ADDISON-WESLEY
Mathematics 10

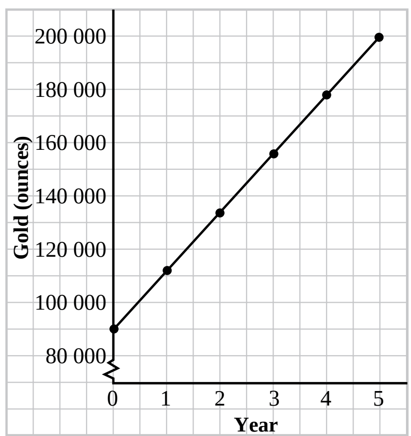
Selected Solutions — Chapter 1

1.1 Exercises, page 7

2. c) Answers may vary. The numbers in Row 3 are multiples of 3.
The numbers in Row 2 are 2 more than multiples of 3.
The numbers in Row 1 are 1 more than multiples of 3.
For example, 100 is $3 \times 33 + 1$, which is 1 more than a multiple of 3, so 100 is in Row 1.
For example, 1001 is $3 \times 333 + 2$, which is 2 more than a multiple of 3, so 1001 is in Row 2.

5. b) Answers may vary.
For part i: The common difference is $11 - 6 = 5$. Continue to add 5 to determine the 7th term.
6, 11, 16, 21, 26, 31, 36
The 7th term of the sequence is 36.

6. b)



9. b) Answers may vary.
For part i: From the two given terms, calculate the common difference: $12 - 7 = 5$. Subtract 5 from 7 to get the first term, 2. Add 5 to 12 to get the 4th term, 17. Add 5 to 17 to get the 5th term, 22.

12. Answers may vary.
If Player A goes first, then player B can always win.
Player A says a single-digit number.
Player B adds a single-digit number to make the sum 10.
Player A adds a single-digit number to make the sum greater than 10 and less than 20.
Player B adds a single-digit number to make the sum 20.
Player A adds a single-digit number to make the sum greater than 20 and less than 30.
Player B adds a single-digit number to make the sum 30.
Player A adds a single-digit number to make the sum greater than 30 and less than 40.
Player B adds a single-digit number to make the sum 40.
Player A adds a single-digit number to make the sum greater than 40 and less than 50.
Player B adds a single-digit number to make the sum 50.

Selected Solutions — Chapter 1

13. Answers may vary.

a) A non-leap year has 52 weeks and 1 day, so January 1 after a non-leap year occurs 1 day later in the week than the year before. A leap year has 52 weeks and 2 days, so January 1 after a leap year will occur 2 days later than January 1 in the leap year.

b) The pattern of skipped days in the first 5 rows is repeated in the next 5 rows, and in all following sets of 5 rows. Each set of 5 rows contains 28 years. Hence, the pattern of skipped days repeats every 28 years.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1984		1985	1986	1987	1988	
1989	1990	1991	1992		1993	1994
1995	1996		1997	1998	1999	2000
	2001	2002	2003	2004		2005
2006	2007	2008		2009	2010	2011
2012		2013	2014	2015	2016	
2017	2018	2019	2020		2021	2022
2023	2024		2025	2026	2027	2028
	2029	2030	2031	2032		2033
2034	2035	2036		2037	2038	2039

c) Only the first 5 rows of the table are needed to determine the day on which January 1 falls for any given year (hence, the following explanations ignore the 6th to 10th rows).

i) The 5th row shows that in 2010, January 1 will be a Friday.

ii) Subtract 28 from 2020 to obtain $2020 - 28 = 1992$. Since January 1 occurred on a Wednesday in 1992, it will occur on a Wednesday in 2020.

iii) $2040 - 28 = 2012$
$2012 - 28 = 1984$, which is in the table
Since January 1 occurred on a Sunday in 1984, it will occur on a Sunday in 2040.

iv) $2100 - 28 = 2072$
$2072 - 28 = 2044$
$2044 - 28 = 2016$
$2016 - 28 = 1988$, which is in the table
Since January 1 occurred on a Friday in 1988, it will occur on a Friday in 2100.

d) For years between 1984 and 2011, use the table.
For years between 2012 and 2100, subtract enough 28s from the given year so that the result is between 1984 and 2011. Use the table for that year.
For years between 1901 and 1983, add enough 28s to the given year so that the result is between 1984 and 2011. Use the table for that year.

