ADDISON-WESLEY Mathematics 10

Selected Solutions — Chapter 2

2.1 Exercises, page 72

9. b) The truncated number will be less than or equal to the rounded number. If the number is rounded up, the truncated number will be less than the rounded number. If the number is rounded down, the truncated number will be equal to the rounded number.

c) If the number was rounded down, this is the same as truncated. If the number was rounded up then, when I square it, the result will be greater than the number for which it is the approximate square root. For example, $\sqrt{3} \doteq 1.732\ 050\ 808$.
I round this to 6 decimal places and get 1.732 051.

The square of that number is 3.000 000 667.
The result is greater than 3.

10. a) Each square root is 10 times the preceding square root because the number under each radical sign is 100 times the number under the preceding radical sign, and $\sqrt{100} = 10$.

b) Each cube root is 10 times the preceding cube root because the number under each radical sign is 1000 times the number under the preceding radical sign, and $\sqrt[3]{1000} = 10$.

c) Each fourth root is 10 times the preceding fourth root because the number under each radical sign is 10 000 times the number under the preceding radical sign, and $\sqrt[4]{10\ 000} = 10$.

12. Answers may vary.
For part a: I know that $64 = 4^3$. Therefore, $\sqrt[3]{64} = 4$.

16. Answers may vary. For part d: To write –5 as a cube root, I first cube –5 to get $(-5)^3 = -125$. This is the number that goes under the cube root sign because $\sqrt[3]{-125} = -5$.

20. Answers may vary. For part e: I worked from the inside out. The square root of 64 is 8, so the exercise becomes $\sqrt[3]{8}$, which I know is 2, since $2^3 = 8$.

24. a) The increase in the area of ice, in square miles, is
$7\ 334\ 000 - 1\ 500\ 000 = 5\ 834\ 000$.

$$\begin{aligned} 6 \text{ months} &= \text{half a year} \\ &= 182.5 \text{ days} \end{aligned}$$

Convert 182.5 days to minutes by multiplying by 24 hours in a day and 60 minutes in an hour.

$$\begin{aligned} 182.5 \text{ days} &= 182.5 \times 24 \times 60 \text{ min} \\ &= 262\ 800 \text{ min} \end{aligned}$$

To find the rate at which ice is growing, divide the area by the time:

$$\frac{5\ 834\ 000 \text{ square miles}}{262\ 800 \text{ min}} \doteq 22.2 \text{ square miles/min}$$

Selected Solutions — Chapter 2

Modelling the Growth of Antarctic Sea Ice

It is assumed that the ice grows at a constant rate, and has a constant thickness of 1 m.
The model in part b uses a square to help you visualize the size of the additional ice per minute.
The model in part c uses a cube to help you visualize the size of the additional ice per minute.
Antarctic ice does not actually grow in squares and cubes, nor does it grow at a constant rate, nor will it be 1 m thick at all places.

Investigate, page 76

1. **b)** The square root of 3

2. **b)** The cube root of 3

3. $x^{\frac{1}{2}}$; the square root of x
$x^{\frac{1}{3}}$; the cube root of x

5. The reciprocal of the square root of x

2.2 Exercises, page 80

5. Answers may vary.
For part b: $4^{\frac{2}{5}}$ — the denominator is the root, and the numerator is the power to which the base, or the root, is raised.
$4^{\frac{2}{5}} = \left(\sqrt[5]{4}\right)^2$ or $\sqrt[5]{4^2} = \sqrt[5]{16}$

9. Answers may vary.
For part a: an exponent of $\frac{1}{2}$ is the same as a square root. To write 3 as a square root, first square it to get 9, then $\sqrt{9} = 3$, or $9^{\frac{1}{2}} = 3$.

11. Answers may vary.
For part d: I wrote $32^{-0.4}$ as $32^{-\frac{4}{10}}$.
The exponent simplifies to $32^{-\frac{2}{5}}$.
To change to a positive exponent, I write the reciprocal of the number: $\frac{1}{32^{\frac{2}{5}}}$
I then take the 5th root of 32 to get 2, and square the result to get 4.
The answer is $\frac{1}{4}$.

20. Answers may vary.
For part e: $\left(\sqrt{x^3}\right)\left(\sqrt[6]{x^2}\right)$, I wrote each term with a rational exponent: $(x^3)^{\frac{1}{2}}(x^2)^{\frac{1}{6}}$.
To raise a power to a power, I multiplied the exponents:
$(x^{\frac{3}{2}})(x^{\frac{2}{6}})$. The bases are the same, So I add the exponents:

Selected Solutions — Chapter 2

$x^{\frac{3}{2}} + \frac{2}{6} = x^{\frac{9}{6}} + \frac{2}{6} = x^{\frac{11}{6}}$

Then I wrote this as a radical $\sqrt[6]{6x^{11}}$.

21. **a)** Each row is a geometric sequence, with first term 1 and common ratio $\sqrt{2}$. In each column, the numbers are equal.

23. **a)** $10^{1.3} = 10^1 \times 10^{0.3}$
$= 10 \times 1.995$
$= 19.95$

b) $10^{2.3} = 10^2 \times 10^{0.3}$
$= 100 \times 1.995$
$= 199.5$

c) $10^{3.3} = 10^3 \times 10^{0.3}$
$= 1000 \times 1.995$
$= 1995$

d) $10^{4.3} = 10^4 \times 10^{0.3}$
$= 10\,000 \times 1.995$
$= 19\,950$

e) $10^{-0.7} = 10^{0.3-1}$
$= 10^{0.3} \div 10^1$
$= 1.995 \div 10$
$= 0.1995$

f) $10^{-1.7} = 10^{0.3-2}$
$= 10^{0.3} \div 10^2$
$= 1.995 \div 100$
$= 0.019\,95$

g) $10^{-2.7} = 10^{0.3-3}$
$= 10^{0.3} \div 10^3$
$= 1.995 \div 1000$
$= 0.001\,995$

h) $10^{-3.7} = 10^{0.3-4}$
$= 10^{0.3} \div 10^4$
$= 1.995 \div 10\,000$
$= 0.000\,199\,5$

Mathematical Modelling: Population Growth and Deforestation, page 84

1. **a)** A geometric sequence is more reasonable for population growth, since parents having children, and the children having children, and so on, is geometric.

Let r be the common ratio.

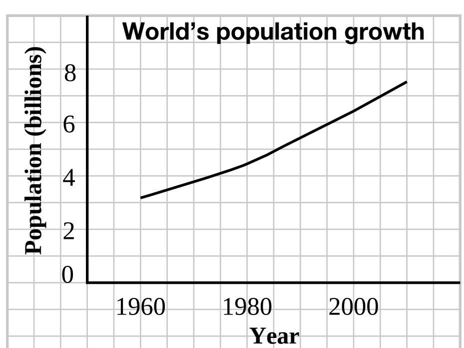
Then the 1st term is 3.2 and the 4th term is 5.3.

So, $3.2 \times r^3 = 5.3$
$r^3 = 1.656\,25$
$r = \sqrt[3]{1.656\,25}$
$\doteq 1.1831$

Use r to complete the table as shown in the student text.

b) 7.5 billion

c)



Selected Solutions — Chapter 2

2. a) Arithmetic; the amount of rain forests lost each year is about the same.

b) Let d be the common difference.

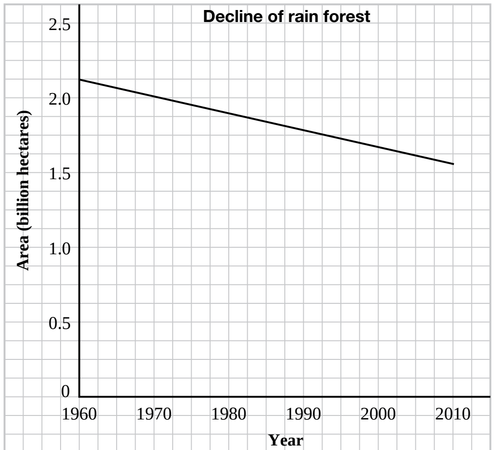
Then the 1st term is 2.10 and the 4th term is 1.75.

