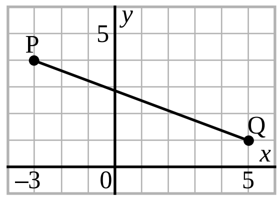
Selected Solutions — Chapter 3

Investigate, page 146

2. a) Answers may vary. I counted the spaces between the endpoints.

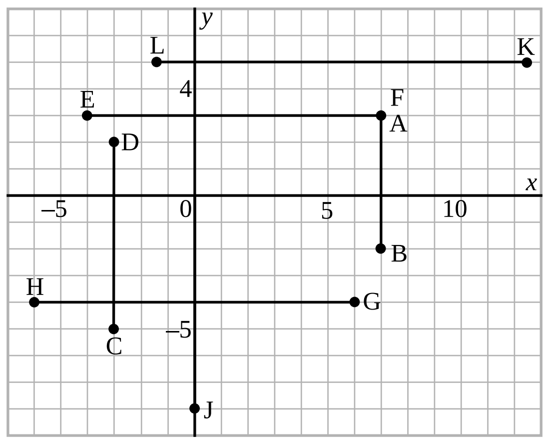
4. Answers may vary. Count the squares between the points.

9.

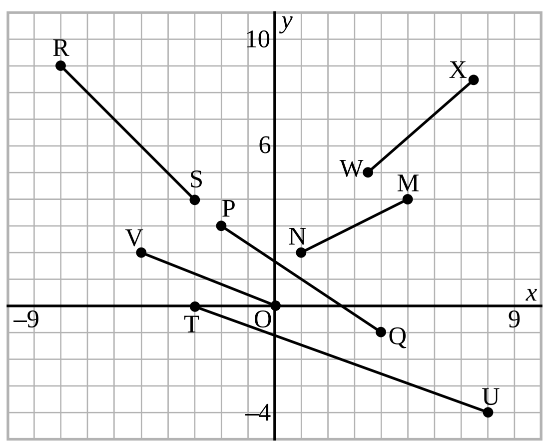


3.1 Exercises, page 149

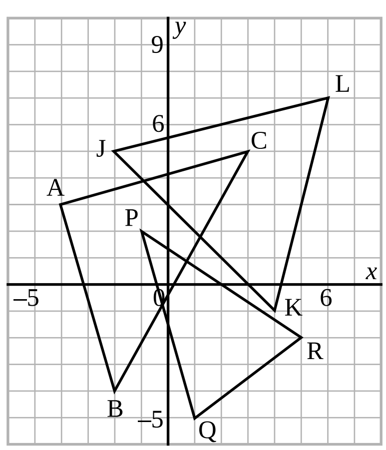
1.



3.



6.



Selected Solutions — Chapter 3

7. Answers may vary. For part a: I used the distance formula to find the length of each side.

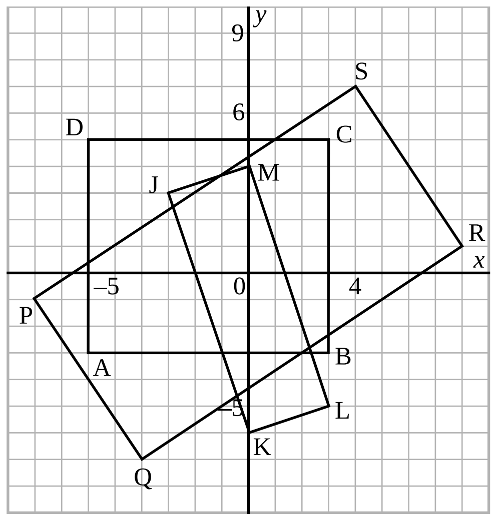
$AB = \sqrt{(-2+4)^2 + (-4-2)^2} = \sqrt{53}$

$BC = \sqrt{(3+2)^2 + (5+4)^2} = \sqrt{106}$

$AC = \sqrt{(3+4)^2 + (5-3)^2} = \sqrt{53}$

Since AB = AC, the triangle is isosceles.

8.



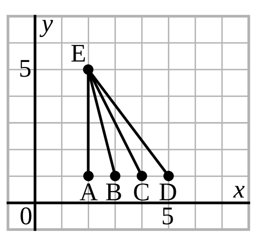
10. A, B, and C are collinear. Since the sum of the lengths of AB and BC is equal to the length of AC, B must lie on the line segment AC.

12. Answers may vary. For part b: If B lies on the circle, then the distance BM must equal the radius 5. I used the distance formula to calculate the distance BM.

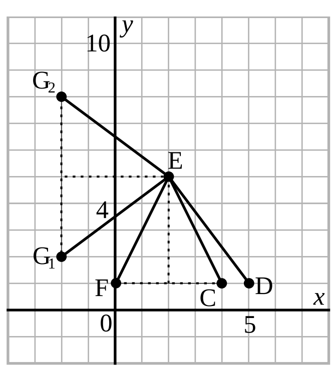
$BM = \sqrt{(3-2)^2 + (6-1)^2} = \sqrt{26}$

Since $\sqrt{26} \neq 5$, B does not lie on the circle.

14. a)



d)



Selected Solutions — Chapter 3

Since F has the same *y*-coordinate as C, it must lie on the same horizontal line. By symmetry, it lies 2 units to the left of the vertical line through E. Hence, the coordinates of F are (0, 1), and $a = 0$. G has *x*-coordinate – 2, so it lies on a vertical line that is 4 units to the left of E.

The right triangle for which ED is the hypotenuse has legs 3 and 4. One leg of the right triangle for which EG is the hypotenuse is 4, so its other leg must be 3.

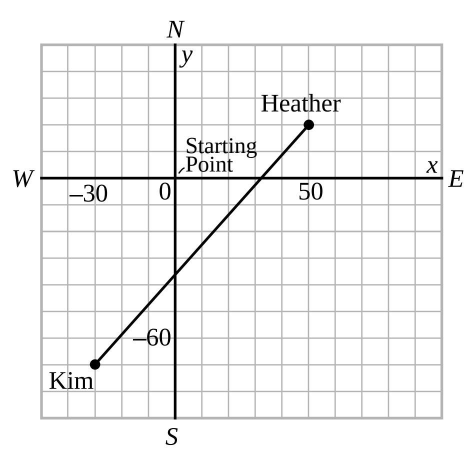
I counted 3 squares up from a horizontal line through E to get G(–2, 8); then I counted 3 squares down from the horizontal line to get a second point G(–2, 2). Hence, $b = 2$ or 8.

e) For part b: To justify that F(0, 1) is the required point, I need to show that FE = EC which, from part a, is $\sqrt{20}$.
Use the distance formula to calculate FE.
$FE = \sqrt{(2-0)^2 + (5-1)^2} = \sqrt{20}$
So, FE = EC
For part c: To justify that points $G_1(-2, 2)$ and $G_2(-2, 8)$ are the required points, I need to show that $G_1E = G_2E = ED$ which, from part a, is 5.
Use the distance formula to calculate G_1E.
$G_1E = \sqrt{(2+2)^2 + (5-2)^2} = \sqrt{16+9} = \sqrt{25} = 5$
Use the distance formula to calculate G_2E.
$G_2E = \sqrt{(2+2)^2 + (5-8)^2} = \sqrt{16+9} = \sqrt{25} = 5$
So, $G_1E = G_2E = ED$

16. b) Answers may vary. For part iv of part a: The endpoints of my segment were A(1, 1) and B(1, 14). My friend chose a vertical line and its endpoints were A(–2, –1) and B(–2, 12). It is unlikely we would choose the same endpoints unless we worked together.

17. b) Answers may vary. For part iii of part a: The endpoints of my segment were A(3, –1) and B(6, –3). I recognized that $\sqrt{13} = \sqrt{9+4} = \sqrt{3^2+2^2}$. So, I chose a line segment that was the hypotenuse of a right triangle with legs 3 and 2. My friend chose a congruent triangle, but she drew her triangle so that the endpoints of the segment were A(4, 1) and B(6, 4). It is unlikely we would choose the same endpoints unless we worked together.

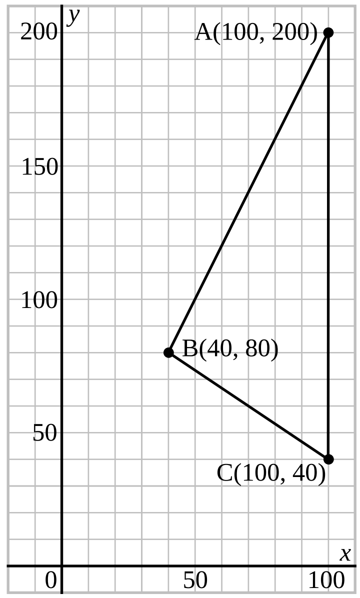
18. c)



Selected Solutions — Chapter 3

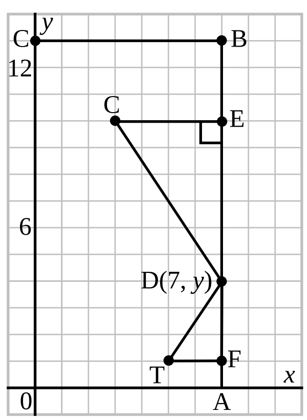
I plotted points on a grid to represent the girls' positions. A straight line from Heather to Kim does not pass through the starting point.

20.



23. a) From exercise 22, the corners of the table are as indicated below at A, B, and C.

Let the point Susan must hit be D with coordinates (7, y).
Draw horizontal lines through C to form right $\triangle$CED, and through T to form right $\triangle$DTF.



For the ball to hit T, $\angle$CDE = $\angle$TDF.
So, $\triangle$CED must be similar to $\triangle$TFD.
Corresponding sides are in the same ratio.

$$\frac{DF}{DE} = \frac{TF}{CE}$$

$$\frac{y-1}{6} = \frac{2}{4}$$

$$y = 4$$

The coordinates of the point are (7, 4).