1.2 Exercises, page 13

4. b) Answers may vary. For part i: The first term a is 5. I subtracted the first term, 5, from the second term, 8, to get the common

difference, 3. Then I substituted $a = 5$ and $d = 3$ in the formula for t_n to get $t_n = 5 + (n - 1)3$.
I expanded the product to get $t_n = 5 + 3n - 3$.
Then I simplified to get $t_n = 3n + 2$.

6. **b)** I assume that the sequence continues until the last row has the least number of bricks possible. There are 6 fewer bricks in each consecutive row. I divided 65 by 6 to get 10 remainder 5. This means I could subtract 6 ten times and have 5 left. This means there are 11 rows, and the last row has 5 bricks.

9. **b)** Answers may vary.
For part i: I used the formula for the general term $t_n = a + (n - 1)d$. The first term a is 5, the common difference d is $8 - 5 = 3$; and t_n is 41. I substituted these numbers into the formula to get $41 = 5 + (n - 1)3$. I expanded the product to get $41 = 5 + 3n - 3$. I simplified to get $39 = 3n$, then solved for n to get $n = 13$. So, 41 is the 13th term.

11. **d)** Answers may vary. The Winter and Summer Olympics are no longer held in the same year. They are now held two years apart. The last year that they were held together was in 1992. When they were split, the Summer Olympics were to be held every 4 years from 1992, and the Winter Olympics every 4 years from 1994.

12. **a)**

2	4	6	8	10	12
7	9	11	13	15	17
12	14	16	18	20	22
17	19	21	23	25	27
22	24	26	28	30	32
27	29	31	33	35	37

b) Answers may vary. The numbers in every row, column, and diagonal form an arithmetic sequence. For example, row 1 has first term 2, common difference 2; column 1 has first term 2, common difference 5; the diagonal from top left to bottom right has first term 2 and common difference 7.

c) Answers may vary. The sequences in column 1 and the diagonal (in part b) have the same first term.

d) Answers may vary. The general term for the sequence in column 1 is $t_n = a + (n - 1)d$, with $a = 2$ and $d = 5$. Substitute to get $t_n = 2 + (n - 1)5$, which simplifies to $t_n = 5n - 3$.
The general term for the sequence in the diagonal is $t_n = a + (n - 1)d$, with $a = 2$ and $d = 7$. Substitute to get $t_n = 2 + (n - 1)7$, which simplifies to $t_n = 7n - 5$.
If I substitute $n = 1$ in each general term, I get $t_1 = 2$, which is the relationship described in part c.

Selected Solutions — Chapter 1

13. a) The first term in each product forms the sequence 1, 3, 5, 7, … , which has general term $2n - 1$. The second term in each product forms the sequence 1, 4, 7, 10, … , which has general term $3n - 2$. The general term of the sequence is $t_n = (2n - 1)(3n - 2)$. The terms of the sequence are 1, 12, 35, 70, … . This is not an arithmetic sequence, as it has no common difference.

b) The first term in each product forms the sequence 2, 4, 6, 8, … , which has general term $2n$. The second term in each product forms the sequence 3, 6, 9, 12, … , which has general term $3n$. The general term of the sequence is $t_n = (2n)(3n) = 6n^2$. The terms of the sequence are 6, 24, 54, 96, … . This is not an arithmetic sequence, as it has no common difference.

c) The numerators form the sequence 1, 2, 3, 4, … , which has general term n. The denominators form the sequence 3, 5, 7, 9, … , which has general term $2n + 1$. The general term of the sequence is $t_n = \frac{n}{2n + 1}$. This is not an arithmetic sequence, as it has no common difference.

d) The first number in each numerator forms the sequence 1, 3, 5, 7, … , which has general term $2n - 1$. The second number in each numerator forms the sequence 3, 5, 7, 9, … , which has general term $2n + 1$. The first number in each denominator forms the sequence 2, 4, 6, 8, … , which has general term $2n$. The second number in each denominator forms the sequence 4, 6, 8, 10, … , which has general term $2n + 2$. The general term of the sequence is $t_n = \frac{(2n - 1)(2n + 1)}{2n(2n + 2)}$. The terms of the sequence are $\frac{3}{8}$, $\frac{15}{24}$, $\frac{35}{48}$, $\frac{63}{80}$, … . This is not an arithmetic sequence, as it has no common difference.

Linking Ideas: Mathematics and Science
Arithmetic Sequences in Astronomy, page 16

3. It could be Halley’s comet. Going back from 1531 and repeatedly subtracting gives 1069. With gravitational forces, the date could have been 1066. Or, the tapestry could have been completed several years after 1066, and included the comet for effect.