So, $2.10 + 3d = 1.75$

$$3d = -0.35$$
$$d = -0.11\overline{6}$$

Use d to complete the table as shown in the student text.

c)



3. No; the prediction is 7.5 billion. There is no effect on the model.

4. Answers may vary. Larger populations need more land to survive, so more forests must be cleared.

5. a) Total volume of wood products is

$(5.3 \times 10^9) \times 0.10\ \text{m}^3 = 5.3 \times 10^8\ \text{m}^3$

b) Anticipated total volume of wood products is

$(7.5 \times 10^9) \times 0.15\ \text{m}^3 = 1.125 \times 10^9\ \text{m}^3$

c) There will be a greater demand for the wood as it becomes more rare.

6. From exercise 1:

Let the population in 1960, which is 3.2 billion, be the 1st term of a geometric sequence.

Let the population in 1970, which is 3.8 billion, be the 11th term of the geometric sequence.

Let the common ratio be r.

Then $3.2r^{10} = 3.8$

$$r^{10} = 1.1875$$
$$r = \sqrt[10]{1.1875}$$
$$\doteq 1.017\ 33$$

This represents a growth of 0.017 33, or about 1.7%.

From exercise 2:

Let the rain forest area in 1960, which is 2.10 billion hectares, be the 1st term of an arithmetic sequence.

Selected Solutions — Chapter 2

Let the rain forest area in 1970, which is 1.98 billion hectares, be the 11th term of the arithmetic sequence.

Let the common difference be d.

Then $2.10 + 10d = 1.98$

$$10d = -0.12$$

$$d = -0.012$$

The rain forests decrease by about 0.012 billion hectares, or about 12 million hectares per year.

In 1970, the decrease as a percent is $\frac{0.012}{2.10} \times 100\% \doteq 0.6\%$.

In 2010, the decrease as a percent is $\frac{0.012}{1.63} \times 100\% = 0.7\%$.

The average decrease is halfway between these: about 0.65%.

The annual rate of population growth is 1.7%.

The annual rate of destruction of the rain forests is 0.65%.

7. In 1990, there were 1.75 billion hectares of rain forests. Use the lower estimate of 16 million hectares (0.016 billion hectares) destroyed annually.

Twenty years later, in 2010, there will be $[1.75 - 20(0.016)]$ billion hectares remaining, or 1.43 billion hectares.

Use the higher estimate of 20 million hectares (0.020 billion hectares) destroyed annually.

Twenty years later, in 2010, there will be $[1.75 - 20(0.020)]$ billion hectares remaining, or 1.35 billion hectares.

These predictions are 90 million hectares and 170 million hectares, respectively, lower than the previous prediction of 1.52 billion hectares for 2010.

8. Answers may vary. The loss of a species environment could lead to the death of that species, which will affect other species that depend on that species.

2.3 Exercises, page 90

Modelling the Brain Mass of a Mammal

This depends on how much of the brain is actually used. Smaller mammals' brains have a greater mass in relation to body mass. This may also have an effect on intelligence.

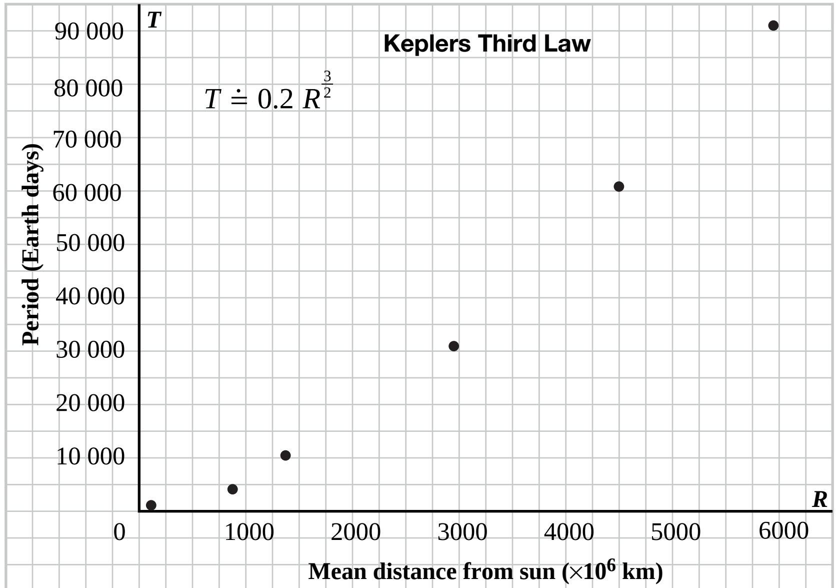
The saying "an elephant never forgets" might have some justification, depending on how much of the elephant's brain is used.

4. b) The brain mass quadruples every time the body mass is multiplied by 8. The percent of body mass halves every time the body mass is multiplied by 8.

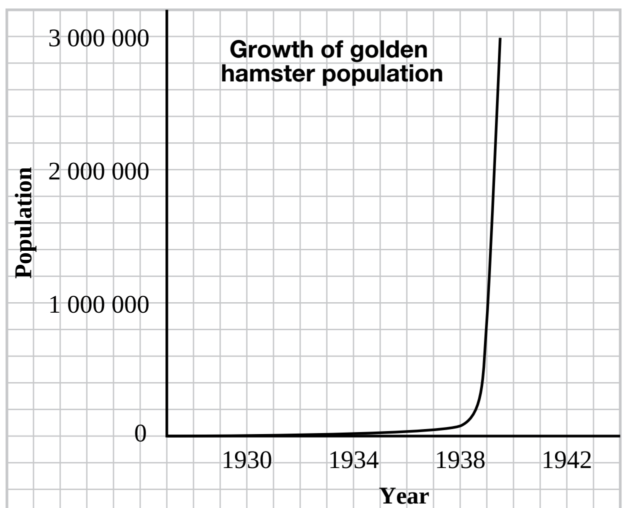
5. d) No, because the number of heart beats per breath is independent of m, the mass.

Selected Solutions — Chapter 2

8. b)



9. b)



11. a) Average lifetime in minutes $= \dfrac{\text{number of beats in a lifetime}}{\text{number of beats per minute}}$

$$= \frac{1.5 \times 10^9}{241m^{-0.25}}$$

$$= \frac{1.5 \times 10^9}{241} m^{0.25}$$

$$\doteq 6.224 \times 10^6 m^{0.25}$$

There are $365 \times 24 \times 60 = 525\,600$ minutes in one year.

Average lifetime in years $\doteq \dfrac{6.224 \times 10^6 m^{0.25}}{525\,600}$

$$\doteq 11.84m^{0.25}$$

b) Use the formula from part a.

Average lifetime in years $\doteq 11.84m^{0.25}$

For the elephant, substitute $m = 6400$

Average lifetime of elephant $\doteq 11.84(6400)^{0.25}$ years

$\doteq 105.9$ years

The elephant lives about 106 years.

For the cat, substitute $m = 6.4$

$$\begin{aligned}\text{Average lifetime of cat} &\doteq 11.84(6.4)^{0.25} \text{ years}\\ &\doteq 18.8 \text{ years}\end{aligned}$$

The cat lives about 19 years.

For the shrew, substitute $m = 0.0064$

$$\begin{aligned}\text{Average lifetime of shrew} &\doteq 11.84(0.0064)^{0.25} \text{ years}\\ &\doteq 3.3 \text{ years}\end{aligned}$$

The shrew lives about 3.3 years.

12. Answers will vary, depending on the current year. If the population continued to grow at the rate it did between 1930 and 1940, in 1997 it would be

$13 \times (230\ 769)^{\frac{67}{10}} \doteq 1.1 \times 10^{37}$.

