 2 solutions.

If the greatest common factor is  $k$ , the number of solutions is  $k + 1$ .

**Exploring with a Graphing Calculator: Sequences and  $Ax + By + C = 0$ , page 240**

2. b) Answers may vary.

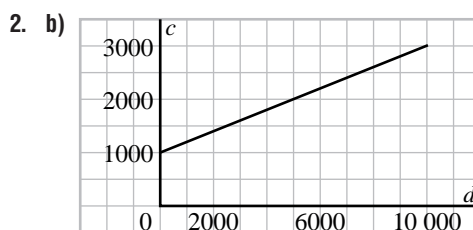
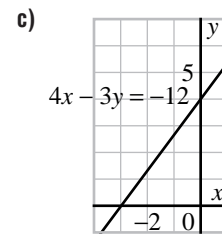
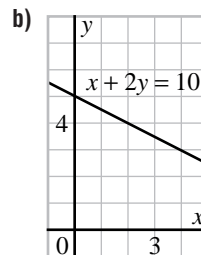
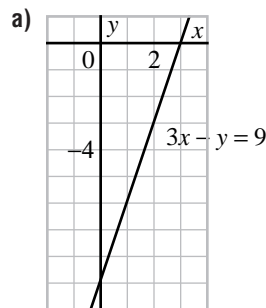
All the graphs of equations with coefficients forming an arithmetic sequence intersect at one point.

3. Answers may vary. You do not see a pattern when you graph the equations at the top right of page 240 because there are not enough equations. Graph additional equations  $x - 2y + 4 = 0$  and  $x - y + 1 = 0$ . They are similar to the second and third equations, with a sign change in the second term. To improve the pattern, graph  $x + 0.5y + 0.25 = 0$  and  $x - 0.5y + 0.25 = 0$ .

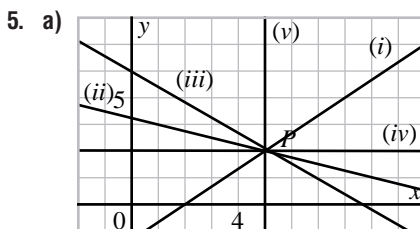
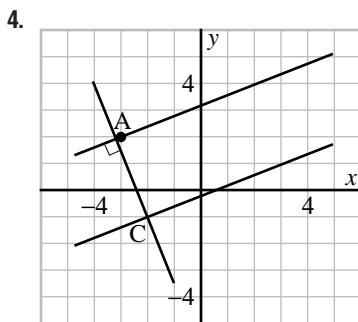
The result is similar to exercise 12 on page 219.

**4 Review, page 241**

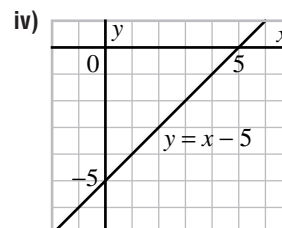
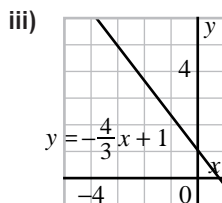
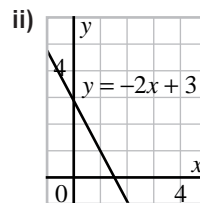
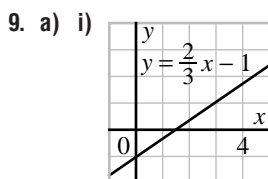
1. Tables of values may vary.



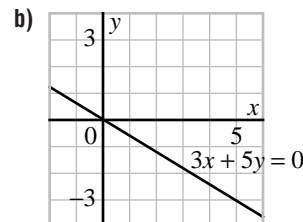
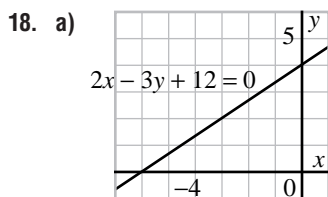
Selected Solutions — Chapter 4



b) Answers may vary. For part iii: From  $P(5, 2)$ , move 5 units up and 7 units left, to reach  $(-2, 7)$ . Draw a line through this point and P.



b) Answers may vary. For part i: The equation is  $y = \frac{2}{3}x - 1$ . The slope is  $\frac{2}{3}$  and the y-intercept is  $-1$ , corresponding to the point  $(0, -1)$ . Plot this point, and move 2 up and 3 right to another point on the line. Draw a straight line through these points.



Selected Solutions — Chapter 4