1.3 Exercises, page 20

3. b) Answers may vary. For part ii: The first term a is 5, the common difference is 6.5, and the number of terms n is 10.
I substituted these numbers in $t_n = a + (n - 1)d$ to find the 10th term.
$t_{10} = 5 + (10 - 1)6.5$, which is 63.5
Then I used the formula for the sum of n terms, $S_n = \left(\frac{a + t_n}{2}\right)n$, and substituted for the variables to get $S_{10} = \frac{(5 + 63.5)(10)}{2}$, or 342.5.
The sum of 10 terms is 342.5.

Selected Solutions — Chapter 1

6. Job A pays \$1260.
(month 1 + month 2 + month 3 = 400 + 420 + 440 = 1260)
Job B pays \$1530.
(week 1 + week 2 + week 3 + · · · + week 12
= 100 + 105 + 110 + · · · + 155 = 1530)
Job B pays more.

8. e) Answers may vary.
For part a, the annual raise is the common difference of an arithmetic sequence with 1st term 25 325 and 7th term 34 445.
To find the common difference, I subtracted the terms and divided by 6. $\frac{34\,445 - 25\,325}{6} = 1520$
The annual raise is \$1520.
For part b, the salary in the fourth year is the 4th term of the sequence; that is, the 1st year's salary plus 3 times the common difference: $25\,325 + 3 \times 1520 = 29\,885$
The salary in the 4th year is \$29 885.
For part c, the salary in the 4th year is \$29 855. I added \$1520 to this to get \$31 405, which is the 5th year's salary. The salary exceeds \$30 000 in the 5th year.
For part d, the total amount Raji earns is the sum of the arithmetic series with 1st term 25 325, 7th term 34 445, and common difference 1520. I substituted these numbers in the formula $S_n = \frac{(a + t_n)n}{2}$, to get $S_7 = \frac{(25\,325 + 34\,445)(7)}{2} = 209\,195$.
Raji earns \$209 195 in 7 years.

9. Assume the sequence continues until there is the least number of bricks greater than 0 in the last row, and the number is part of the sequence.

12. b) Answers may vary.
For part iii: I first calculated the number of terms. I used the formula for the general term, $t_n = a + (n - 1)d$, and substituted $t_n = 133$, $a = 3$, and $d = 2.5$, to get $133 = 3 + (n - 1)2.5$. This simplifies to $133 = 2.5n + 0.5$. I solved the equation to get $n = 53$.
Then I used the formula for the sum of n terms, $S_n = \frac{(a + t_n)}{2}n$, and substituted for the variables I know.
I got $S_n = \frac{(3 + 133)(53)}{2}$, which simplified to 3604.
The sum of the series is 3604.

13. a) Answers may vary. Each word has one more letter than the previous word. The size of the words increases like a snowball rolling down a hill.

15. The 6th term is the difference between the sum of 6 terms and the sum of 5 terms.

$$\begin{aligned} t_6 &= S_6 - S_5 \\ &= 123 - 85 \\ &= 38 \end{aligned}$$

Selected Solutions — Chapter 1

Use the formula for the sum of 6 terms to calculate the first term, a.

Substitute $S_6 = 123$, $t_6 = 38$, and $n = 6$ in $S_n = \frac{(a + t_n)n}{2}$.

$$123 = \frac{(a + 38)6}{2}$$
$$123 = (a + 38) \times 3$$
$$41 = a + 38$$
$$a = 3$$

Use the formula for t_n to calculate the common difference.

$t_n = a + (n - 1)d$

Substitute $t_6 = 38$, $a = 3$, and $n = 6$.

$$38 = 3 + (6 - 1)d$$
$$5d = 35$$
$$d = 7$$

The series has first term 3 and common difference 7. The first four terms are $3 + 10 + 17 + 24$.

16. **a)** To determine t_n, use the formula $t_n = a + (n - 1)d$.

Substitute $a = 3$ and $d = 4$.

$$t_n = 3 + (n - 1)4$$
$$= 4n - 1$$

To determine t_{20}, substitute $n = 20$.

$$t_{20} = 4(20) - 1$$
$$= 79$$

b) To determine S_n, use the formula for the sum of n terms,

$$S_n = \frac{(a + t_n)n}{2}$$

Substitute $a = 3$ and $t_n = 4n - 1$.

$$S_n = \frac{(3 + 4n - 1)(n)}{2}$$
$$= \frac{n(4n + 2)}{2}$$
$$= \frac{2n(2n + 1)}{2}$$
$$= n(2n + 1)$$

To determine S_{20}, substitute $n = 20$ in the formula for S_n.

$$S_n = n(2n + 1)$$
$$S_{20} = 20(2(20) + 1)$$
$$= 820$$

c) Write an inequality involving the general term.

$$4n - 1 < 500$$
$$4n < 501$$
$$n < 125.25$$

125 terms are less than 500.

d) Write an inequality involving the formula for the sum of n terms.

$$n(2n + 1) < 500$$

Use guess and check to solve this inequality.