The model gives an unreasonable number for the hamster population in 1997. The hamster population cannot continue to increase as it did between 1930 and 1940. The number 1.1×10^{37} means that there could be about 10^{27} hamsters for every person in the world. We could compare the mass of the hamsters with the mass of Earth, which is about 6×10^{27} g. We can make statements such as this: If there were 10^{37} hamsters, and they were so light that one billion of them had a mass of only 1 g, all the hamsters would still have a total mass of almost double the mass of Earth.

13. a) i) At 200 km, $T = 1.66 \times 10^{-0.4}(6370 + 200)^{1.5}$

$\doteq 88$ min

$\doteq 1.5$ h

ii) At 600 km, $T = 1.66 \times 10^{-0.4}(6370 + 600)^{1.5}$

$\doteq 97$ min

$\doteq 1.6$ h

iii) At 35 848.52 km, $T = 1.66 \times 10^{-0.4}(6370 + 35\ 848.52)^{1.5}$

$\doteq 1440$ min

$\doteq 24$ h

b) It takes 24 h to complete one revolution, which is how long it takes for Earth to rotate. The satellite will stay over one area, so that it can receive and transmit to one area on the surface. To a person on Earth, the satellite appears to be stationary.

Linking Ideas: Mathematics and Science
Bird Eggs, page 94

5. f) From exercise 2a, $\frac{\text{egg mass}}{\text{bird mass}}$ is $0.277m^{-0.23}$. As m increases, this fraction decreases because we are dividing by mass. From exercise 3a, $\frac{\text{shell mass}}{\text{egg mass}}$ is $0.0482e^{0.132}$. From equation (1), m increases, e increases, so this fraction increases.

Selected Solutions — Chapter 2

2.4 Exercises, page 99

8. Answers may vary. For part a: The hypotenuse is 5, so the square of the hypotenuse is 25. I needed to find two numbers that add to make 25. I chose 20 and 5. Each of these numbers is the square of a leg. So, the legs have length $\sqrt{20}$ cm and $\sqrt{5}$ cm.
To check, $(\sqrt{20})^2 + (\sqrt{5})^2 = 20 + 5 = 25$
$\sqrt{25} = 5$

9. a) Use indirect proof. Assume that $\sqrt{3}$ is a rational number. Then there are natural numbers m and n such that $\sqrt{3} = \frac{m}{n}$, where m and n are in lowest terms.
Square each side to obtain: $(\sqrt{3})^2 = \left(\frac{m}{n}\right)^2$

$$3 = \frac{m^2}{n^2}$$
$$3n^2 = m^2$$

Since the left side of this equation is divisible by 3, the right side is divisible by 3. Hence, m must be divisible by 3. Substitute $3p$ for m.

$$3n^2 = (3p)^2$$
$$3n^2 = 9p^2$$
$$n^2 = 3p^2$$

Since the right side of this equation is divisible by 3, the left side is divisible by 3. Hence, n must be divisible by 3. That is, both m and n are divisible by 3. This means that the fraction $\frac{m}{n}$ is not in lowest terms, although we assumed in Step 1 that it is in lowest terms. This contradicts the assumption in Step 1 that $\sqrt{3}$ can be written as a fraction in lowest terms.
The assumption in Step 1 that $\sqrt{3}$ is a rational number is incorrect. Hence, $\sqrt{3}$ is not a rational number.

b) Use indirect proof. Assume that $\sqrt{5}$ is a rational number. Then there are natural numbers m and n such that $\sqrt{5} = \frac{m}{n}$, where m and n are in lowest terms.
Square each side to obtain: $(\sqrt{5})^2 = \left(\frac{m}{n}\right)^2$

$$5 = \frac{m^2}{n^2}$$
$$5n^2 = m^2$$

Since the left side of this equation is divisible by 5, the right side is divisible by 5. Hence, m must be divisible by 5. Substitute $5p$ for m.

$$5n^2 = (5p)^2$$
$$5n^2 = 25p^2$$
$$n^2 = 5p^2$$

Since the right side of this equation is divisible by 5, the left side is divisible by 5. Hence, n must be divisible by 5. That is, both m and n are divisible by 5. This means that the fraction $\frac{m}{n}$ is not in lowest terms, although we assumed in Step 1 that it is in lowest terms. This contradicts the assumption in Step 1 that $\sqrt{5}$ can be written as a fraction in lowest terms.

Selected Solutions — Chapter 2

The assumption in Step 1 that $\sqrt{5}$ is a rational number is incorrect. Hence, $\sqrt{5}$ is not a rational number.

11. a) No, because $\sqrt{4}$ is a rational number.

b) Use indirect proof. Assume that $\sqrt{4}$ is a rational number. Then there are natural numbers m and n such that $\sqrt{4} = \frac{m}{n}$, where m and n are in lowest terms.

Square each side to obtain: $(\sqrt{4})^2 = \left(\frac{m}{n}\right)^2$

$$4 = \frac{m^2}{n^2}$$

$$4n^2 = m^2$$

Since the left side of this equation is even, the right side is even. Hence, m must be an even number. Substitute $2p$ for m.

$$4n^2 = (2p)^2$$
$$4n^2 = 4p^2$$
$$n^2 = p^2$$

This is where the proof breaks down. The argument in Step 4 no longer applies. We cannot say that n is even, so we cannot obtain the contradiction that $\frac{m}{n}$ is not in lowest terms.

Note that instead of getting a contradiction, we get something that is consistent with what was written previously. In Step 3, we get $n^2 = p^2$, which means that $n = p$ (since everything is positive, we don't have to worry about signs). But we substituted $2p$ for m, so we also know that $m = 2p$. From the last two equations, we see that $m = 2n$. But this is just another way of writing what we had at the beginning when we assume that there are natural numbers m and n such that $\sqrt{4} = \frac{m}{n}$, since this equation can be written as $m = 2n$.

If we try to use indirect proof to prove something that is false, we do not get a contradiction. We just get something that we would have known to begin with.

12. a) Let the side of the square be s, and the radius of the circle be r.

Area of square = area of circle

$$s^2 = \pi r^2$$
$$\frac{s^2}{r^2} = \pi$$
$$\frac{s}{r} = \sqrt{\pi}\text{, or } s : r = \sqrt{\pi} : 1$$

b) Use the Pythagorean Theorem to calculate the length of a diagonal.

$$(\text{Diagonal})^2 = s^2 + s^2$$
$$= 2s^2$$
$$\text{Diagonal} = \sqrt{2}s$$
$$\frac{\text{Diagonal}}{\text{Radius}} = \frac{\sqrt{2}s}{r}$$
$$= \sqrt{2}\frac{s}{r}$$

Substitute $\sqrt{\pi}$ for $\frac{s}{r}$.

$$\frac{\text{Diagonal}}{\text{Radius}} = (\sqrt{2})(\sqrt{\pi}) = \sqrt{2\pi}$$

Or diagonal : radius $= \sqrt{2\pi} : 1$

2.5 Exercises, page 104

2. a) The number is rational because it is a terminating decimal.

b) The number is irrational because 434 is not a perfect square.

3. b) No. If you continue calculating, you find the 9s do not repeat any more.

6. Answers may vary.
For part a: $\sqrt{21}$ is irrational because 21 is not a perfect square.

10. a) Yes, all integers can be written in the form $\frac{a}{b}$, where a and b are integers. For example: $-1 = \frac{-1}{1}$

b) Yes, natural numbers are positive integers.

c) No, 0 is the only exception. Zero is a whole number but not a natural number.

d) Yes, –5 is a rational number and an integer.

e) No, a number cannot be both rational and irrational. Rational and irrational numbers do not overlap.

12.