Selected Solutions — Chapter 3

b) The total distance travelled is CD + DF.
Use the Pythagorean Theorem in △CED.
$CD = \sqrt{4^2 + 6^2} = \sqrt{52}$
Use the Pythagorean Theorem in △DTF.
$DT = \sqrt{2^2 + 3^2} = \sqrt{13}$
The total distance is $(\sqrt{52} + \sqrt{13})$ units.
But 1 unit = 0.2 m
So, total distance is $(\sqrt{52} + \sqrt{13})(0.2)$ m $\doteq$ (2.2) m

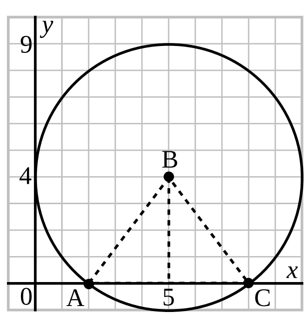
24. a) For P to be equidistant from A and B, PA = PB.
Use the distance formula to calculate each length.
$PA = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = 5$
$PB = \sqrt{0^2 + 5^2} = 5$
P is equidistant from A and B.

b) For Q to be equidistant from A and B, QA = QB. Use the distance formula.
$QA = \sqrt{1^2 + (-7)^2} = \sqrt{50}$
$QB = \sqrt{(-3)^2 + (-5)^2} = \sqrt{34}$
Q is not equidistant from A and B.

c) For R to be equidistant from A and B, AR = BR. Use the distance formula.
$RA = \sqrt{(-1)^2 + (-7)^2} = \sqrt{50}$
$RB = \sqrt{(-5)^2 + (-5)^2} = \sqrt{50}$
R is equidistant from A and B.

d) For S to be equidistant from A and B, AS = BS. Use the distance formula.
$SA = \sqrt{2^2 + (-1)^2} = \sqrt{5}$
$SB = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$
S is equidistant from A and B.

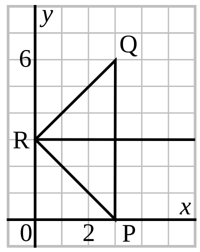
25. Draw a diagram. Plot point B. Use compasses with radius set to 5 units.
Draw a circle, centre B.
The circle intersects the x-axis at A(2, 0) and C(8, 0).



By inspection, we check that AB is the hypotenuse of a 3, 4, 5 triangle, as is BC.

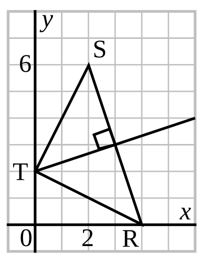
Selected Solutions — Chapter 3

26. a) Draw a diagram.
By symmetry, the point lies on the horizontal line through the midpoint of PQ.



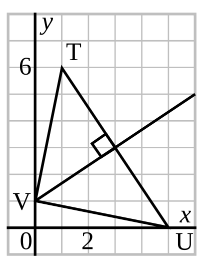
The required point has coordinates (0, 3).
We can check by inspection that RQ = RP since they are hypotenuses of triangles with corresponding legs equal.

b) Draw a diagram. Using the results of part a, we consider the required point as the vertex of an isosceles triangle with base SR.
Draw the perpendicular through the midpoint of SR.
It intersects the *y*-axis at T(0, 2).



We can check by inspection that ST = RT, as in part a.

c) Draw a diagram.
Draw the perpendicular through the midpoint of UT to meet the *y*-axis at V(0, 1).



We can check by inspection that UV = VT.

27. Answers may vary. See exercise 26 above.

28. Use the distance formula in each case.

a) Ottawa to Vancouver:

$\sqrt{(4500 - 2241)^2 + (3000 - 4388)^2}$

$= \sqrt{2259^2 + (-1388)^2}$

$= \sqrt{7\,029\,625}$

$\doteq 2651$

Selected Solutions — Chapter 3

Ottawa to Quebec City:

$\sqrt{(1930 - 2241)^2 + (3638 - 4388)^2}$

$= \sqrt{(-311)^2 + (-750)^2}$

$= \sqrt{659\ 221}$

$\doteq 812$

Ottawa to Washington:

$\sqrt{(1489 - 2241)^2 + (5605 - 4388)^2}$

$= \sqrt{(-752)^2 + 1217^2}$

$= \sqrt{2\ 046\ 593}$

$\doteq 1431$

b) Calculate the distance between Vancouver and Calgary, and Vancouver and Washington.

Vancouver to Calgary:

$\sqrt{(3850 - 4500)^2 + (2800 - 3000)^2}$

$= \sqrt{(-650)^2 + (-200)^2}$

$= \sqrt{462\ 500}$

$\doteq 680.1$

Vancouver to Washington:

$\sqrt{(1489 - 4500)^2 + (5605 - 3000)^2}$

$= \sqrt{(-3011)^2 + (2605)^2}$

$= \sqrt{15\ 852\ 146}$

$\doteq 3981.5$

It is cheaper to call Calgary since it is closer to Vancouver.

Linking Ideas: Mathematics and Technology

Shortest Networks, page 154

3. b) Cell C3: The formula in this cell calculates the length of AB, which is twice the distance from B to the centre of the square, which is at (50, 50). The distance from B(C2, 50) to the centre of the square is C2 – 50.

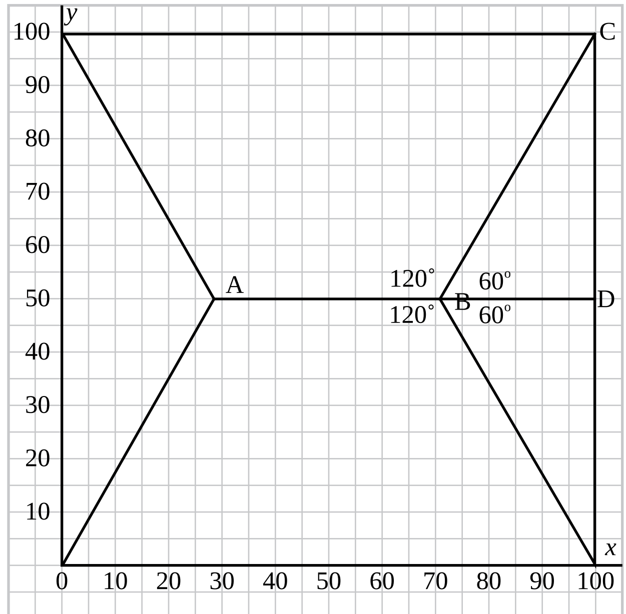
Cell C4: The formula in this cell calculates the length of BC. It uses the Pythagorean Theorem in $\triangle$BDC, with the lengths of BD and CD as the legs.

Cell C5: The formula in this cell calculates the length of the cable, which is made up of AB (cell C3), and four lengths equal to BC (cell C4).

4. The x-coordinate of B is 71.129; the length of AB is 42.258, the length of BC is 57.737, and the length of cable is 273.205.

Selected Solutions — Chapter 3

5.



Investigate, page 156

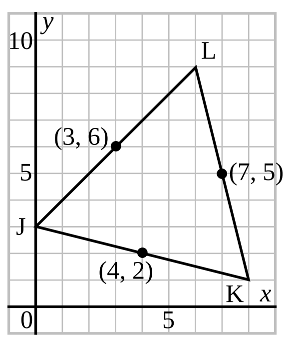
2. **a)** Answers may vary. By inspection

   b) By finding the mean of 9 and 1

4. Yes. Both midpoints can be found by inspection. They can also be found by determining the mean of the x-coordinates for A and B and the y-coordinates for B and C.

3.2 Exercises, page 158

3.



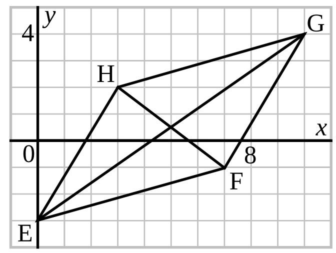
4. The method in the text was to calculate the coordinates of the centre of the circle, then use these coordinates along with the coordinates of C or D, and the distance formula, to calculate the radius.
An alternative method is to use the distance formula to calculate the diameter CD, then halve this length.

$$\begin{aligned} CD &= \sqrt{(5+1)^2 + (5+3)^2} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

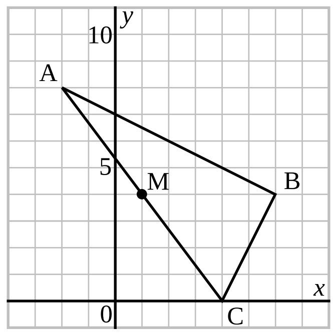
The radius is $\frac{10}{2} = 5$.

Selected Solutions — Chapter 3

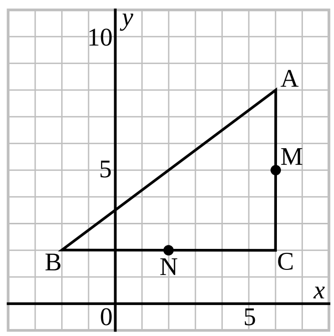
6.



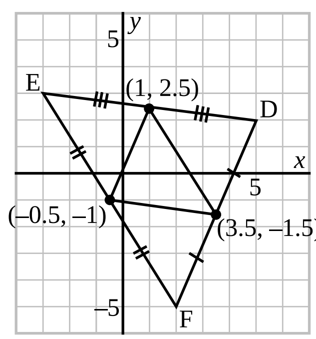
7. a)



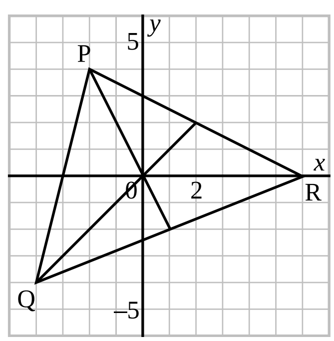
8. a)



9. a)

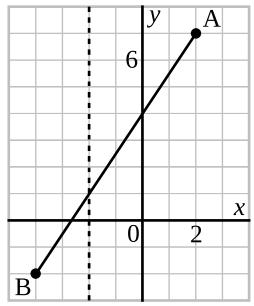


10.



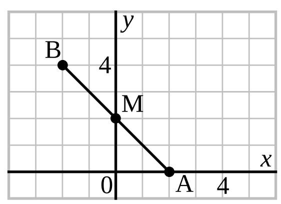
Selected Solutions — Chapter 3

12. I drew a diagram.

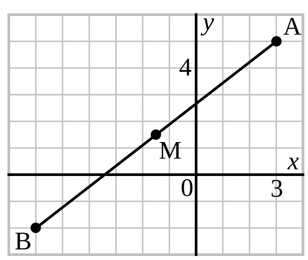


I counted squares to calculate the horizontal distance between B and A, which was 6. I divided this distance into 3 parts, and drew vertical dotted lines 2 units from B and 4 units from B. The required point was where each line intersected AB. The points had coordinates (–2, 1) and (0, 4).

14. Diagrams may vary.



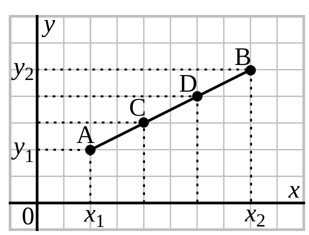
16. Answers may vary. For part c: I drew a graph, and plotted A and M.



I counted the numbers of squares horizontally and vertically between A and M.
There are 4.5 squares horizontally and 3.5 vertically. From M, I counted 4.5 left, then 3.5 down, to reach point B(–6, –2). This is the required endpoint.

17. **b)** There are two whole number answers to each part, so student responses may differ.