Try $n = 10$, $n(2n + 1) = 10(21) = 210$ — too low

Selected Solutions — Chapter 1

Try $n = 20,\ n(2n + 1) = 20(41) = 820$ — too high

Try $n = 15,\ n(2n + 1) = 15(31) = 465$ — close

Try $n = 16,\ n(2n + 1) = 16(33) = 528$ — too high

The sum of 15 terms is less than 500.

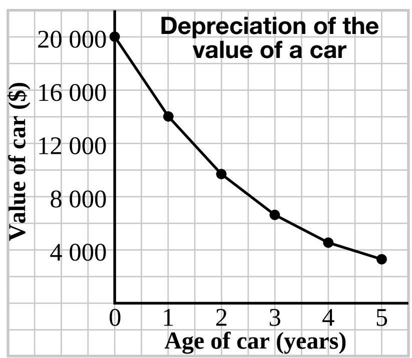
Investigate, page 23

2. b) In the second column, each number is double the number above it. In the third column, each number is one-half the number above it.

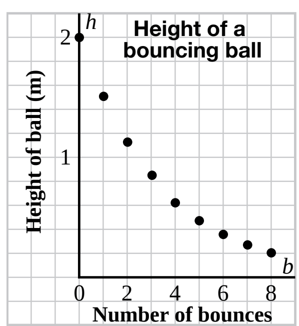
1.4 Exercises, page 27

3. b) Answers may vary.
For part i: The common ratio is $6 \div 3 = 2$. Multiply the third term and each successive term by 2 to determine the terms up to the 6th term.
3, 6, 12, 24, 48, 96
The 6th term of the sequence is 96.

4. b)



5. c)



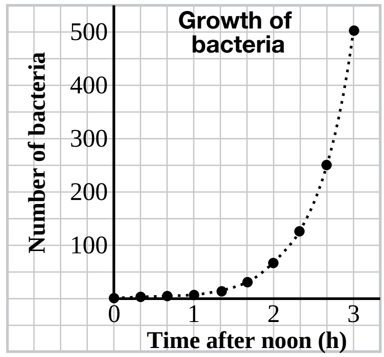
6. Explanations may vary.

a) The y-intercept would be greater, the graph would have a similar shape and be higher on the grid.

b) The graph would fall more sharply to the right.

Selected Solutions — Chapter 1

7. b)



Modelling the Growth of Bacteria

The body's disease-fighting mechanism would be activated and start slowing down the growth. The growth may continue until treatment is sought, but it may be slower than the doubling. The growth could not continue indefinitely because the bacteria might eventually kill their host.

9. c) Answers may vary. For part a, let r represent the common ratio.
To go from the 1st term to the 4th term, I multiply by r three times: $2, 2r, 2r^2, 2r^3, \ldots$
The 4th term in this series is equal to 54.
I write an equation.
$2r^3 = 54$
Divide each side by 2.
$r^3 = 27$
Take the cube root of each side.
$r = 3$
The geometric sequence is $2, 2 \times 3, 2 \times 3^2, 2 \times 3^3$, or 2, 6, 18, 54.

11. Assume GST and PST are each 7%.
The total tax is 14%.
To find the cost with tax, multiply the retail price by 1.14.
The retail price is x dollars.
The original cost is $1.14x$ dollars.
After one week, the price decreases by 10%.
Hence, the new price is 90% of the previous price.
The retail price in Week 1 is $(0.9)(1.14x)$ dollars, or $\$1.026x$.
Similarly, in Week 2, the retail price is $(0.9)^2(1.14x)$ dollars, or $\$0.9234x$.
In Week 3, the retail price is $(0.9)^3(1.14x)$ dollars, or about $\$0.8311x$.
In Week 4, the retail price is $(0.9)^4(1.14x)$ dollars, or about $\$0.7480x$.

12. a) Multiply 40 000 by 1.07 five times.
$40\ 000(1.07)^5 = 56\ 102$
Its value is $56 102.