Rational numbers	Irrational numbers
0.35 $\frac{4}{3}$ –32.141414 $\sqrt{\frac{1}{4}}$	$\sqrt{7}$ -2π $\frac{\pi}{2}$
Integers ⊃ Whole numbers ⊃ Natural numbers: 23 10^6 $\sqrt{9}+\sqrt{16}$	

13. Only the square root of a perfect square can be a rational number. An irrational number cannot be a perfect square.

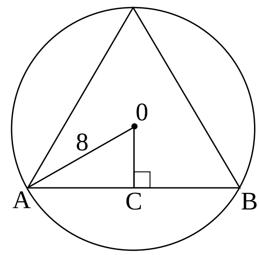
2.6 Exercises, page 110

4. Answers may vary. For part c: The hypotenuse is 10, so the shorter leg is one-half of 10, which is 5. The longer leg is $\sqrt{3}$ times the shorter leg, which is $5\sqrt{3}$. I used my calculator to write the mixed radical as a decimal, to 2 decimal places: 8.66

10. Answers may vary. For part d: The longer leg is $\frac{\sqrt{3}}{4}$ cm. The longer leg is $\sqrt{3}$ times the shorter leg. I can write $\frac{\sqrt{3}}{4}$ as $\sqrt{3} \times \frac{1}{4}$, so the shorter leg is $\frac{1}{4}$ cm, or 0.25 cm.
The hypotenuse is twice the shorter leg, which is 2×0.25 cm, or 0.5 cm.

Selected Solutions — Chapter 2

14. I drew a diagram.

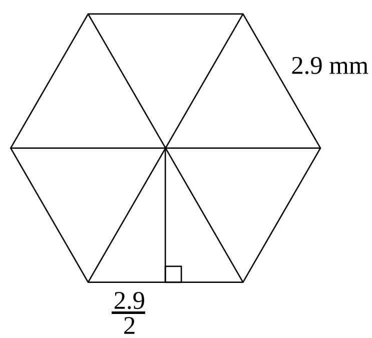


I drew the radius to A, one vertex of the triangle.
I labelled the radius 8.
I dropped the perpendicular from the centre O to the side AB of the triangle, at C.
Then ΔAOC is a 30-60-90 triangle, with $\angle A = 30^\circ$.
OC is the shorter leg, so it's 4.
AC is the longer leg, so it's $4\sqrt{3}$.
AC is one-half the base of the equilateral triangle.
The height of the equilateral triangle is $4 + 8 = 12$.

$$\begin{aligned}\text{The area of the equilateral triangle} &= \tfrac{1}{2}\text{base} \times \text{height}\\ &= 4\sqrt{3} \times 12\\ &= 48\sqrt{3}\end{aligned}$$

The area is $48\sqrt{3}$ cm^2, or 83.1 cm^2.

20.



A hexagon consists of 6 equilateral triangles. Each equilateral triangle has side length 2.9 mm.
The height of the triangle is $\frac{2.9}{2}\sqrt{3} = 1.45\sqrt{3}$.

$$\begin{aligned}A &= 6\left(\tfrac{1}{2}bh\right)\\ &= 3bh\\ &= (3 \times 2.9 \times 1.45\sqrt{3})\text{ mm}^2\\ &\doteq 21.85\text{ mm}^2\end{aligned}$$

The area of the honeycomb is
$(43.8 \times 20.6)\text{ cm}^2 = 902.28\text{ cm}^2 = 90\,228\text{ mm}^2$
There are about $90\,228 \div 21.85 \doteq 4129$ cells on one side of the honeycomb, and 8258 on both sides.

Selected Solutions — Chapter 2

Modelling the Storage of Honey in a Comb

The area of the comb is 902.28 cm^2. The volume, V, contained by the cells is equal to the area times the height times 2 (for 2 sides of the comb).

$$\begin{aligned} V &= 902.28 \times 1.1 \times 2 \\ &= 1985.016 \end{aligned}$$

The volume is about 1985 cm^3.

90% of 1985 is about 1787.

95% of 1985 is about 1886.

Between 1787 cm^3 to 1886 cm^3 of the honeycomb is filled with honey.

If the wall is 0.1 mm thick, the side length of each cell is reduced to 2.7 mm. This reduces the area of each cell.

$$\begin{aligned} A &= 3bh \\ &= 3(2.7)(1.35\sqrt{3}) \\ &\doteq 18.94 \end{aligned}$$

The area of each cell is about 18.94 mm^2.

There are still 4129 cells on each side of the honeycomb.

The total area of cells on one side of the honeycomb is

$$\begin{aligned} 4129 \times 18.94\ mm^2 &\doteq 78\ 203\ mm^2 \\ &\doteq 782\ cm^2 \end{aligned}$$

$$\begin{aligned} V &= 782 \times 1.1 \times 2 \\ &= 1720.4 \end{aligned}$$

The volume contained by the cells is 1720 cm^3.

90% of 1720 is about 1548.

95% of 1720 is about 1634.

Between 1548 cm^3 to 1634 cm^3 of the honeycomb is filled with honey.

Problem Solving: Squaring the Circle, page 115

1. **a)** $$\begin{aligned} A &= \pi r^2 \\ &= \pi(2.5)^2 \\ &= 6.25\pi \\ &\doteq 19.6 \end{aligned}$$

The area of the circle is 19.6 cm^2.

b) $$\begin{aligned} A &= s^2 \\ 6.25\pi &= s^2 \\ s &= \sqrt{6.25\pi} \\ &\doteq 4.4 \end{aligned}$$

The side length of the square is 4.4 cm.

2. **a)** $A_{square} = A_{circle} = \pi r^2$

The area of the square is πr^2 cm^2.

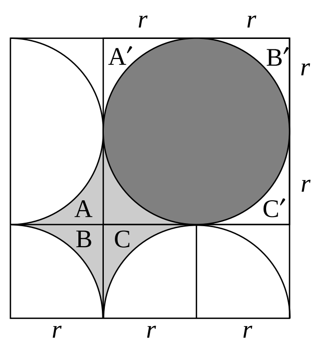
b) Let s represent the side length of the square.

Selected Solutions — Chapter 2

$$A_{\text{square}} = A_{\text{circle}}$$
$$s^2 = \pi r^2$$
$$s = \sqrt{\pi}r$$

The side length of the square is $\sqrt{\pi}r$ cm.

3. a)



From the diagram, the side length of the square is $3r$ cm. Thus, the area of the square is $9r^2$ cm^2.

b) The shaded areas A, B, and C are equal to the areas A′, B′, and C′. These fill the square with side length $2r$ cm. The total shaded area is $4r^2$ cm^2.

4. b) Each figure has the same shape as the shaded figure in exercise 3. The centres are 1 cm apart, so the radius of each circle is 0.5 cm. From exercise 3, the total shaded area is $4r^2$ cm^2. So, the shaded area is $4(0.5)^2$ cm^2 = 1 cm^2.

5. Let d represent the diameter of the circle, and s the side length of the square.

$$s = \frac{8}{9}d$$

$$A_{\text{circle}} = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$$

$$A_{\text{square}} = s^2 = \frac{64}{81}d^2$$

$$\frac{\text{Area of square}}{\text{Area of circle}} = \frac{\frac{64}{81}d^2}{\frac{\pi d^2}{4}} = \frac{64}{81}d^2 \times \frac{4}{\pi d^2} \doteq 1.006$$

The area of the square is very close to, but greater than, the area of the circle.

Investigate, page 116

1. Yes; $\sqrt{4}$ is 2, $\sqrt{9}$ is 3, so $\sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$

$4 \times 9 = 36$, and $\sqrt{36}$ is 6

So, $\sqrt{4} \times \sqrt{9} = \sqrt{4 \times 9}$

Selected Solutions — Chapter 2

2.7 Exercises, page 119

4. Determine factors of the number in which one factor is a perfect square. Find the square root of this number. Write the other factor as a radical. For example, $\sqrt{12}$. We know $4 \times 3 = 12$, where 4 is a perfect square. $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$

6. Answers may vary. For part i: to write $3\sqrt{4}$ as an entire radical, I squared 3 to get 9, then wrote 3 as $\sqrt{9}$. I then multiplied $\sqrt{9}$ and $\sqrt{4}$ to get $\sqrt{36}$.