22. **a)**



Selected Solutions — Chapter 3

Let the points that divide AB into 3 parts be C and D.
The x-coordinate of C is
$x_1 + \frac{1}{3}(x_2 - x_1)$, or $\frac{x_2 + 2x_1}{3}$.
The y-coordinate of C is
$y_1 + \frac{1}{3}(y_2 - y_1)$, or $\frac{y_2 + 2y_1}{3}$.
The x-coordinate of D is
$x_1 + \frac{2}{3}(x_2 - x_1)$, or $\frac{x_1 + 2x_2}{3}$.
The y-coordinate of D is
$y_1 + \frac{2}{3}(y_2 - y_1)$, or $\frac{y_1 + 2y_2}{3}$.

b) Let D be the point that divides AB into 2 parts.
Then the coordinates of D are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{3}\right)$.
Let C be the point that divides AD into 2 parts.
Then the coordinates of C are

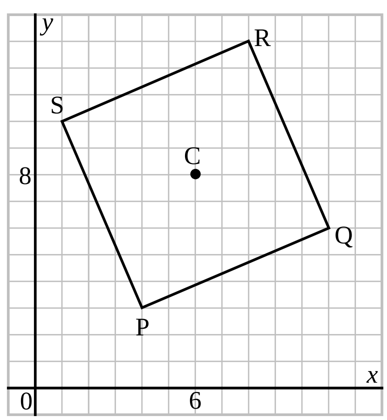
$$\left(\frac{x_1 + \frac{x_1 + x_2}{2}}{2}, \frac{y_1 + \frac{y_1 + y_2}{2}}{2}\right),$$

or $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right)$.
Let E be the point that divides DB into 2 parts.
Then the coordinates of E are

$$\left(\frac{\frac{x_1 + x_2}{2} + x_2}{2}, \frac{\frac{y_1 + y_2}{2} + y_2}{2}\right), \text{ or}$$

$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right)$.
C, D, and E divide AB into 4 parts.

23.

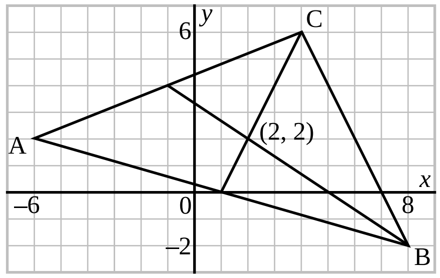


a) The mean of x-coordinates is $\frac{4 + 11 + 8 + 1}{4} = 6$.
The mean of y-coordinates is $\frac{3 + 6 + 13 + 10}{4} = 8$.

b) C is the midpoint of the square. The mean of the coordinates produces the coordinates of the point that is equidistant from each vertex.

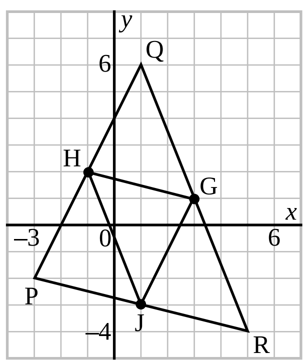
Selected Solutions — Chapter 3

24. From the diagram in exercise 10 on page 159, the centroid is the point of intersection of the medians. Plot the given points, draw the triangle, then draw the line segment from each vertex to the midpoint of the opposite side.



The centroid has coordinates (2, 2).
I could have determined the centroid by calculating the mean of the *x*-coordinates and the mean of the *y*-coordinates of points A, B, and C.

25. Plot the given points on a grid.
From the results of exercise 9, the triangle formed by the midpoints of the sides is similar to the original triangle and equal to $\frac{1}{4}$ of its area.
I draw a triangle similar to $\triangle$GHJ on each side of $\triangle$GHJ.



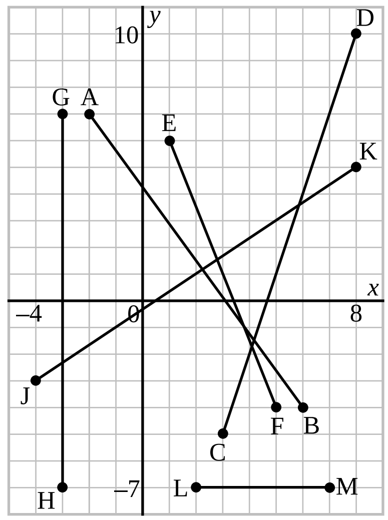
Then I check that G, H, and J are the midpoints of the sides of the large triangle formed, $\triangle$PQR.
The coordinates of the vertices are (1, 6), (5, –4), and (–3, –2).

3.3 Exercises, page 166

4. For each line segment, I counted squares to calculate the vertical and horizontal distances between the ends of the line segment. For example, for part a, for line segment AC, the vertical distance is 3, and the horizontal distance (which is the same for all line segments in part a) is 8. The slope of AC is the vertical distance divided by the horizontal distance, which is $\frac{3}{8}$.

Selected Solutions — Chapter 3

7.



8. I used the formula for the slope of a line segment that uses the coordinates of the endpoints of the line segment.

That is, slope is $\frac{y_2 - y_1}{x_2 - x_1}$.

I substituted the coordinates of E for x_1 and y_1, and the coordinates of F for x_2 and y_2.

I know the slope is 5, so I wrote an equation, then solved for k.

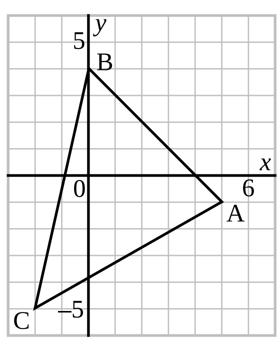
$$5 = \frac{k - 3}{4 - 6}$$

$$5 = \frac{k - 3}{-2}$$

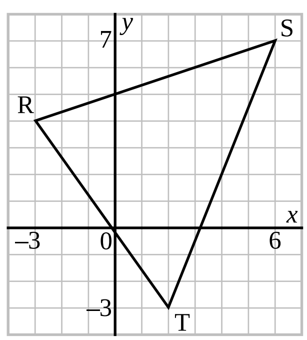
$$-10 = k - 3$$

$$k = -7$$

9. a)

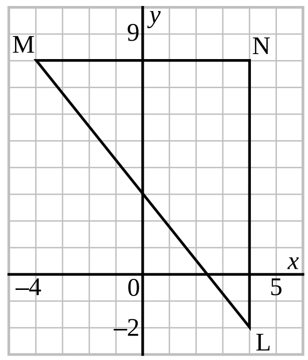


b)

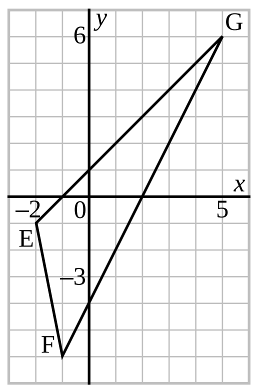


Selected Solutions — Chapter 3

c)



d)



11. b) Answers may vary. For part i: (1, 1) and (2, 4) could be the endpoints. However, as long as $\frac{y_2 - y_1}{x_2 - x_1} = 3$, any corresponding endpoints will suffice.

12. c) Answers may vary. For part a: The run must be $\frac{1}{4}$ of the rise. That is, rise must be 4 times the run; hence, the slope is 4.
For part b: Use the Pythagorean Theorem to calculate the run.

$$\text{Run}^2 = 2.6^2 - 2.5^2$$
$$\text{Run} = \sqrt{0.51}$$
$$\doteq 0.714$$

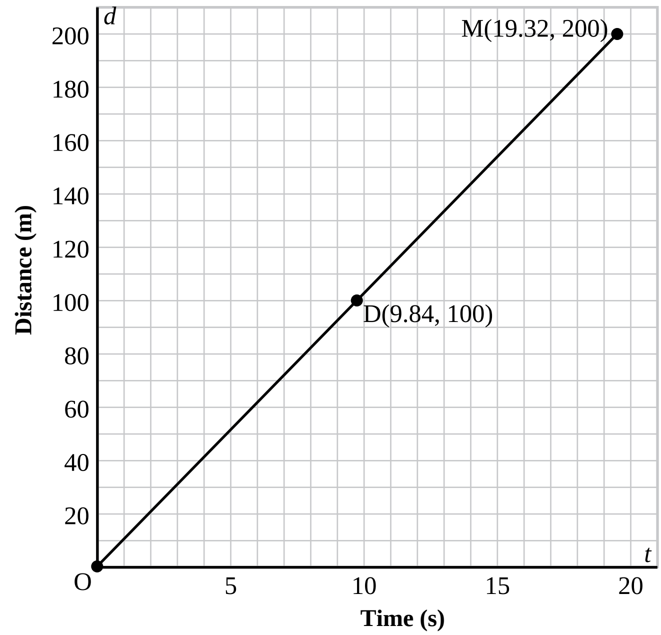
The slope is $\frac{\text{rise}}{\text{run}} \doteq \frac{2.5}{0.714}$
$\doteq 3.5$

The slope is approximately 3.5, which is less than 4. So, the ladder is not safe.

13. c) No, the helicopter with the greatest rate of climb is represented by the graph with the greatest slope, which is C. It reaches its maximum altitude after about 11 min. But helicopter A reaches its maximum altitude after 8 min.

Selected Solutions — Chapter 3

14.a), b)

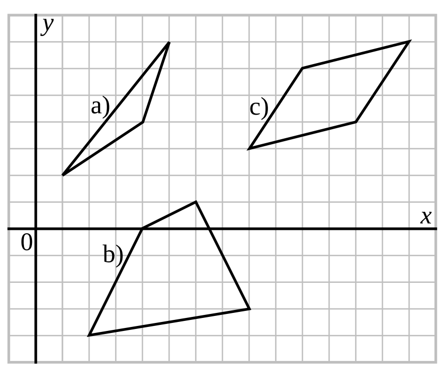


c) Answers may vary. Johnson had the greater average speed because the slope of the line representing his race is greater than that of Bailey's. But a comparison of average speeds might not be the best indicator of who was faster.

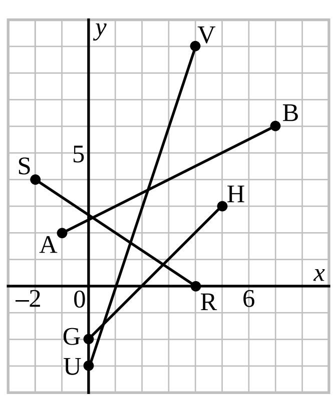
Modelling to Determine the World's Fastest Human

No; the graphs of the actual races would not be straight lines, but curves. No; each runner is trained for a certain distance, and it might not be fair to compare them over different distances.

16. Answers may vary.

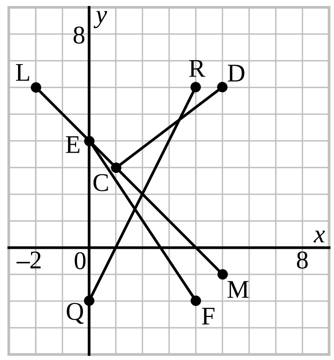


17. c)



Selected Solutions — Chapter 3

18.