Selected Solutions — Chapter 1

b) Multiply 40 000 by 1.07 ten times.
$40\ 000(1.07)^{10} = 78\ 686$
Its value is \$78 686.

c) Multiply 40 000 by 1.07 n times.
$40\ 000(1.07)^n$
Its value is $\$40\ 000(1.07)^n$.

13. a) Use a pattern to express t_n in terms of a, r, and n.

$$t_1 = a$$
$$t_2 = t_1 \times r = ar$$
$$t_3 = t_2 \times r = ar \times r = ar^2$$
$$t_4 = t_3 \times r = ar^2 \times r = ar^3$$
$$\vdots$$
$$t_n = t_{n-1} \times r = ar^{n-2} \times r = ar^{n-1}$$

b) The general term of an arithmetic sequence is $t_n = a + (n - 1)d$. In an arithmetic sequence, you start with the first term, and add $(n - 1)$ times the common difference. In a geometric sequence, you start with the first term, and multiply by the $(n - 1)$th power of the common ratio.

Mathematical Modelling: How Many Links Are Needed?, page 30

1. a) D = 40 000 × 20 = 800 000

b) E = 800 000 × 20 = 16 000 000

c) F = 16 000 000 × 20 = 320 000 000

d) G = 320 000 000 × 20 = 6 400 000 000

2.

Person	Number of links	Number of people
A	0	100
B	1	2 000
C	2	40 000
D	3	800 000
E	4	16 000 000
F	5	320 000 000
G	6	6 400 000 000

3. Yes, the answer was 6.4 billion.

4. Answers may vary.

a) Relatives, friends, neighbours, co-workers, people who play tennis, local MP, and so on

b) Yes. For example, my friend knows a professional tennis player. This tennis player’s manager knows someone from Monica Seles’ entourage. This member of Monica Seles’ entourage knows Monica. Therefore, I am separated from Monica Seles by 4 degrees of separation.

Selected Solutions — Chapter 1

5. Answers may vary.

a) Yes, it is a reasonable assumption. Or, no, it is not reasonable because I don't know 100 people.

b) For example, assume that you know 50 people, and each person knows 10 people who do not know any of the previous people. Then you would have to recalculate the number of people linked to you after each step. For 6 degrees of separation, this is $50 \times 10^6 = 50$ million. I would need 8 degrees of separation, $(50 \times 10^8 = 5$ billion) or even 9 degrees (50 billion) to ensure a link to anyone on Earth.

6. Answers may vary.

a) I need 1 link to link me with someone in school, because 1 link links me to 2000 people.

b) I need 4 links to link me with someone in my province, because 4 links link me to 16 million people.

c) I need 5 links to link me with someone in Canada, because 5 links link me to 320 million people.

1.5 Exercises, page 35

3. Answers may vary. For exercise 2e: To simplify $\left(-\frac{3}{5}\right)^{-2}$, I wrote the reciprocal of the fraction and changed the sign of the exponent to get $\left(-\frac{5}{3}\right)^{2}$. Then I squared the numerator and the denominator to get $\frac{25}{9}$.

8. Answers may vary. For exercise 6d: To simplify $(xy^3)^2$, I raised each variable to the power 3, by multiplying the exponents to get x^2y^6.

14. b) Answers may vary. For part i: To simplify $(x^3y^2)(x^2y^3)$, I multiplied terms with the same base by adding their exponents to get $x^{3+2}y^{2+3} = x^5y^5$. Then I substituted $x = -1$ and $y = 2$ to get $(-1)^5(2)^5 = -32$.

16. a)
$$\begin{aligned}\left(\frac{c^4}{d}\right)^{-2} \times \frac{c^8}{d} &= \left(\frac{d}{c^4}\right)^{2} \times \frac{c^8}{d}\\ &= \frac{d^2}{c^8} \times \frac{c^8}{d}\\ &= d\end{aligned}$$

b)
$$\begin{aligned}\left(\frac{-5x^3}{2y}\right)^{-2} \times \left(\frac{x}{y^{-4}}\right)^{2} &= \left(\frac{2y}{-5x^3}\right)^{2} \times \frac{x^2}{(y^{-4})^2}\\ &= \frac{2^2y^2}{(-5)^2(x^3)^2} \times \frac{x^2}{y^{-8}}\\ &= \frac{4y^2}{25x^6} \times \frac{x^2y^8}{1}\\ &= \frac{4y^{10}}{25x^4}\end{aligned}$$

Selected Solutions — Chapter 1

c) $\left(\frac{a^{-3}b}{c^2}\right)^{-4} \times \left(\frac{c^5}{a^4b^{-3}}\right)^{-1} = \left(\frac{c^2}{a^{-3}b}\right)^4 \times \left(\frac{a^4b^{-3}}{c^5}\right)$

$= \frac{c^8}{a^{-12}b^4} \times \frac{a^4b^{-3}}{c^5}$

$= a^{16}b^{-7}c^3$

Investigate, page 37

1. a) To calculate the hourly rate of pay, I divided the regular pay by the hours worked.