8. **a)** Since the dots are vertices of squares, ΔAEB and ΔADC are similar, with ∠A common to both triangles, and ∠E = ∠D = 90°.

Similar triangles have corresponding sides in the same ratio.

$$\frac{AD}{AE} = \frac{AC}{AB}$$
$$\frac{2}{1} = \frac{AC}{AB}$$
$$AC = 2AB$$

Use the Pythagorean Theorem in ΔABE.

$$AB^2 = AE^2 + BE^2$$
$$= 1^2 + 1^2$$
$$= 2$$
$$AB = \sqrt{2}$$

Use the Pythagorean Theorem in ΔACD.

$$AC^2 = AD^2 + CD^2$$
$$= 2^2 + 2^2$$
$$= 8$$
$$AC = \sqrt{8}$$

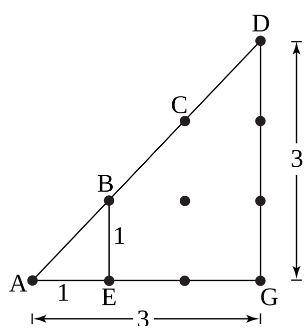
We know that $AC = 2AB$.

So, $AC = 2\sqrt{2}$.

We know that $AC = \sqrt{8}$.

Hence, $\sqrt{8} = 2\sqrt{2}$.

b)



ΔABE and ΔADG are similar, with ∠A common and ∠E = ∠G = 90°.

Similar triangles have corresponding sides in the same ratio.

Selected Solutions — Chapter 2

$$\frac{DG}{BE} = \frac{DA}{BA}$$
$$\frac{3}{1} = \frac{DA}{BA}$$
$$DA = 3BA$$

Use the Pythagorean Theorem in ΔADG.

$$AD^2 = AG^2 + DG^2$$
$$= 3^2 + 3^2$$
$$= 18$$
$$AD = \sqrt{18}$$

From part a, we know that $AB = \sqrt{2}$.

We know that $DA = 3BA$.

So, $DA = 3\sqrt{2}$.

We know that $DA = \sqrt{18}$.

Hence, $\sqrt{18} = 3\sqrt{2}$.

9. a) The 3rd number is 10 times the 1st. The 4th number is 10 times the 2nd. Each number is $\sqrt{10}$ times the preceding number.

b) The 3rd number is 10 times the 1st. The 4th number is 10 times the 2nd. Each number is $\sqrt{10}$ times the preceding number.

c) The 3rd number is twice the 1st. The 4th number is twice the 2nd. Each number is $\sqrt{2}$ times the preceding number.

13. Answers may vary. For part c: to calculate $-7\sqrt{\frac{6}{35}} \times 2\sqrt{\frac{5}{9}}$, I multiplied -7 by 2 to get -14. Then I multiplied the radicands $\sqrt{\frac{6 \times 5}{35 \times 9}}$, but first I divided the numerator and denominator by common factors of 5 and 3 to get $\sqrt{\frac{2 \times 1}{7 \times 3}}$, or $\sqrt{\frac{2}{21}}$. The product is $-14\sqrt{\frac{2}{21}}$.

17. The answers are probably different, but one person is not necessarily wrong since there are often many ways to write a number as a product of mixed radicals.

18. a) Each term is multiplied by 2, or $\sqrt{4}$ to get the next term. So, the sequence is geometric with first term $\sqrt{3}$ and common ratio 2, or $\sqrt{4}$.

b) Each term is multiplied by $\sqrt{2}$ to get the next term. So, the sequence is geometric with first term $\sqrt{43}$, and common ratio $\sqrt{2}$.

c) Each term is multiplied by $\sqrt{3}$ to get the next term. So, the sequence is geometric with first term $\sqrt{2}$ and common ratio $\sqrt{3}$.

d) Each term is multiplied by $\sqrt{10}$ to get the next term. So, the sequence is geometric with first term $\sqrt{5}$ and common ratio $\sqrt{10}$.

Selected Solutions — Chapter 2

20. Answers may vary. For part a: I wrote each radical as an entire radical.

$7\sqrt{2} = \sqrt{49} \times \sqrt{2} = \sqrt{98}$

$3\sqrt{7} = \sqrt{9} \times \sqrt{7} = \sqrt{63}$

$2\sqrt{15} = \sqrt{4} \times \sqrt{15} = \sqrt{60}$

$4\sqrt{6} = \sqrt{16} \times \sqrt{6} = \sqrt{96}$

I then ordered the entire radicals:

$\sqrt{60}, \sqrt{63}, \sqrt{96}, \sqrt{98}$

Then I rewrote each radical as a mixed radical:

$2\sqrt{15}, 3\sqrt{7}, 4\sqrt{6}, 7\sqrt{2}$

22. Write the radicals as entire radicals, then order the entire radicals, beginning with the largest, then rewrite the entire radicals as mixed radicals.

24. b) Each side length is $\sqrt{2}$ times the preceding side length. Each area is twice the preceding area.

25. $\sqrt{3} \doteq 1.7321$

a) $\sqrt{300} = \sqrt{100} \times \sqrt{3} \doteq 10 \times 1.7321 = 17.321$

$\sqrt{30\,000} = \sqrt{10\,000} \times \sqrt{3} \doteq 100 \times 1.7321 = 173.21$

$\sqrt{3\,000\,000} = \sqrt{1\,000\,000} \times \sqrt{3} \doteq 1000 \times 1.7321 = 1732.1$

b) $\sqrt{0.03} = \sqrt{0.01} \times \sqrt{3} \doteq 0.1 \times 1.7321 = 0.17321$

$\sqrt{0.0003} = \sqrt{0.0001} \times \sqrt{3} \doteq 0.01 \times 1.7321 = 0.017\,321$

c) $\sqrt{12} = \sqrt{4} \times \sqrt{3} \doteq 2 \times 1.7321 = 3.4642$

$\sqrt{27} = \sqrt{9} \times \sqrt{3} \doteq 3 \times 1.7321 = 5.1963$

$\sqrt{48} = \sqrt{16} \times \sqrt{3} \doteq 4 \times 1.7321 = 6.9284$

$\sqrt{75} = \sqrt{25} \times \sqrt{3} \doteq 5 \times 1.7321 = 8.6605$

d) $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \doteq \frac{1.7321}{2} = 0.866\,05$

$\sqrt{\frac{1}{3}} = \sqrt{\frac{3}{9}} = \frac{\sqrt{3}}{3} \doteq \frac{1.7321}{3} = 0.577\,3\overline{6}$

26. The rectangle comprises one square and six half squares; that is, 4 squares in all.

By symmetry, the area of WXYZ is one-quarter the area of the rectangle, or one-half the area of one-half the rectangle.

The area of one-half the rectangle is $\sqrt{6} \times \sqrt{6} = 6$.

So, the area of WXYZ $= \frac{1}{2}$ of $6 = 3$.

The area of WXYZ is 3 square units.

Selected Solutions — Chapter 2

2.8 Exercises, page 127

4. a) By inspection, each term is multiplied by $\frac{1}{\sqrt{4}}$, or $\frac{1}{2}$ to get the next term. So the sequence is geometric, with first term $\frac{1}{\sqrt{5}}$ and common ratio $\frac{1}{2}$.

b) By inspection, each term is multiplied by $\frac{1}{\sqrt{9}}$, or $\frac{1}{3}$ to get the next term. So, the sequence is geometric with first term $\frac{2}{\sqrt{2}}$ and common ratio $\frac{1}{3}$.