19. Answers may vary. For exercise 18a: I wrote an expression for slope, using the given coordinates, and made it equal to the given slope, $\frac{3}{4}$.

$$\frac{y-3}{5-1} = \frac{3}{4}$$

$$\frac{y-3}{4} = \frac{3}{4}$$

Since the denominators are the same, the numerators must be the same.

$$y - 3 = 3$$

$$y = 6$$

21. In each case, one endpoint is A(4, 6). Let the other endpoint be B(x, 0). The slope of AB is $\frac{0-6}{x-4}$. In each case, equate this expression for slope to the given slope.

a) $\frac{0-6}{x-4} = 1$

$x - 4 = -6$

$x = -2$

The endpoint on the x-axis has coordinates (–2, 0).

b) $\frac{0-6}{x-4} = 2$

$-6 = 2x - 8$

$2x = 2$

$x = 1$

The endpoint on the x-axis has coordinates (1, 0).

c) $\frac{0-6}{x-4} = 3$

$-6 = 3x - 12$

$3x = 6$

$x = 2$

The endpoint on the x-axis has coordinates (2, 0).

d) $\frac{0-6}{x-4} = \frac{1}{2}$

$-12 = x - 4$

$x = -8$

The endpoint on the x-axis has coordinates (–8, 0).

e) $\frac{0-6}{x-4} = -2$

$-6 = 2x + 8$

$2x = 14$

$x = 7$

The endpoint on the x-axis has coordinates (7, 0).

Selected Solutions — Chapter 3

f) $\frac{0-6}{x-4} = -\frac{1}{2}$

$12 = -4 + x$

$x = 16$

The endpoint on the *x*-axis has coordinates (16, 0).

22. In each case, one endpoint is B(–3, 4). Let the other endpoint be C(0, y). The slope of BC is

$\frac{y-4}{0-(-3)}$, or $\frac{y-4}{3}$.

In each case, equate this expression for slope to the given slope.

a) $\frac{y-4}{3} = 3$

$y - 4 = 9$

$y = 13$

The endpoint on the *y*-axis has coordinates (0, 13).

b) $\frac{y-4}{3} = -2$

$y - 4 = -6$

$y = -2$

The endpoint on the *y*-axis has coordinates (0, –2).

c) $\frac{y-4}{3} = 1$

$y - 4 = 3$

$y = 7$

The endpoint on the *y*-axis has coordinates (0, 7).

d) $\frac{y-4}{3} = \frac{1}{2}$

$2y - 8 = 3$

$2y = 11$

$y = \frac{11}{2}$

$= 5.5$

The endpoint on the *y*-axis has coordinates (0, 5.5).

e) $\frac{y-4}{3} = -\frac{1}{4}$

$4y - 16 = -3$

$4y = 13$

$y = \frac{13}{4}$

$= 3.25$

The endpoint on the *y*-axis has coordinates (0, 3.25).

23. Let the line segment with slope 2 have endpoint A(a, 0). The other endpoint is given: N(0, 4)

Then, slope of AN is $\frac{4-0}{0-a} = 2$

$4 - 0 = -2a$

$a = -2$

The line segment with slope 2 has an endpoint with coordinates (–2, 0). Let the line segment with slope $-\frac{1}{2}$ have endpoint B(b, 0). The other endpoint is given: N(0, 4)

Selected Solutions — Chapter 3

Then, slope of BN is $\frac{4-0}{b-0} = -\frac{1}{2}$

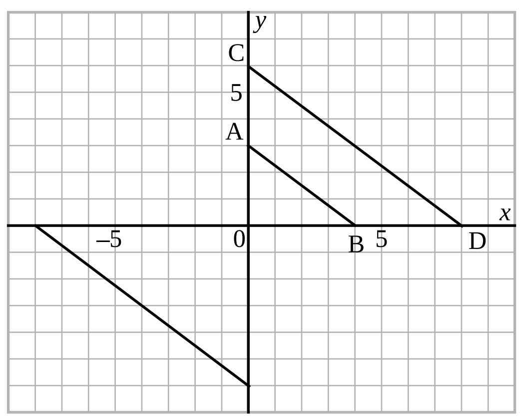
$$-8 = -b$$

$$b = 8$$

The line segment with slope $-\frac{1}{2}$ has an endpoint with coordinates (8, 0).

24. On a grid, draw a line segment with its endpoints on the axes, so that it has a slope of $-\frac{3}{4}$; that is, rise is –3 and run is 4.

Using the Pythagorean Theorem, $AB = \sqrt{3^2 + 4^2}$, or 5. We want a line segment twice as long. So, draw another segment, CD, so that $OC = 2OA$ and $OD = 2OB$. Using the Pythagorean Theorem, $CD = \sqrt{6^2 + 8^2}$, or 10. By symmetry, we can draw another segment in the 3rd quadrant that also has slope $-\frac{3}{4}$. There are two possible segments; the coordinates of their endpoints are (0, 6), (8, 0), and (0, –6), (–8, 0).



Linking Ideas: Mathematics and Construction

Building the Best Staircase, page 170

1. b) Answers may vary; staircase B because it's the one with rise closest to run

c) Staircase C because it takes up the least floor space

d) Answers may vary; the staircase with which most people are familiar — this is probably staircase B

2. c) No, because people have different stride lengths

d) Answers may vary; for example, ornamental garden

Modelling Staircase Design

If we did not round, we would need one step to be different (that is, different rise and different run) from all other steps, so that the staircase will fit — this could cause people to stumble.

4. Answers may vary. The staircase would not fit into the plan, or the top step would be higher than the upper floor.

Selected Solutions — Chapter 3

Linking Ideas: Mathematics and Technology

Length, Midpoint, and Slope of a Line Segment, page 172

Coordinates of P		Midpoint of OP		Slope of OP	Length of OP
x-coord	*y*-coord	*x*-coord	*y*-coord		
4	0	2	0	0	4
4	1	2	0.5	0.25	4.12310563
4	2	2	1	0.5	4.47213595
4	3	2	1.5	0.75	5
4	4	2	2	1	5.65685425
4	5	2	2.5	1.25	6.04312424
4	6	2	3	1.5	7.21110255
4	7	2	3.5	1.75	8.06225775
4	8	2	4	2	8.94427191
4	9	2	4.5	2.25	9.8488578
4	10	2	5	2.5	10.7703296

b) The midpoint of OP always has *x*-coordinate 2; the *y*-coordinate increases by 0.5 when the *y*-coordinate of P increases by 1. The slope of OP increases by 0.25 when the *y*-coordinate of P increases by 1. The length of OP increases and it gets closer to the value of the *y*-coordinate of P.

6. b) Angle POA is 60° because △OAP is a 30-60-90 triangle. That is, its sides have lengths 4, $4\sqrt{3}$, and 8.

10. a) As the number of sides of the polygon increases, its perimeter gets closer to the circumference of the circle. An approximation of π is the perimeter of the 28-sided polygon divided by the diameter of the circle, which is 10.

b) The approximation is less than π because the perimeter of the polygon is less than the circumference of the circle. Each side of the polygon (for example, AB) is a straight line, while each part of the circle is an arc, which is slightly longer than the straight line.

Investigate, page 175

1. a) The segments are parallel because they lie on opposite sides of a square.

d) The slopes of the segments are equal. They have equal rises and runs; therefore, the slopes must be equal.

2. No; they are still two parallel line segments, and their corresponding rises and runs would be equal.

Selected Solutions — Chapter 3

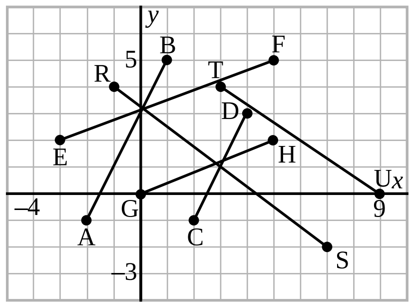
3.4 Exercises, page 177

1. a) Parallel; slope AB = slope CD = $\frac{1}{2}$

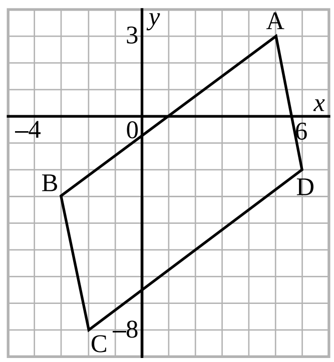
b) Parallel; slope RS = slope PQ = $-\frac{2}{3}$

c) Not parallel; slope JK = $\frac{7}{3}$, slope LM = $\frac{5}{2}$

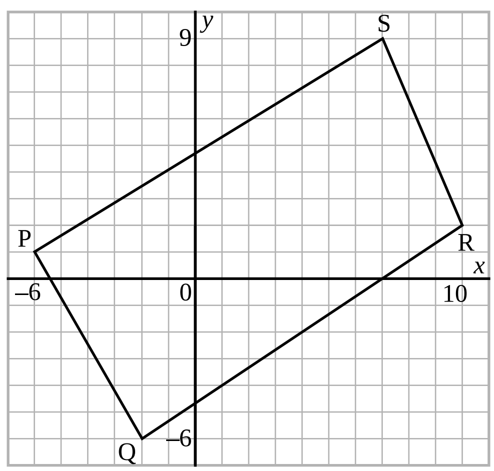
2.



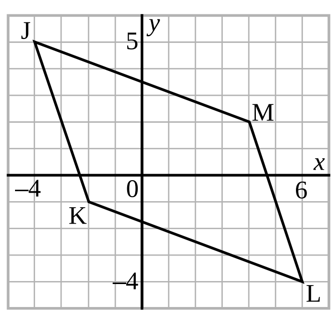
3. a)



b)



c)



Selected Solutions — Chapter 3

4. Answers may vary. For part a: The quadrilateral is a parallelogram if its opposite sides are parallel. I calculated the slope of each side, using the slope formula.