$\frac{\$288.00}{40\text{ h}} = \$7.20/\text{h}$

3. She gets 1.5 times her regular pay for the hours worked overtime; that is, she is paid $\$7.20 \times 1.5$ or \$10.80 per overtime hour.

1.6 Exercises, page 40

1. a) Multiply the price by 0.07.

b) Multiply the price by 0.107.

e) Answers may vary. For part c, add the two numbers in the GST column.

3. a) Multiply the number of wins by 2 and add the number of ties.

b) Anaheim and Phoenix would be tied. The rest of the standings would be the same.

c) Phoenix would move ahead of Anaheim to tie with Detroit. Anaheim, St. Louis, and Edmonton would be tied. Vancouver would move ahead of Chicago.

6. Answers may vary. In my province, $\text{GST} = \text{PST} = 7\%$.
To calculate cost price, I multiply selling price by 1.14.
So, to calculate the selling price, I divide the cost price by 1.14.
In this case, the cost price is \$20.00.

$\text{Selling price} = \frac{\$20.00}{1.14}$
$\doteq \$17.54$

Because I rounded, I check by multiplying, to ensure the cost price is below \$20.00.

$\$17.54 \times 1.14 = \$19.9956$ — this rounds to \$20.00.

7. b) Answers may vary. You cannot simply multiply Canada's data by 10. Other factors, such as how well a dealership is doing, can people afford to buy a car in that particular area, and so on, should be considered.

9. d) Steps may vary. Find the currency you need across the top of the table. Look down the column to the currency you have. Multiply this by the amount of the currency you need.

10. a) Let x dollars represent the cost price.
Let the discount be 20%. Assume taxes of 14%.
Apply the discount first.
Then the sale price is 80% of $x = 0.80x$.

Then the cost price is $1.14 \times 0.80x = 0.912x$.
Apply the taxes first.
Selling price is $1.14x$.
Then apply the discount.
Cost price is $(0.80)(1.14x) = 0.912x$.
The customer pays the same in each case.
For example:

Apply discount first: $\$15.99 \times 0.8 = \$12.79$

Then apply taxes: $\$12.79 \times 1.14 = \$14.58$

Apply taxes first: $\$15.99 \times 1.14 = \$18.23$

Then apply discount: $\$18.23 \times 0.8 = \$14.58$

b) Use the example in part a.
Apply the discount first.
The store receives 80% of original price = $\$0.80x$.
Apply the taxes first.
Government receives 14% of original price = $\$0.14x$.
Cost price is unchanged at $\$0.912x$.
Store receives the difference between cost price and the amount the government receives: $\$0.912x - \$0.14x = \$0.772x$.
The store receives more if the discount is calculated before taxes.

Problem Solving: How Many Kernels of Wheat?, page 44

1., 2.a), 6. a)

Square number	Kernels on square	Total kernels
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255
9	256	511
10	512	1023

b) The first column is an arithmetic sequence with first term 1, and common difference 1.
The second column is a geometric sequence with first term 1 and common ratio 2.

d) $2^{64-1} = 2^{63}$
$= 9.223\ 372 \times 10^{18}$

3. **a)** Find n for which $2n - 1$ is about 1 000 000 (the number of kernels in a bushel).

Selected Solutions — Chapter 1

Try some values of n.

$$n = 15,\ 2^{15-1} = 2^{14} = 16\ 384$$
$$n = 20,\ 2^{20-1} = 2^{19} = 524\ 288$$
$$n = 21,\ 2^{21-1} = 2^{20} = 1\ 048\ 576$$

One bushel of wheat would be needed on the 21st square.

b) $n = 22,\ 2^{22-1} = 2^{21} = 2\ 097\ 152$

$$n = 23,\ 2^{23-1} = 2^{22} = 4\ 194\ 304$$
$$n = 24,\ 2^{24-1} = 2^{23} = 8\ 388\ 608$$

A total of approximately 14.7 bushels

4. a) Canada's annual production is about 30 000 000 t. There are about 32 500 000 kernels in one tonne. There are about $30\ 000\ 000 \times 32\ 500\ 000 = 9.75 \times 10^{14}$ kernels in Canada's annual production.