5. Answers may vary. For part b:

The sequence is $\frac{2}{\sqrt{2}}, \frac{2}{\sqrt{18}}, \frac{2}{\sqrt{162}}$.

For the sequence to be geometric, each term has to be multiplied by the common ratio to get the next term.

The numerators are the same, so they have been multiplied by 1.

I looked at the first two denominators: $\sqrt{2}$ and $\sqrt{18}$. To get $\sqrt{18}$, I multiply $\sqrt{2}$ by $\sqrt{9}$.

To check, I multiply $\sqrt{18}$ by $\sqrt{9}$ to get $\sqrt{162}$, the denominator of the 3rd term.

So, the common ratio is $\frac{1}{\sqrt{9}}$, or $\frac{1}{3}$.

The fourth term is $\frac{2}{\sqrt{162}} \times \frac{1}{\sqrt{9}} = \frac{2}{\sqrt{1458}}$

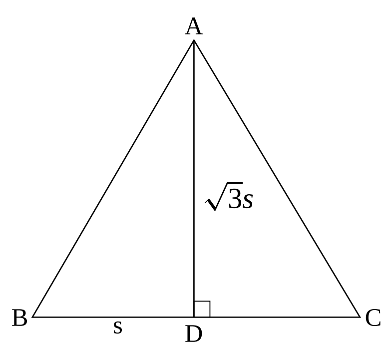
The fifth term is $\frac{2}{\sqrt{1458}} \times \frac{1}{\sqrt{9}} = \frac{2}{\sqrt{13\,122}}$

7. Since there are many possible quotients of mixed radicals to get each mixed radical in this exercise, it is likely that two people would get different answers for each radical, but both people could be correct.

8. Answers may vary. For part d: The radical is $2\sqrt{10}$. I multiplied $2\sqrt{10}$ by $\frac{4\sqrt{3}}{4\sqrt{3}}$. This doesn't change the radical.

Then the quotient is $\frac{2\sqrt{10} \times 4\sqrt{3}}{4\sqrt{3}} = \frac{8\sqrt{30}}{4\sqrt{3}}$.

12.



Draw equilateral ΔABC. Drop the perpendicular from A to BC at D.

Selected Solutions — Chapter 2

Then BC is the base and AD is the height.

ΔADC is a 30-60-90 triangle.

Let BD be s centimetres. Then AD is $\sqrt{3}s$ centimetres.

The area of ΔABC is $\frac{1}{2}$(BC)(AD).

$$30 = s\left(\sqrt{3}s\right)$$
$$30 = s^2\sqrt{3}$$
$$s^2 = \frac{30}{\sqrt{3}}$$
$$s = \sqrt{\frac{30}{\sqrt{3}}}$$
$$\doteq 4.1618$$

Then AD is $\sqrt{3}(4.1618)$, or approximately 7.2084.
And BC is 2(4.1618), or approximately 8.3236.

The base of the triangle is 8.32 cm and the height is 7.21 cm.

14. No, for example:

$\frac{\sqrt{2}}{\sqrt{2}} = 1$; $\frac{\sqrt{27}}{\sqrt{3}} = 3$

15. Answers may vary.

a) Yes. Find two numbers whose product is a perfect square.
For example:
$\sqrt{27} \times \sqrt{3} = \sqrt{81} = 9$
$\frac{\sqrt{27}}{\sqrt{3}} = 3$

b) Yes. Find two numbers whose product is not a perfect square.
For example:
$\sqrt{3} \times \sqrt{2} = \sqrt{6}$
$\frac{\sqrt{3}}{\sqrt{2}}$
Both results are irrational.

c) No. If $\sqrt{a} \times \sqrt{b}$ is rational, then $ab = n^2$, and
$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b} = \frac{\sqrt{n^2}}{b} = \frac{n}{b}$, which is rational.
If $\frac{\sqrt{a}}{\sqrt{b}}$ is rational, then $ab = n^2$, and $\sqrt{a} \times \sqrt{b} = \sqrt{ab} = \sqrt{n^2} = n$,
which is rational.

Investigate, page 129

1. No, $\sqrt{9}$ is 3, $\sqrt{16}$ is 4, so $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$
$\sqrt{25} = 5$; so, $\sqrt{9} + \sqrt{16} \neq \sqrt{25}$

2-9 Exercises, page 132

4. Answers may vary. For part a: $\sqrt{40} + \sqrt{90}$

I considered $\sqrt{40}$ and looked for 2 factors of 40, one of which is a perfect square: $40 = 4 \times 10$

I considered $\sqrt{90}$ and did the same: $90 = 9 \times 10$

I then wrote each radical as a product of radicals.

$\sqrt{40} + \sqrt{90} = \sqrt{4} \times \sqrt{10} + \sqrt{9} \times \sqrt{10}$

I simplified the radicals: $2\sqrt{10} + 3\sqrt{10}$

I combined the two like radicals: $5\sqrt{10}$

5. Using the Pythagorean Theorem, the hypotenuse is $\sqrt{10}$. The sum of the lengths of two sides of a triangle must be greater than the length of the third side. Therefore, $\sqrt{2} + \sqrt{8} > \sqrt{10}$.

7. Answers may vary. For part b: $5\sqrt{32} - 2\sqrt{8}$.

For part i, I used my calculator. I keyed in:
32 [2nd] [x^2] [×] 5 [–] 8 [2nd] [x^2] [×] 2 [=]
to get 22.627 417.
I rounded this number to 3 decimal places: 22.627

For part ii, I wrote each radical as a product of two radicals, so that in each product one number was a perfect square.

$5\sqrt{32} - 2\sqrt{8} = 5 \times \sqrt{16} \times \sqrt{2} - 2 \times \sqrt{4} \times \sqrt{2}$

I simplified each product.

$$= 5 \times 4\sqrt{2} - 2 \times 2\sqrt{2}$$
$$= 20\sqrt{2} - 4\sqrt{2}$$

Then I subtracted the radicals: $16\sqrt{2}$

For part iii, I used my calculator and keyed in
2 [2nd] [x^2] [×] 16 [=] to get 22.627 417.
I rounded this number to 3 decimal places: 22.627

11. Answers may vary. For exercise 10c: $8\sqrt{7}$

I first wrote the radical as the difference of two like radicals.

$8\sqrt{7} = 12\sqrt{7} - 4\sqrt{7}$

Then I wrote the whole number part of the first mixed radical as a product: $12\sqrt{7} = 6 \times 2\sqrt{7}$

Then I wrote 2 as a radical: $12\sqrt{7} = 6 \times \sqrt{4} \times \sqrt{7}$

Then I multiplied the radicals: $12\sqrt{7} = 6\sqrt{28}$

Then $8\sqrt{7} = 6\sqrt{28} - 4\sqrt{7}$

17. Answers may vary. For part f: $7\sqrt{24} + 3\sqrt{28} + 9\sqrt{54} + 6\sqrt{175}$

I first wrote each radical as the product of two radicals, so one number under each radical sign was a perfect square.

$7 \times \sqrt{4} \times \sqrt{6} + 3 \times \sqrt{4} \times \sqrt{7} + 9 \times \sqrt{9} \times \sqrt{6} + 6 \times \sqrt{25} \times \sqrt{7}$

I then simplified the radicals and multiplied the whole numbers.

$14\sqrt{6} + 6\sqrt{7} + 27\sqrt{6} + 30\sqrt{7}$

Selected Solutions — Chapter 2

I then added like radicals: $14\sqrt{6} + 27\sqrt{6} = 41\sqrt{6}$

$6\sqrt{7} + 30\sqrt{7} = 36\sqrt{7}$

The result was $41\sqrt{6} + 36\sqrt{7}$.