Slope of AB is $\frac{3-(-3)}{5-(-3)} = \frac{6}{8}$, or $\frac{3}{4}$

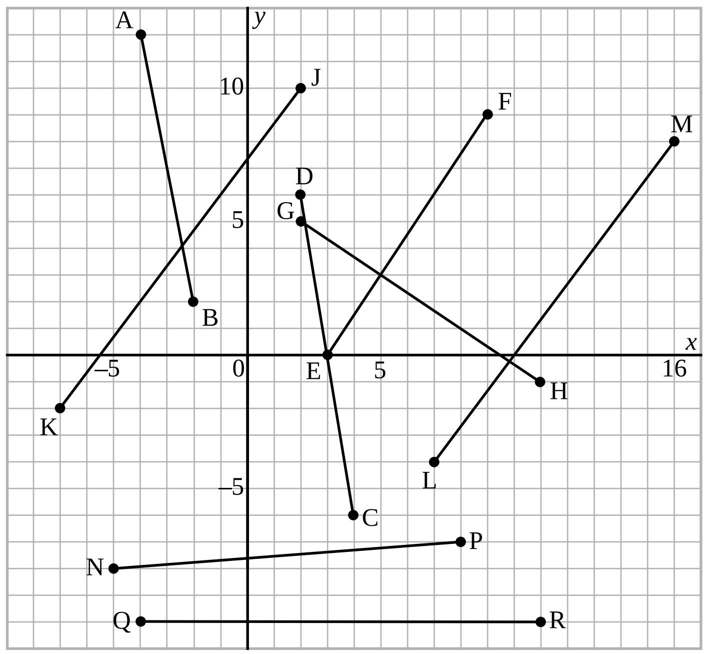
Slope of BC is $\frac{-3-(-8)}{-3-(-2)} = \frac{5}{-1}$, or -5

Slope of CD is $\frac{-8-(-2)}{-2-(-6)} = \frac{-6}{-8}$, or $\frac{3}{4}$

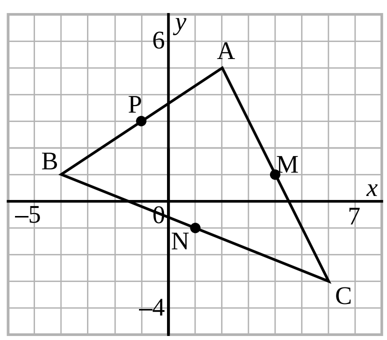
Slope of DA is $\frac{-2-(-3)}{6-5)} = -\frac{5}{1}$, or -5

Since opposite sides are parallel, the quadrilateral is a parallelogram.

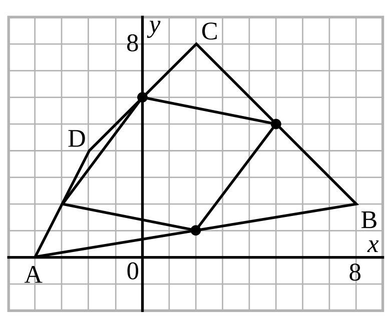
6.



7. a)



8. a), b)



Selected Solutions — Chapter 3

11. a) Answers may vary.
Assume a reaction time of 0.20 s.

b) Answers may vary.
Each line segment would have one endpoint at (0.2, 0).
Bailey's running time: $9.84 - 0.20 = 9.64$
Slope of graph $\frac{100}{9.64} \doteq 10.37$
Johnson's running time: $19.32 - 0.20 = 19.12$
Slope of graph $\frac{200}{19.12} \doteq 10.46$
Johnson has the faster average speed.

c) Answers may vary.
Assume a reaction time of 0.40 s.
Bailey's running time: $9.84 - 0.40 = 9.44$
Slope of graph $\frac{100}{9.44} \doteq 10.59$
Johnson's running time: $19.32 - 0.40 = 18.92$
Slope of graph $\frac{200}{18.92} \doteq 10.57$
Bailey has the faster average speed.

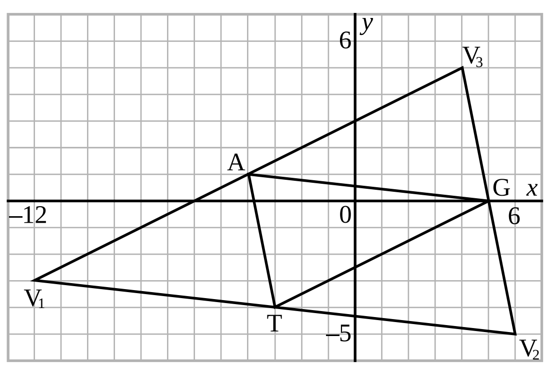
Modelling to Determine the World's Fastest Human

Answers may vary. The method in exercise 11 is a different model, but not necessarily a fairer one.

12. b) Answers may vary. For part i: The slope is $\frac{3}{13}$, so any line segment with endpoints such as C(1, 1) and D(14, 4) will have the same slope. It is possible that all students have different segments that are all correct.

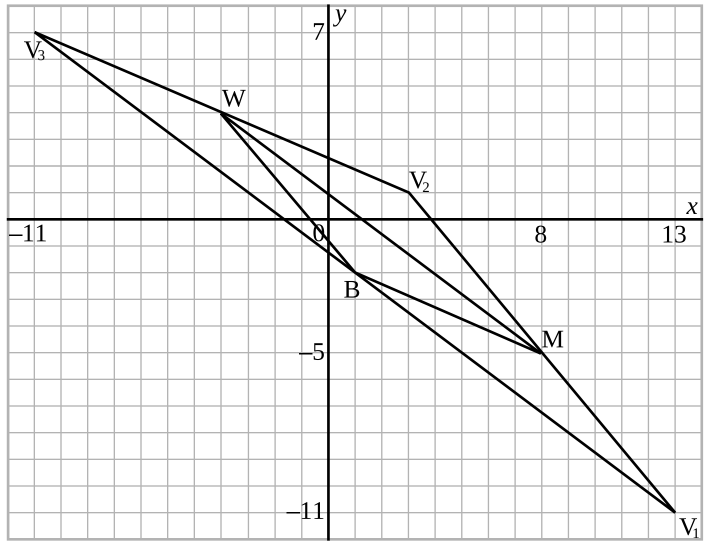
15. Plot the given points. Draw the possible parallelograms. Consider each segment as a side of a parallelogram and as a diagonal. Read the coordinates of the possible vertices from the diagram.

a) The coordinates are (–12, –3), (4, 5), and (6, –5).

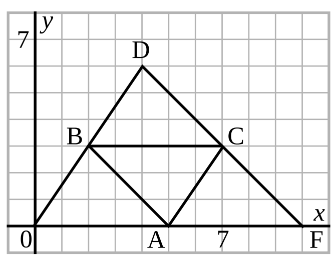


Selected Solutions — Chapter 3

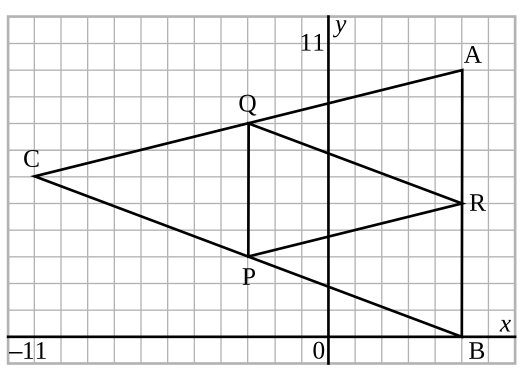
b) The coordinates are (–11, 7), (3, 1), and (13, –11).



16. a) Plot the given points A(5, 0), B(2, 3) and C(7, 3).
Each side of △ABC is parallel to and equal to one-half the length of each side of the required triangle.
Draw a line through B parallel to AC.
Draw a line through C parallel to AB.
These lines intersect at D(4, 6).
Draw a line through A parallel to BC.
This line is the *x*-axis and it intersects the other 2 lines at O(0, 0) and F(10, 0). △DOF is the required triangle.



b)

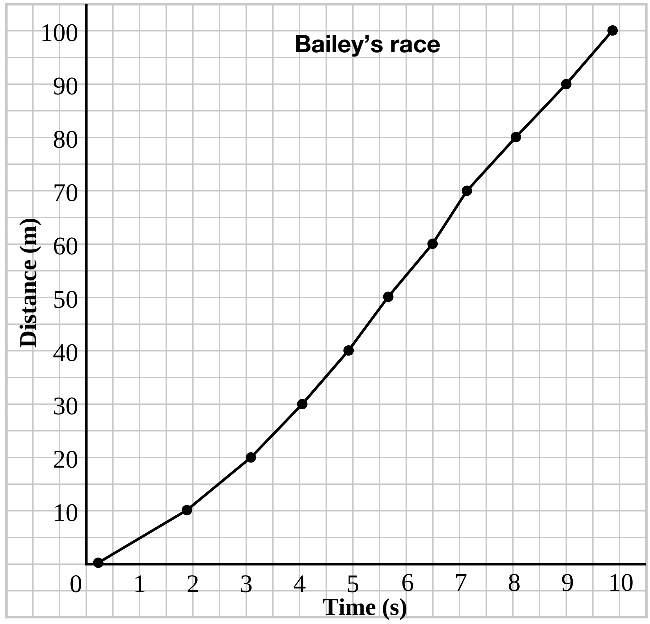


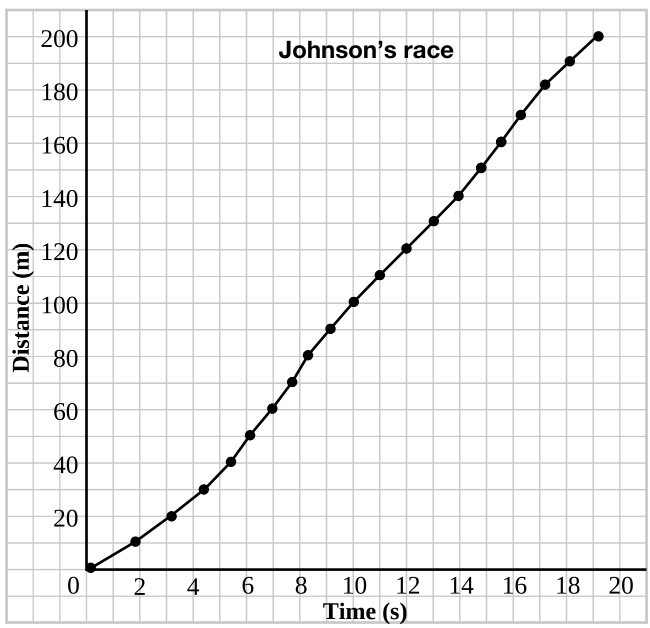
Plot the given points P(–3, 3), Q(–3, 8), and R(5, 5).
Draw a line through Q parallel to PR. Draw a line through R parallel to PQ. Draw a line through P parallel to QR. These lines intersect at A(5, 10), B(5, 0), and C(–11, 6). △ABC is the required triangle.

Selected Solutions — Chapter 3

Mathematical Modelling: Who Is the World's Fastest Human?, page 180

1.