Find n for which 2^{n-1} is about 9.75×10^{14}.

Try some values of n.

$$n = 50,\ 2^{50-1} = 2^{49} \doteq 5.63 \times 10^{14}$$
$$n = 51,\ 2^{51-1} = 2^{50} \doteq 1.13 \times 10^{15}$$

On the 51st square, Canada's annual wheat production would be needed.

b) $n = 52,\ 2^{52-1} = 2^{51} \doteq 2.25 \times 10^{15}$

$$n = 53,\ 2^{53-1} = 2^{52} \doteq 4.50 \times 10^{15}$$
$$n = 54,\ 2^{54-1} = 2^{53} \doteq 9.01 \times 10^{15}$$

This is a total of approximately 1.58×10^{16} t. The numbers of years' production is $\frac{1.58 \times 10^{16}}{9.75 \times 10^{14}}$, or about 16 years.

5. a) The world's production was 581 000 000 t. There are $30\ 000\ 000 \times 581\ 000\ 000 = 1.743 \times 10^{16}$ kernels in the world's production.

Find n for which 2^{n-1} is about 1.743×10^{16}.

Try some values of n.

$$n = 55,\ 2^{55-1} = 2^{54} \doteq 1.80 \times 10^{16}$$

The world's annual production of wheat would be needed on the 55th square.

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b) For the last square, $n = 64,\ 2^{64-1} = 2^{63}$
$\doteq 9.223 \times 10^{18}$

The number of years' production is $\frac{9.223 \times 10^{18}}{1.743 \times 10^{16}} \doteq 529$.
About 529 years would be needed for the last square.

6. b) Each number in the third column is 1 less than twice the corresponding number in the second column.

c) The number of kernels needed to fill a square is 1 more than the total so far.
For exercise 3b, the number of kernels needed to fill the 22nd square is 1 more than all the kernels needed to fill from the 1st to the 21st square. We can approximate this: the number needed to fill any square is equal to the total required so far.

7. a) There are 9.223×10^{18} kernels on the 64th square. The servant can count 10 kernels per second.
Number of seconds: $\frac{9.223 \times 10^{18}}{10} = 9.223 \times 10^{17}$
Number of minutes: $\frac{9.223 \times 10^{17}}{60} = 1.537 \times 10^{16}$
Number of hours: $\frac{1.537 \times 10^{16}}{60} = 2.562 \times 10^{14}$
Number of days: $\frac{2.562 \times 10^{14}}{24} = 1.068 \times 10^{13}$
Number of years: $\frac{1.068 \times 10^{13}}{365} = 2.926 \times 10^{10}$
It would take about 30 billion years.

b) There are $60 \times 60 \times 24 \times 365 = 3.1536 \times 10^{7}$ seconds in one year. The servant can count 10 kernels per second, or 3.1536×10^{8} kernels per year.
Find n such that 2^{n-1} is about 3.1536×10^{8}.
Try some values of n.
$n = 25,\ 2^{25-1} = 2^{24}$
$= 16\ 777\ 216$
$n = 29,\ 2^{29-1} = 2^{28}$
$= 268\ 435\ 456$
$n = 30,\ 2^{30-1} = 2^{29}$
$= 536\ 870\ 912$, or 5.36×10^{8}
The 30th square would take more than one year to fill.

8. There are 3.1536×10^{7} seconds in one year. The total number of kernels is $2^{64} - 1$, or 1.8447×10^{19}. Number of kernels per second is $\frac{1.8447 \times 10^{19}}{3.1536 \times 10^{7}} = 5.85 \times 10^{11}$
About 5.85×10^{11} kernels would have to be counted each second.

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1. e) The opening balance of a row is equal to the closing balance of the previous row.

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1.7 Exercises, page 50

3. e) Answers may vary.
For part a, subtract the opening balance in Year 1 from the closing balance in Year 4 to get $656.19.
For part b, the interest rate is 6%, which is 0.06 as a decimal. The opening balance is A dollars.
The interest earned is the opening balance multiplied by the interest rate.
Interest earned = 0.06A$
The closing balance is opening balance + interest
= $$A$ + 0.06A$
= 1.06A$
For part c, in Year 4, the opening balance is $2977.54.
The interest rate is 9%, or 0.09.
The interest earned is (0.09)($2977.54) = $267.98.
The closing balance is $2977.54 + $267.98 = $3245.52.
For part d, in Year 3, the opening balance is $2809.00.
The interest rate is 9%, or 0.09.
The closing balance is (1.09)($2809.00) = $3061.81.
In Year 4, the opening balance is $3061.81.
The interest rate is 0.09.
The closing balance is (1.09)($3061.81) = $3337.37.