20. No. Consider a negative value of x.

Let $x = -2$, then $x^2 = 4$, and $\sqrt{x^2} = \sqrt{4} = 2$

In general, when $x < 0$, $\sqrt{x^2} \neq x$

21. Each triangle has a longer leg of $2\sqrt{3}$, since these are 30-60-90 triangles. The short lengths of the perimeter P are therefore $2\sqrt{3} - 2$.

$$\begin{aligned} P &= 4(4) + 4(2\sqrt{3} - 2) \\ &= 16 + 8\sqrt{3} - 8 \\ &= 8 + 8\sqrt{3} \end{aligned}$$

The perimeter is $(8 + 8\sqrt{3})$ cm.

22. **a)** If $\angle BAC$ is 90°, then the Pythagorean Theorem is satisfied and $BC^2 = AB^2 + AC^2$.

Check to find out.
In ΔABD, use the Pythagorean Theorem to calculate AB.

$$\begin{aligned} AB^2 &= BD^2 + AD^2 \\ &= (4\sqrt{2})^2 + (5\sqrt{3})^2 \\ &= 32 + 75 \\ &= 107 \end{aligned}$$

In ΔADC,

$$\begin{aligned} DC &= 2\sqrt{50} - 4\sqrt{2} \\ &= 10\sqrt{2} - 4\sqrt{2} \\ &= 6\sqrt{2} \end{aligned}$$

Use the Pythagorean Theorem in ΔADC.

$$\begin{aligned} AC^2 &= DC^2 + DA^2 \\ &= (6\sqrt{2})^2 + (5\sqrt{3})^2 \\ &= 72 + 75 \\ &= 147 \end{aligned}$$

Then, $AB^2 + AC^2 = 107 + 147 = 254$

$BC^2 = (2\sqrt{50})^2 = 200$

Since $AB^2 + AC^2 \neq BC^2$, $\angle BAC \neq 90^\circ$

b) Construct P on AD and draw ΔBPC.

Use the Pythagorean Theorem in ΔBPC.

$BP^2 + PC^2 = BC^2 = 200$

Use the Pythagorean Theorem in ΔBPD.

$BP^2 = BD^2 + PD^2 = 32 + PD^2$ ①

Use the Pythagorean Theorem in ΔPCD.

$PC^2 = DC^2 + PD^2 = 72 + PD^2$ ②

Add equations ① and ②.

$BP^2 + PC^2 = 32 + PD^2 + 72 + PD^2$

But $BP^2 + PC^2 = BC^2 = 200$

$200 = 32 + PD^2 + 72 + PD^2$

$2PD^2 = 96$

$PD^2 = 48$

$PD = 4\sqrt{3}$

The length of PD is $4\sqrt{3}$ cm.

23. If $\sqrt{a} + \sqrt{b} = \sqrt{a + b}$,

Then $\left(\sqrt{a} + \sqrt{b}\right)^2 = \left(\sqrt{a + b}\right)^2$

But $\left(\sqrt{a} + \sqrt{b}\right)^2 = a + 2\sqrt{a}\sqrt{b} + b$

and $\left(\sqrt{a + b}\right)^2 = a + b$

These two expressions are equal only if $2\sqrt{a}\sqrt{b} = 0$; that is, if one of a or b is 0, and the other is a positive real number. For example, $a = 0$ and $b = 9$

2.10 Exercises, page 138

6. Answers may vary. For part f: $\left(\sqrt{6} - \sqrt{3}\right)\left(\sqrt{12} - \sqrt{6}\right)$

I used FOIL to multiply the binomials:

$\sqrt{6}\sqrt{12} - \sqrt{6}\sqrt{6} - \sqrt{3}\sqrt{12} + \sqrt{3}\sqrt{6}$

Then I multiplied the radicals.

$\sqrt{72} - 6 - \sqrt{36} + \sqrt{18}$

Then I wrote the entire radicals as mixed radicals.

$6\sqrt{2} - 6 - 6 + 3\sqrt{2}$

Then I combined like radicals and constants to get $9\sqrt{2} - 12$.

7. a) The whole numbers in the products are 5, 9, 15, 23, 33, 45, … .

The difference between any pair of terms is 2 more than the preceding difference.

The mixed radicals in the products are $3\sqrt{3}$, $5\sqrt{3}$, $7\sqrt{3}$, $9\sqrt{3}$, $11\sqrt{3}$, $13\sqrt{3}$, … .

This is an arithmetic sequence with first term $3\sqrt{3}$ and common difference $2\sqrt{3}$.

b) The whole numbers in the products are 3, 4, 5, 6, 7, 8, … .

This is an arithmetic sequence with first term 3, and common difference 1.

The mixed radicals in the products are

$2\sqrt{2}, 2\sqrt{3}, 2\sqrt{4}, 2\sqrt{5}, 2\sqrt{6}, 2\sqrt{7}$, … .

The number under each radical is 1 more than the number under the preceding radical.

Selected Solutions — Chapter 2

10. Answers may vary. For part c: $\left(\sqrt{8}+\sqrt{2}\right)^2$

I wrote the square as a product of two equal binomials:
$\left(\sqrt{8}+\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right)$

Then I used FOIL: $\left(\sqrt{8}\right)^2+\sqrt{8}\sqrt{2}+\sqrt{2}\sqrt{8}+\left(\sqrt{2}\right)^2$

Then I multiplied the radicals: $8+\sqrt{16}+\sqrt{16}+2$

Then I simplified the radicals: $8+4+4+2$

Then I combined like terms to get 18.

For part d: $\left(\sqrt{8}-\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right)$

I noticed that the binomials had the same terms as in part c, but one sign was different.

I used FOIL: $\sqrt{8}\sqrt{8}+\sqrt{8}\sqrt{2}-\sqrt{2}\sqrt{8}-\sqrt{2}\sqrt{2}$

I used the products from part c, with different signs where appropriate: $8+4-4-2$

Then I combined like terms to get 6.

12. a) When the quotients are written with rational denominators, the numerator of each quotient is the sum of two radicals; one radical is always $\sqrt{3}$, the other radical is such that the number under each root sign increases by 1. The denominators of the quotients increase by 1 each time.

b) Each quotient is the difference of a whole number and a radical. The whole numbers increase by 1 each time. The number under the root sign is the product of the whole number, and 1 less than the whole number.

16. Answers may vary. For part e: $\frac{5\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}$

I multiplied numerator and denominator by $2\sqrt{3}+\sqrt{2}$ to make the denominator a rational number:

$$\frac{5\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}\times\frac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}}$$

Then I multiplied the binomials in the denominator, and the binomials in the numerator:

$$\frac{10\sqrt{9}+5\sqrt{6}+2\sqrt{6}+\sqrt{4}}{4\sqrt{9}-\sqrt{4}}$$

Then I simplified the radicals and combined like radicals:

$$\frac{30+7\sqrt{6}+2}{12-2}=\frac{32+7\sqrt{6}}{10}$$

18. Answers may vary. For part h: $\frac{3\sqrt{2}}{\sqrt{12}}-\frac{5\sqrt{3}}{\sqrt{8}}$

I looked at each fraction to see if I could simplify it.

For the first fraction, I divided numerator and denominator by

$\sqrt{2} : \frac{3}{\sqrt{6}} - \frac{5\sqrt{3}}{\sqrt{8}}$

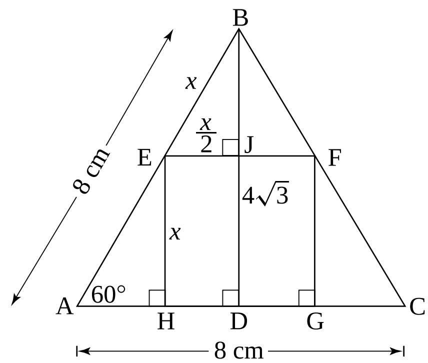
I then determined a common denominator ($\sqrt{24}$) and multiplied numerator and denominator by a radical that made the denominators $\sqrt{24}$:

$\frac{3\sqrt{4}}{\sqrt{24}} - \frac{5\sqrt{9}}{\sqrt{24}}$

I then simplified the numerators and wrote them over a common denominator: $\frac{6 - 15}{\sqrt{24}} = \frac{-9}{\sqrt{24}}$

I then simplified the radical in the denominator (but this is not necessary for the answer to be correct): $\frac{-9}{2\sqrt{6}}$

21. **a)** Copy the diagram. Label it as indicated, then drop the perpendicular from B to AC at D.