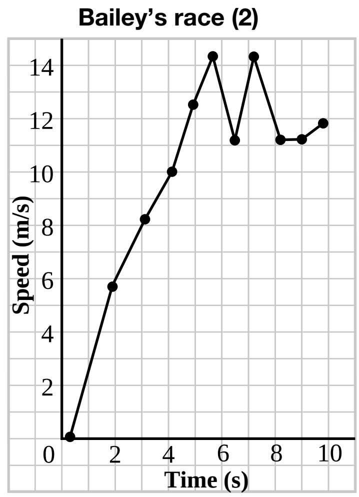


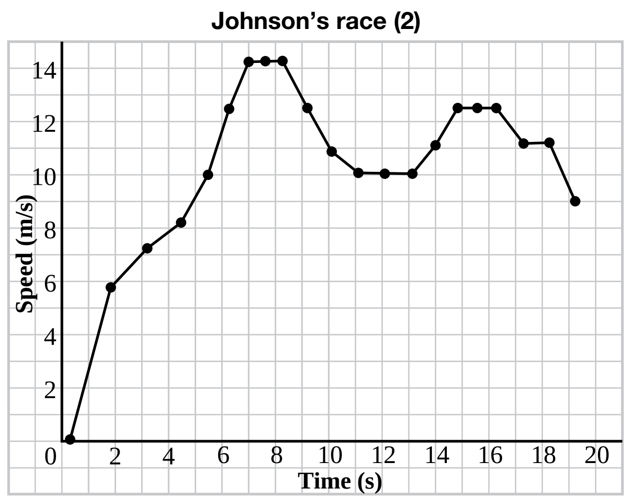
2. **a)** The steepest segment on Bailey's graph is between 4.9 s and 5.6 s; and between 6.5 s and 7.2 s. The slope is 14.29 m/s. The steepest segment on Johnson's graph is between 6.3 s and 8.4 s. The slope is 14.29 m/s.

b) The slope represents the average speed for the 10-m segment for Bailey and the 30-m segment for Johnson.

Selected Solutions — Chapter 3

3.





4. Answers may vary.
No, the segment slopes vary from segment to segment within the graph. The maximum average speed was determined over a short time interval only.

5. Answers may vary.
From the graphs of exercise 3, both runners have the same maximum average speed of 14.29 m/s.
From the graphs of exercise 1, Michael Johnson has an average speed of 10.35 m/s, while Donovan Bailey has an average speed of 10.16 m/s. Michael Johnson is the faster runner.

6. Answers may vary.
Johnson should have won the race.
Use the average speeds calculated for Johnson and Bailey in exercise 5.
Johnson's estimated time for the 150-m race: $\frac{150 \text{ m}}{10.35 \text{ m/s}} \doteq 14.49$ s
Bailey's estimated time for the 150-m race: $\frac{150 \text{ m}}{10.16 \text{ m/s}} \doteq 14.76$ s
Johnson maintained his maximum speed for longer and had a higher average speed over a longer distance.

Selected Solutions — Chapter 3

7. a) Answers may vary.
The result is surprising. Donovan Bailey's average speed was $\frac{150\text{ m}}{14.99\text{ m/s}} \doteq 10.00$ m/s. This speed is less than Michael Johnson's average speed over a longer distance.

b) Johnson did not finish the race and may have beaten Bailey if he had finished the race.

8. Answers may vary.

Mathematics File: Solving Equations in the Form $\frac{a}{b} = \frac{c}{d}$, page 182

3. Answers may vary. You cannot use the shortcut with more than 2 fractions. For example, $\frac{5}{2} = \frac{n}{4}$ can be solved by using the shortcut since it has the form $\frac{a}{b} = \frac{c}{d}$. But $1 + \frac{r}{4} = \frac{2}{3}$ cannot be solved by using the shortcut because there is a whole number and a fraction on the left side.

7. This equation can be solved by using the shortcut since it has the form $\frac{a}{b} = \frac{c}{d}$, where $a = y + 1$, $b = -3$, $c = -3$, and $d = 2$.

Investigate, page 184

1. a) The line segments are perpendicular because they are adjacent sides of a square.

d) The slopes are negative reciprocals.

$$\text{rise AB} = -\text{run DE}$$

$$\text{run AB} = \text{rise DE}$$

$$\begin{aligned}\text{slope AB} &= \frac{\text{rise AB}}{\text{run AB}} \\ &= \frac{-\text{run DE}}{\text{rise DE}} \\ &= -\frac{1}{\text{slope DE}}\end{aligned}$$

2. No; the slopes would remain the same if the lengths changed.

3-5 Exercises, page 187

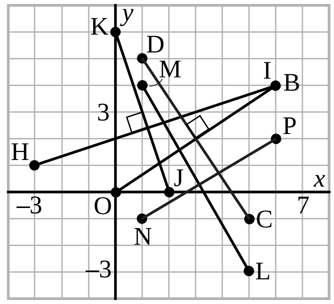
1. a) AB and CD are perpendicular.
$(\text{slope AB})(\text{slope CD}) = \frac{2}{3} \times -\frac{3}{2} = -1$

b) PQ and RS are not perpendicular.
$(\text{slope RS})(\text{slope PQ}) = -\frac{4}{5} \times \frac{4}{3} \neq -1$

c) GF and EF are perpendicular.
$(\text{slope EF})(\text{slope FG}) = -\frac{1}{2} \times -2 = -1$

Selected Solutions — Chapter 3

4.



5. Answers may vary. For part a: I calculated squares to determine rise and run.

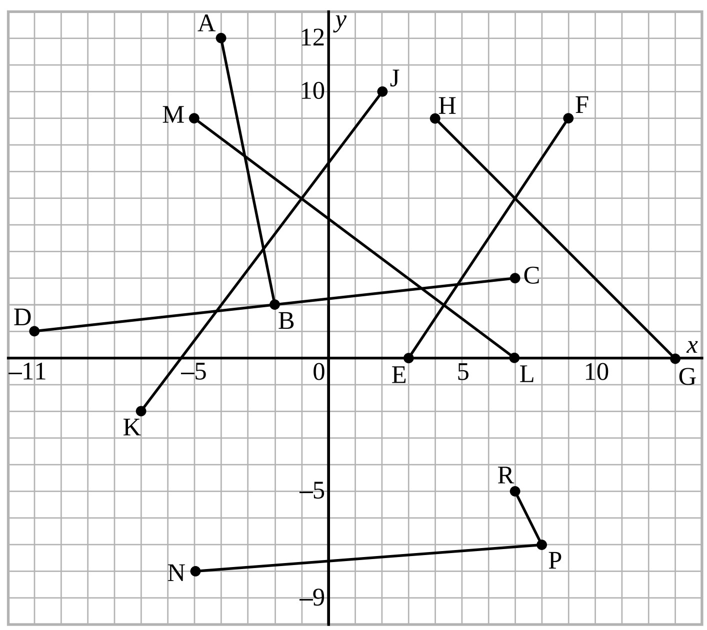
$$\text{Slope OB} = \frac{4}{6}$$
$$= \frac{2}{3}$$
$$\text{Slope CD} = -\frac{6}{4}$$
$$= -\frac{3}{2}$$

I then multiplied the slopes.

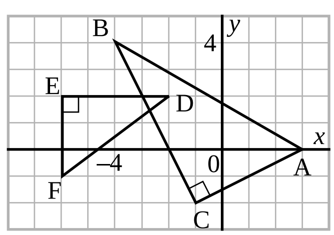
$$(\text{slope OB})(\text{slope CD}) = \frac{2}{3} \times -\frac{3}{2} = -1$$

Since the product of the slopes is –1,
OB is perpendicular to CD.

6.

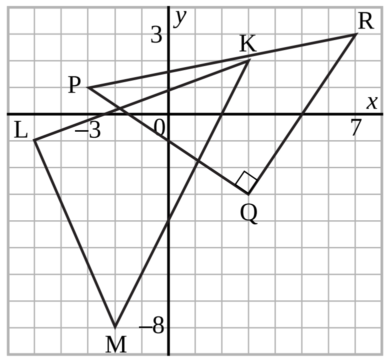


7.a), b)

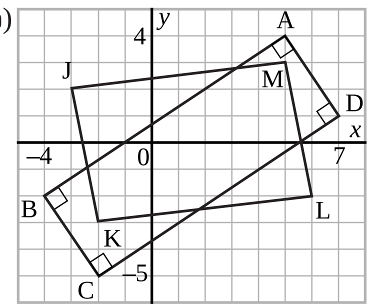


Selected Solutions — Chapter 3

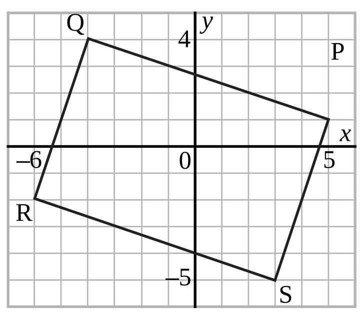
c), d)



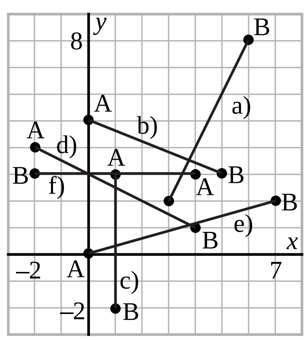
8.a), b)



c)



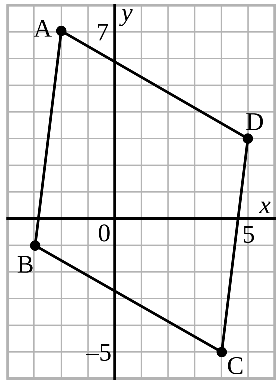
12.



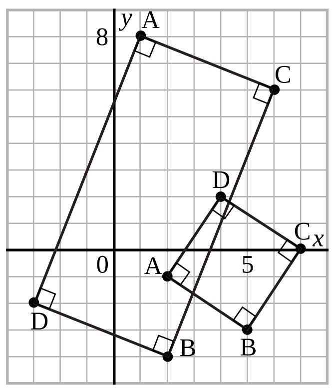
13. Answers may vary. For part b: For AB, I counted squares to determine the rise –2 and run 5. For a perpendicular line segment with one endpoint A, I used a rise of 5 and a run of 2. I counted squares from A to reach C(2, 10).

Selected Solutions — Chapter 3

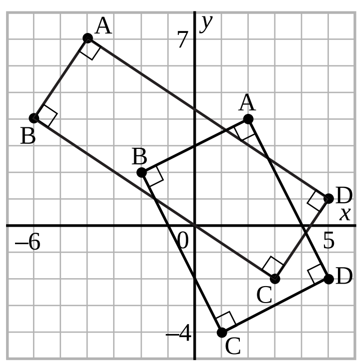
15.



16.a), b)



c), d)

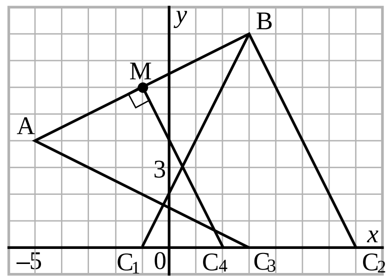


17. Answers may vary. For part d: I calculated the slope of BA by counting squares for the rise (2) and the run (4) from B to A. These are also the rise and run for CD from C to D. From C, I counted 2 squares up and 4 squares right to reach D(5, –2)

18. **b)** Answers may vary.
It is likely that students will have different endpoints, but their answers will be correct. There are an infinite number of line segments that can be drawn perpendicular to a given segment.