6. I multiplied the opening balance for Year 3 by 0.0725 to get the interest earned. I added the interest earned, the annual investment of $500, and the opening balance to get the closing balance for Year 3.

Modelling the Growth of an Investment

If interest rates rise, then the investments will earn more.

If you thought the rates would go lower, then you would invest long-term. If you expected them to rise, then you would not invest long-term.

7. a) The interest rate in fraction form is $\frac{i}{100}$.

Year	Opening balance ($)	Interest rate (%)	Interest earned ($)	Closing balance ($)
1	A	i	$A\left(\frac{i}{100}\right)$	$A + A\left(\frac{i}{100}\right) = A\left(1+\frac{i}{100}\right)$
2	$A\left(1+\frac{i}{100}\right)$	i	$A\left(1+\frac{i}{100}\right)\left(\frac{i}{100}\right)$	$A\left(1+\frac{i}{100}\right) + A\left(1+\frac{i}{100}\right)\left(\frac{i}{100}\right) = A\left(1+\frac{i}{100}\right)^2$
3	$A\left(1+\frac{i}{100}\right)^2$	i	$A\left(1+\frac{i}{100}\right)^2\left(\frac{i}{100}\right)$	$A\left(1+\frac{i}{100}\right)^2 + A\left(1+\frac{i}{100}\right)^2\left(\frac{i}{100}\right) = A\left(1+\frac{i}{100}\right)^3$
4	$A\left(1+\frac{i}{100}\right)^3$	i	$A\left(1+\frac{i}{100}\right)^3\left(\frac{i}{100}\right)$	$A\left(1+\frac{i}{100}\right)^3 + A\left(1+\frac{i}{100}\right)^3\left(\frac{i}{100}\right) = A\left(1+\frac{i}{100}\right)^4$

b) Continue the pattern to obtain $A\left(1+\frac{i}{100}\right)^n$ as the value of the investment in dollars after n years.

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1. b) Cell D5: calculates the interest earned on the opening balance by multiplying the opening balance (cell B5) by the interest rate (cell C5)
Cell E5: calculates the closing balance by adding the interest earned (cell D5) to the opening balance (cell B5)
Cell A6: fills in the year by adding 1 to the previous year (cell A5)
Cell B6: the opening balance (cell B6) is equal to the closing balance of the previous year (cell E5)
Cell C6: the interest rate is the same for every year
Cell D6: copies the formula in cell D5, to calculate the interest earned
Cell E6: copies the formula in cell E5, to calculate the closing balance

d)

	A	B	C	D	E
1	Calculating the Value of Investment				
2					
3	Year	Opening balance ($)	Interest rate	Interest earned ($)	Closing balance ($)
4					
5	1	1000.00	7.00%	70.00	1070.00
6	2	1070.00	7.00%	74.90	1144.90
7	3	1144.90	7.00%	80.14	1225.04
8	4	1225.04	7.00%	85.75	1310.80
9	5	1310.80	7.00%	91.76	1402.55
10	6	1402.55	7.00%	98.18	1500.73
11	7	1500.73	7.00%	105.05	1605.78
12	8	1605.78	7.00%	112.40	1718.19
13	9	1718.19	7.00%	120.27	1838.46
14	10	1838.46	7.00%	128.69	1967.15

1.8 Exercises, page 58

15. a) Start with the closing balance of 500. For each day, starting with the 3rd day, work backward to find the starting balance by completing the inverse operations: multiply by 2 and subtract 50.

	Closing balance ($)	Starting balance ($)
Day 3	500	950
Day 2	950	1850
Day 1	1850	3650

Amira started with $3650.

b) Amira started with $3650, and finished with $500. The difference is $3150. But she also deposited $150, so she actually withdrew $\$3150 + \$150 = \$3300$.

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2. b) Answers may vary.

For part ii: The sequence is the same as in part i. The first term is 1 and the common ratio is 2. To go from the 3rd term, which is 4, to the 12th term, I multiply by r nine times.

$$\begin{aligned} t_{12} &= 4 \times 2^9 \\ &= 2048 \end{aligned}$$

3. a) t_{12} is not twice t_6 since the numbers are increasing geometrically; that is, every number is twice the preceding number. For example, t_6 is twice t_5.

b) $t_{14} = 2t_{13}$
$t_{13} = 2t_{12}$
In all geometric sequences, any term is the product of the common ratio and the preceding term.