Let x represent the side length of the square.
Since ΔAEH is a 30-60-90 triangle,

$AH = \frac{x}{\sqrt{3}}$ and $AE = \frac{2x}{\sqrt{3}}$.

Then, $EJ = \frac{x}{2}$, since it is one-half the side length of the square.

ΔBEJ is also a 30-60-90 triangle.

Thus, $BE = 2EJ = x$

$$AB = AE + BE$$
$$8 = \frac{2x}{\sqrt{3}} + x$$
$$8\sqrt{3} = 2x + \sqrt{3}x$$
$$8\sqrt{3} = x(2 + \sqrt{3})$$
$$x = \frac{8\sqrt{3}}{2 + \sqrt{3}}$$

The side length of the square is $\frac{8\sqrt{3}}{2 + \sqrt{3}}$ cm, or 3.713 cm.

b) The area of the square is $\left(\frac{8\sqrt{3}}{2 + \sqrt{3}}\right)^2 = \frac{64(3)}{4 + 4\sqrt{3} + 3}$
$= \frac{192}{7 + 4\sqrt{3}}$

The area of the square is $\frac{192}{7 + 4\sqrt{3}}$ cm^2 or 13.785 cm^2.

Selected Solutions — Chapter 2

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3. Answers may vary. For part d: $\left(\frac{16}{0.0001}\right)^{-\frac{1}{4}}$

I wrote the reciprocal of the fraction and the exponent became positive: $\left(\frac{0.0001}{16}\right)^{\frac{1}{4}}$

Then I took the 4th root: $\sqrt[4]{\frac{0.0001}{16}} = \frac{0.1}{2}$

0.1 is $\frac{1}{10}$, so I multiplied $\frac{1}{2}$ by $\frac{1}{10}$: $\frac{1}{20}$

5. Assume the populations every 5 years are terms of a geometric sequence.

Let the common ratio be r.

Then the 5th term is $36r^4$.

But the 5th term is 250.

Hence, $36r^4 = 250$

$$r^4 = \frac{250}{36}$$

$$r = \sqrt[4]{\frac{250}{36}}$$

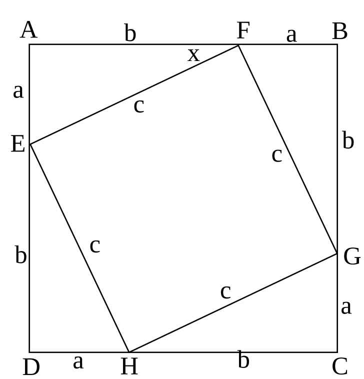
The second term is $36 \times \sqrt[4]{\frac{250}{36}}$.

I used my calculator and keyed in:
250 [÷] 36 [=] [2nd] [x^y] 4 [×] 36 [=]
to display 58.440 224 79.

I then multiplied this number by $\sqrt[4]{\frac{250}{36}}$ to get the next term: 94. 868 329 81

I continued in this way to get each term by multiplying the preceding term by the common ratio, then rounded each number to the nearest whole number.

7.



The small right triangles are all congruent by SAS, or SSS.
Let ∠AFE = x. Then ∠AEF = 90˚ – x.
Thus, ∠GFB = 90˚ – x (congruent triangles)

$$\angle AFE + \angle EFG + \angle GFB = 180^\circ \text{ (straight line)}$$
$$x + \angle EFG + 90^\circ - x = 180^\circ$$
$$\angle EFG = 90^\circ$$

The rest of the angles in the small quadrilateral can be proved to be 90˚ in a similar way. The sides of the small quadrilateral are all equal due to the congruent triangles. Thus, the small quadrilateral is a square. Its area is c^2.

The area of the small square can also be determined by subtracting the areas of the triangles from the area of the large square.

Area of triangle $= \frac{1}{2}ab$

Area of large square $= (a + b)^2$

$$\text{Area of small square} = (a+b)^2 - 4 \times \tfrac{1}{2}ab$$
$$= a^2 + 2ab + b^2 - 2ab$$
$$= a^2 + b^2$$

Thus, $c^2 = a^2 + b^2$; this is the Pythagorean Theorem.

8. **a)** $\sqrt{5}$ is irrational, since 5 is not a perfect square.

b) Use indirect proof. Assume that $\sqrt{5}$ is a rational number. Then there are natural numbers m and n such that $\sqrt{5} = \frac{m}{n}$, where m and n are in lowest terms.

Square each side to obtain: $(\sqrt{5})^2 = \left(\frac{m}{n}\right)^2$

$$5 = \frac{m^2}{n^2}$$
$$5n^2 = m^2$$

Since the left side of this equation is divisible by 5, the right side is divisible by 5. Hence, m must be divisible by 5.

Substitute $5p$ for m.

$$5n^2 = (5p)^2$$
$$5n^2 = 25p^2$$
$$n^2 = 5p^2$$

Since the right side of this equation is divisible by 5, the left side is divisible by 5. Hence, n must be divisible by 5. That is, m and n are both divisible by 5. This means that the fraction $\frac{m}{n}$ is not in lowest terms, although we assumed in Step 1 that it is in lowest terms. This contradicts the assumption in Step 1 that $\sqrt{5}$ can be written as a fraction in lowest terms.

The assumption in Step 1 that $\sqrt{5}$ is a rational number is incorrect. Hence, $\sqrt{5}$ is not a rational number.

12. Answers may vary. For part b: $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{5}}, \frac{\sqrt{3}}{5}, \ldots$

I check to see if the series is geometric.

Divide the 2nd term by the 1st term: $\dfrac{\frac{1}{\sqrt{5}}}{\frac{1}{\sqrt{3}}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{3}}{1} = \frac{\sqrt{3}}{\sqrt{5}}$

Divide the 3rd term by the 2nd term: $\dfrac{\frac{\sqrt{3}}{5}}{\frac{1}{\sqrt{5}}} = \dfrac{\sqrt{3}}{5} \times \dfrac{\sqrt{5}}{1} = \dfrac{\sqrt{3}}{\sqrt{5}}$

These two calculations for the common ratio are equal, so the sequence is geometric, with common ratio $\dfrac{\sqrt{3}}{\sqrt{5}}$.

The 4th term is $\dfrac{\sqrt{3}}{5} \times \dfrac{\sqrt{3}}{\sqrt{5}} = \dfrac{3}{5\sqrt{5}}$.

The fifth term is $\dfrac{3}{5\sqrt{5}} \times \dfrac{\sqrt{3}}{\sqrt{5}} = \dfrac{3\sqrt{3}}{25}$.

17. Answers may vary. For part d: $\dfrac{3\sqrt{2} - 5\sqrt{3}}{\sqrt{5} - \sqrt{3}}$

I multiplied the numerator and denominator by $\sqrt{5} + \sqrt{3}$, so the denominator becomes a rational number:

$$\frac{3\sqrt{2} - 5\sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$$

Then I multiplied the expressions in the numerator and in the denominator:

$$\frac{3\sqrt{10} + 3\sqrt{6} - 5\sqrt{15} - 5\sqrt{9}}{5 - 3}$$

Then I simplified the radicals where possible, and collected like terms:

$$\frac{3\sqrt{10} + 3\sqrt{6} - 5\sqrt{15} - 15}{2}$$

2 Cumulative Review, page 143

7. **e)** Each radius is double the preceding radius.