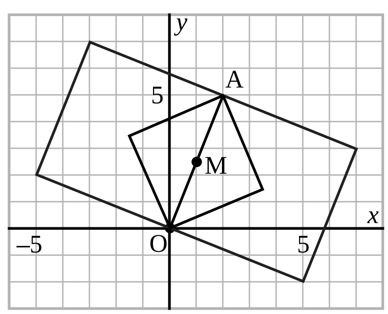
Selected Solutions — Chapter 3

21. Plot the points on a grid.



Suppose AB is one of the two equal sides. Then BC is the other equal side. Put compasses point on B and draw an arc whose radius is equal to BA. Mark the points where the arcs intersect the *x*-axis. Their coordinates are (–1, 0) and (7, 0). These are two possible vertices.
Then put compasses point on A and draw an arc whose radius is equal to AB. Mark the points where the arcs intersect the *x*-axis. One point is (–13, 0), which is collinear with A and B. The other point is (3, 0). This is a possible vertex.
Suppose AB is the base. Determine the midpoint M. M has coordinates $\left(\frac{-5+3}{2}, \frac{4+8}{2}\right)$, or (–1, 6).
Draw the perpendicular bisector of AB.
AB has slope 2, so its perpendicular bisector has slope $-\frac{1}{2}$. It intersects the *x*-axis at (2, 0). This is a possible vertex.

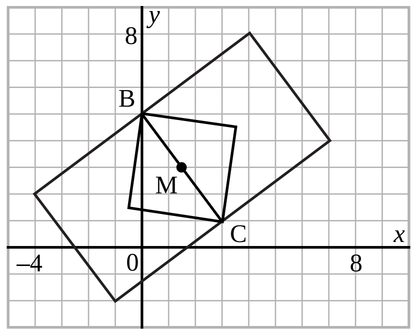
22. a) Plot the points on a grid.



Suppose OA is one side of the square. It has slope $\frac{5}{2}$ and length $\sqrt{5^2 + 2^2}$, or $\sqrt{29}$.
Draw a line perpendicular to OA at A, and at O.
Since the rise and run for OA are 5 and 2, respectively, the rise and run for the two perpendicular sides are –2 and 5 respectively. From A, move 2 down and 5 right to (7, 3). From A, move 2 up and 5 left to (–3, 7). From O, move 2 down and 5 right to (5, –2). From O, move 2 up and 5 left to (–5, 2). For one square, the other two vertices are (5, –2) and (7, 3). For another square, the other two vertices are (–5, 2) and (–3, 7).

Suppose OA is a diagonal. The diagonals of a square bisect each other at right angles. The midpoint of OA is M, where M has coordinates (1, 2.5). Since the rise and run for MA are 2.5 and 1 respectively, the rise and run for the segments from M to the unknown vertices are –1 and 2.5, respectively. From M, move 1 down, then 2.5 right to (3.5, 1.5) and, from M, move 1 up then 2.5 left to (–1.5, 3.5). These are the coordinates of the vertices of the third square.

b) Plot the points on a grid.



Suppose BC is one side of the square. It has slope $-\frac{4}{3}$ and length $\sqrt{3^2 + 4^2}$, or 5.

Draw a perpendicular to BC at B and at C. Since the rise and run of BC are –4 and 3 respectively, the rise and run for the two perpendicular sides are 3 and 4 respectively. From B, move 3 up and 4 right to (4, 8). From C, move 3 up and 4 right to (7, 4). These two points are the vertices of one square. From B, move 3 down and 4 left to (–4, 2). From C, move 3 down and 4 left to (–1, –2). These two points are the vertices of another square.

Suppose BC is a diagonal. The midpoint of BC is M, where M has coordinates (1.5, 3). Since the rise and run for MC are –2 and 1.5, respectively, the rise and run for the segments from M to the unknown vertices are 1.5 and 2 respectively. From M, move 1.5 up and 2 right to (3.5, 4.5) and, from M, move 1.5 down and 2 left to (–0.5, 1.5). These are the coordinates of the vertices of the third square.

23. a) See student text, page 189, for the diagram.

b) i) From the diagram, the coordinates are P(–1, 3), Q(1, –1), and R(4, 2).

Count squares.

Slope of AP $= \frac{3}{-3}$, or -1

Slope of QR $= \frac{3}{1}$, or 1

Since the slopes are negative reciprocals, AP is perpendicular to QR.

Count squares.

$$\text{Length of AP} = \sqrt{3^2 + 3^2} = \sqrt{18}$$

$$\text{Length of QR} = \sqrt{3^2 + 3^2} = \sqrt{18}$$

Hence, AP = QR

Selected Solutions — Chapter 3

ii) Similarly, slope BQ $= \frac{5}{1}$, or 5

Length BQ $= \sqrt{5^2 + 1^2}$, or $\sqrt{26}$

Slope PR $= \frac{1}{-5}$, or $-\frac{1}{5}$

Length PR $= \sqrt{1^2 + 5^2}$, or $\sqrt{26}$

Hence, BQ is perpendicular to PR and BQ = PR.

iii) Similarly, slope OR $= \frac{1}{2}$

Length OR $= \sqrt{4^2 + 2^2}$, or $\sqrt{20}$

Slope PR $= \frac{4}{-2}$, or -2

Length PQ $= \sqrt{2^2 + 4^2}$, or $\sqrt{20}$

Hence, OR is perpendicular to PQ and OR = PQ.

Problem Solving: Square Patterns on a Grid, page 190

1. a) There are: 4 squares with sides 1 unit; 4 squares with sides $\sqrt{2}$ units; 5 squares with sides 2 units; 1 square with sides $2\sqrt{2}$ units; and 1 square with sides 4 units. There are 15 squares in all.

b) 12

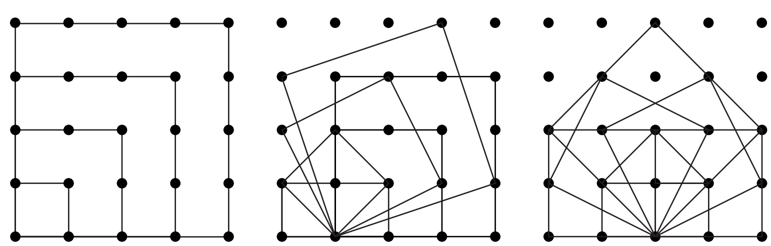
c) In 3 cases, 4 squares combine to form another square that does not have a vertex at the centre.

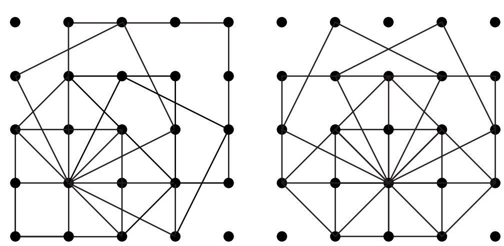
2. a) From exercise 1a, there are 5 different squares; hence, 5 different areas.

b) The areas are 1, 2, 4, 8, 16; the side lengths are 1, $\sqrt{2}$, 2, $2\sqrt{2}$, 4.

c) Answers may vary, depending on the order in which the squares are listed. From smallest to largest: both are geometric sequences, with common ratios 2 and $\sqrt{2}$ respectively. If the side lengths are written as 1, $\sqrt{2}$, $\sqrt{4}$, $\sqrt{8}$, $\sqrt{16}$, it is more obvious that they form a geometric sequence, and that the common ratio is $\sqrt{2}$.

3. a) Here are all the possibilities, apart from orientation. Coordinates are used for the dot, starting with (0, 0) at the top left.





b) Refer to the art in part a. Four out of 5 have line symmetry.

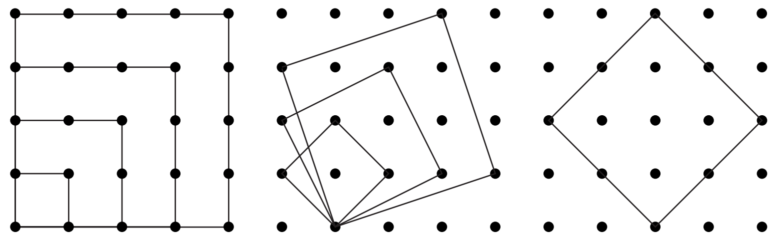
Selected Solutions — Chapter 3

4. a) Answers may vary. All the squares have area 5 square units.

b) Four other squares with area 5 square units can be drawn on the grid.

c) Answers may vary.

5.a), b) Eight squares can be drawn. They have side lengths 1, $\sqrt{2}$, 2, $\sqrt{5}$, $\sqrt{8}$, 3, $\sqrt{10}$, 4. They can be arranged systematically, as shown below.



6. a) From exercise 5, there are 8 non-congruent squares.

b) Use the pattern from exercise 5. There will be 6 squares with side lengths 1, 2, 3, 4, 5, 6. There will be 5 squares with side lengths $\sqrt{2}$, $\sqrt{5}$, $\sqrt{10}$, $\sqrt{17}$, $\sqrt{26}$. There will be 3 squares with side lengths $\sqrt{8}$, $\sqrt{13}$, $\sqrt{20}$. There will be 1 square with side length $\sqrt{18}$.

Look at the patterns — each time 2 dots are added to the side length of each grid, 2 more squares in each configuration are possible, with one extra square in one more configuration.

For a 9 by 9 grid, there will be $8 + 7 + 5 + 3 + 1 = 24$ non-congruent squares.

c) Notice the pattern for the total number of squares; each total is 1 less than a perfect square.

For a 5 by 5 grid, the total is $3^2 - 1$, or

$$\left(\frac{5+1}{2}\right)^2 - 1$$

For a 7 by 7 grid, the total is $4^2 - 1$, or

$$\left(\frac{7+1}{2}\right)^2 - 1$$

For a 9 by 9 grid, the total is $5^2 - 1$, or

$$\left(\frac{9+1}{2}\right)^2 - 1$$

Continuing this pattern, for an n by n grid, the total number of non-congruent squares will be

$\left(\frac{n+1}{2}\right)^2 - 1$, which can be written

as $\frac{n^2 + 2n + 1 - 4}{4}$

$$= \frac{n^2 + 2n - 3}{4}$$

$$= \frac{(n+3)(n-1)}{4}$$

Selected Solutions — Chapter 3

7. a) There are 25 dots in the pattern, which is the sum of $1 + 3 + 5 + 7 + 9$. Each time a new square is added, the number of extra dots is the preceding number plus 2.

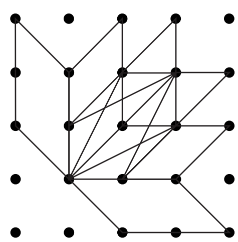
b) $1 + 3 + 5 + 7 + 9 = 25$, or 5^2
We can write the sum as $1 + 3 + 5 + 7 + 9 = \left(\frac{9+1}{2}\right)^2$.
For the sum $1 + 3 + 5 + 7 + 9 + \cdots + n$, the sum is $\left(\frac{n+1}{2}\right)^2$.

8. a) With (0, 0) as the coordinates of the lower left dot, the common vertex has coordinates (1, 1).

b) The side lengths are: the diagonal of a 1 by 1 square, which is $\sqrt{2}$; and 2 units.

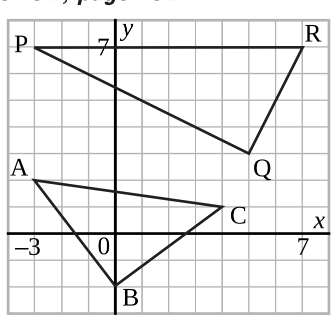
c) Each parallelogram has area 2 square units, and the area is the product of the base and height.
When the base is 2 units, the height is 1 unit, since $2 \times 1 = 2$.
When the base is $\sqrt{2}$ units, the height is $\sqrt{2}$ units, since $\sqrt{2} \times \sqrt{2} = 2$.

d)



3 Review, page 192

3.



4. Answers may vary. For part a: I calculated the lengths of the sides and the slopes of the sides that look as if they may form a right angle.

$$\begin{aligned} AB &= \sqrt{4^2 + 3^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned} \qquad \begin{aligned} BC &= \sqrt{3^2 + 4^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Selected Solutions — Chapter 3

So, AB = BC

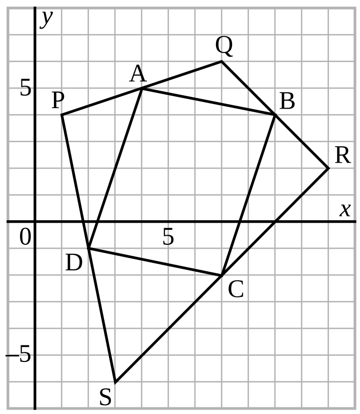
Slope of AB $= -\frac{4}{3}$ Slope of BC $= \frac{3}{4}$

Since the slopes are negative reciprocals, $\angle ABC = 90^\circ$.

$\triangle ABC$ is a right isosceles triangle.

(I did not need to calculate the length of the third side, because I knew it could not be an equilateral triangle, since it contains a right angle.)

9. Draw a diagram



From the diagram:

The midpoint of PQ is A, with coordinates (4, 5).

The midpoint of QR is B, with coordinates (9, 4).

The midpoint of RS is C, with coordinates (7, –2).

The midpoint of PS is D, with coordinates (2, –1).

From the diagram:

The slope of AB is $-\frac{1}{5}$.

The slope of BC is $\frac{6}{2} = 3$.

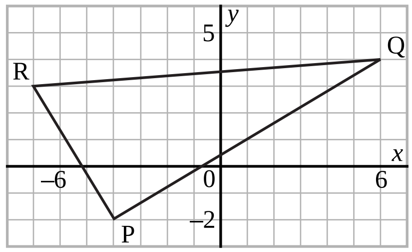
The slope of CD is $-\frac{1}{5}$.

The slope of DA is $\frac{6}{2} = 3$.

Since the opposite sides of quadrilateral ABCD are parallel, the quadrilateral is a parallelogram.

Selected Solutions — Chapter 3

12. I plotted the points on a grid, then drew the triangle.



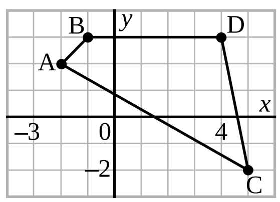
From the diagram, I counted squares to calculate the slopes of PQ and PR.

The slope of PQ is $\frac{6}{10} = \frac{3}{5}$.

The slope of RP is $-\frac{5}{3}$.

Since the slopes are negative reciprocals, the line segments RP and PQ are perpendicular.
Hence, $\angle RPQ = 90^\circ$.

19.



PQ

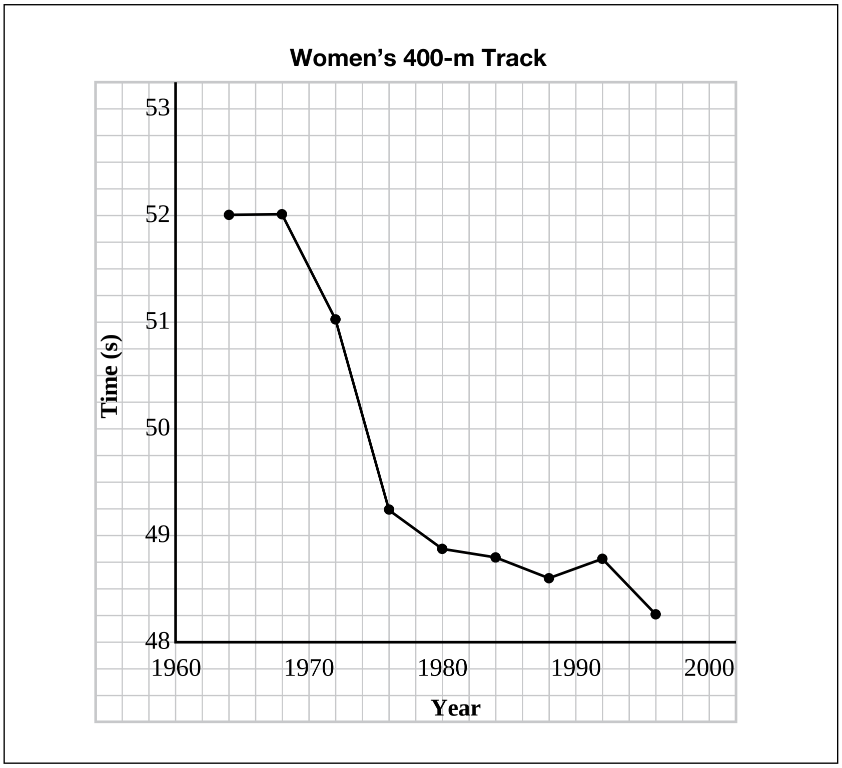
By inspection, the figure is not a rectangle, as $AB \neq CD$ and $BD \neq AC$. The diagram is not a parallelogram because AB is not parallel to CD and BD is not parallel to AC.

Selected Solutions — Chapter 3

3 Cumulative Review, page 194

9. a)

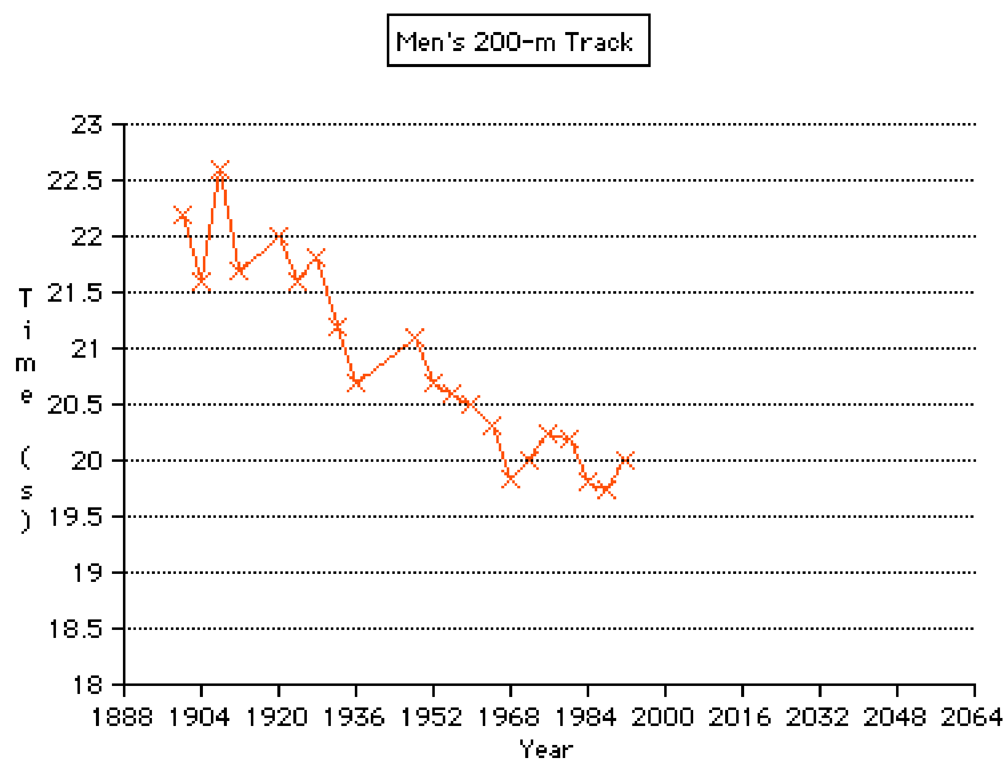
	A	B	C	D
1	1964	52	Betty Cuthbert	
2	1968	52.03	Colette Besson	
3	1972	51.08	Monica Zehrt	
4	1976	49.29	Irena Szewinska (Kirszestein)	
5	1980	48.88	Marita Koch	
6	1984	48.83	Valerie Brisco-Hooks	
7	1988	48.65	Olga Bryzgina	
8	1992	48.83	Marie-José Pérec	
9	1996	48.25	Marie-José Pérec	
10				



c) It is not possible to estimate the fastest possible time — it depends on the physiology of the human body.

e)

	A	B	C	D	E
1	1990	22.2	John Walter Tewksburry		
2	1904	21.6	Archie Hahn		
3	1908	22.6	Robert Kerr		
4	1912	21.7	Ralph Graig		
5	1920	22	Allen Woodring		
6	1924	21.6	Jackson Scholz		
7	1928	21.8	Percy williams		
8	1932	21.2	Tomas "Eddie" Tolan		
9	1936	20.7	James "Jesse' Owens		
10	1948	21.1	Melvin Patton		
11	1952	20.7	Andrew Stanfield		
12	1956	20.6	Bobby Joe Morrow		
13	1960	20.5	Livio Berruti		
14	1964	20.3	Henry Carr		
15	1968	19.83	Tommie Smith		
16	1972	20	Valery Borzov		
17	1976	20.23	Donald Quarrie		
18	1980	20.19	Pietro Mennea		
19	1984	19.8	F. Carlton Lewis		
20	1988	19.75	Joseph DeLoach		
21	1992	20.01	Michael Marsh		
22	1996	19.32	Michael Johnson		
23					



Selected Solutions — Chapter 3

10.